

MPGN – AN APPROACH FOR DISCOVERING CLASS ASSOCIATION RULES*

Iliya Mitov

ABSTRACT. The article briefly presents some results achieved within the PhD project R1876 *Intelligent Systems' Memory Structuring Using Multi-dimensional Numbered Information Spaces*, successfully defended at Hasselt University. The main goal of this article is to show the possibilities of using multi-dimensional numbered information spaces in data mining processes on the example of the implementation of one associative classifier, called MPGN (Multi-layer Pyramidal Growing Networks).

1. Introduction. The knowledge discovery process consists of many steps and data mining is one of them [1]. Data Mining is the process of analyzing a large set of raw data in order to extract hidden information which can be

ACM Computing Classification System (1998): H.2.8, D.4.3.

Key words: Data mining, classification, associative classifiers, MPGN, multidimensional numbered information spaces, ArM 32.

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predicted [2]. The questions related to data mining present several aspects, such as: classification, estimation, clustering, associations, regularities, etc. From the whole data mining area we focus on classification algorithms and especially on associative classifiers, or so-called class-association rules (CAR) classifiers.

Associative classifiers appeared initially within the field of market basket analysis for discovering interesting rules from large data collections. As mentioned in [3], the five major advantages of associative classifiers can be highlighted:

- The training is very efficient regardless of the size of the training set;
- Training sets with high dimensionality can be handled with ease and no assumptions are made on dependence or independence of attributes;
- The classification is very fast;
- Classification based on association methods presents higher accuracy than traditional classification methods [4] [5] [6] [7];
- The classification model is a set of rules easily interpreted by human beings and can be edited [8].

On the other hand, the process of creating classification models inevitably touches upon the use of appropriate access methods. We focus on the memory organization called Multi-Dimensional Numbered Information Spaces which allows operating with context-free multidimensional data structures [9]. Our goal is to use such structures and operations in the implementation of one class association rule classifier MPGN in order to see the vitality of the idea of using context-free multidimensional data structures as a powerful tool for knowledge discovery.

The rest of the article is organised as follows: Section 2 makes a brief overview of the associative classifiers; Section 3 presents the algorithm of the proposed classifier MPGN; Section 4 draws attention to the program realization; Section 5 contains the results of the sensitivity analysis made with MPGN; and finally, in the conclusion, steps for future development are highlighted.

2. Associative classifiers. Generally the structure of CAR-classifiers consists of three major data mining steps: (1) Association rule mining; (2) Pruning (optional); and (3) Recognition.

The mining of association rules is a typical data mining task that works in an unsupervised manner. A major advantage of association rules is that they are theoretically capable of revealing all interesting relationships in a database.

But for practical applications the number of mined rules is usually too large to be exploited entirely. This is why the pruning phase is stringent in order to build accurate and compact classifiers. The fewer rules a classifier needs to approximate the target concept satisfactorily, the more human-interpretable is the result.

In 1998, CBA is introduced in [4], often considered to be the first associative classifier. During the last decade, various other associative classifiers were introduced, such as CMAR [5], ARC-AC and ARC-BC [10], CPAR [7], CorClass [11], ACRI [12], TFPC [13], HARMONY [14], MCAR [6], 2SARC1 and 2SARC2 [15], CACA [16], ARUBAS [17], etc.

2.1. Association Rule Mining. Several techniques for creating association rules are used, mainly based on:

- Apriori algorithm [18] (used by CBA, ARC-AC, ARC-BC, ACRI, ARUBAS): Apriori makes multiple rounds over the data with patterns of length for 1 to k . Each pass consists of 2 steps – first: creating candidate patterns with length k ; and second: remaining only patterns which support exceeds given support;
- FP-tree algorithm [19] (used by CMAR): FP-tree is an extended prefix-tree structure storing quantitative information about frequent patterns. Only frequent length-1 instances have nodes in the tree, and the tree nodes are arranged in such a way that more frequently occurring nodes will have better chances of sharing nodes than less frequently occurring ones;
- FOIL algorithm [20] (used by CPAR): FOIL is a sequential covering algorithm that learns first-order logic rules. It learns new rules one at a time, removing the positive examples covered by the latest rule before attempting to learn the next rule;
- Morishita & Sese Framework [21] (used by CorClass): It efficiently computes significant association rules according to common statistical measures such as chi-squared value or correlation coefficient.

Generating association rules can be made:

- from all training transactions together (applied in CBA, CMAR, ARC-AC);
- for transactions grouped by class label (as is made in ARC-BC).

2.2. Pruning. In order to reduce the produced association rules, pruning in parallel with (pre-pruning) or after (post-pruning) creating association rules

is performed. Different heuristics for pruning during rule generation are used, mainly based on minimum support, minimum confidence and different kinds of error pruning [22]. At the post-pruning phase, criteria such as data coverage (ACRI) or correlation between consequent and antecedent (CMAR) are also used.

During the pruning phase or at the classification stage, different ranking criteria for ordering the rules are used. The most common ranking mechanisms are based on the support, confidence and cardinality of the rules, but other techniques such as cosine measure and coverage measure (ACRI) also exist; we can mention amongst them:

- pruning by confidence: retain more general rules with higher accuracy: $|R_1| < |R_2|$ and $conf(R_1) < conf(R_2)$, then R_1 is pruned (used in ARC-AC, ARCBC);
- pruning by precedence: special kind of ordering using “precedence” (CBA and MCAR);
- correlation pruning: statistical measuring of the rules’ significance using weighted χ^2 (CMAR).

2.3. Recognition. Different approaches can be discerned in the recognition stage [17]:

- using a single rule (CBA);
- using a subset of rules (CPAR);
- using all rules (CMAR).

Different order-based combined measures for a subset or all rules can be applied also: select all matching rules; group rules per class value; order rules per class value according to some criterion; calculate a combined measure for the best Z rules; etc.

3. MPGN Algorithm. In [23] two associative classifiers are proposed: PGN and MPGN.

The algorithm of the PGN classifier is already presented in [24]. The PGN classifier has several advantages and experiments show very good benefits [23]. It possesses all advantages of CAR classifiers, such as creating a compact pattern set used in the recognition stage, easy interpretation of the results, and

very good accuracy for clear datasets. But one disadvantage is seen, connected with exponential growth of operations during the process of creating the pattern set.

In order to overcome this bottleneck, the MPGN algorithm is created. MPGN is an abbreviation of “Multi-layer Pyramidal Growing Networks of information spaces”. The main goal is to use a special kind of multi-layer memory structures called “pyramids”, which permits defining and realizing new opportunities.

As an example we will use the following dataset for showing different steps of the MPGN algorithm (the separation of two classes in primary dataset is made only for increasing readability):

Class 1		Class 2	
R1:	(1 5,5,5,5)	R9:	(2 4,2,3,1)
R2:	(1 5,3,5,4)	R10:	(2 3,2,3,1)
R3:	(1 5,4,5,3)	R11:	(2 2,1,2,1)
R4:	(1 1,1,1,1)	R12:	(2 4,1,2,1)
R5:	(1 4,1,3,1)	R13:	(2 2,2,2,1)
R6:	(1 1,2,1,1)		
R7:	(1 1,2,2,2)		
R8:	(1 4,2,4,1)		

3.1. Training process. The training process in MPGN consists of:

- preprocessing step;
- generalization step;
- pruning step.

3.1.1. Preprocessing step. MPGN deals with instances and patterns separately for each class. This allows the MPGN algorithm to be implemented for use on parallel computers which could be particularly helpful within the current trend of using cloud services and grid infrastructures.

The preprocessing steps aim to convert the learning set in a standard form for further steps. It consists of:

- discretization of numerical attributes [25];
- numbering the values of attributes and converting nominal vectors to numerical vectors.

3.1.2. Generalization step. The process of generalization is a chain of creating the patterns of the upper layer as intersection between patterns from the lower layer until new patterns are generated. For each class, the process starts from layer 1 that contains the instances of the training set. Patterns generated as intersections between instances of the training set, are stored in layer 2. Layer N is formed by patterns generated as intersections between patterns of the layer N-1. This process continues until further intersections are not possible.

During generalization, for every class a separate pyramidal network structure is built. The process of generalization creates “vertical” interconnections between patterns from neighbourhood layers. These interconnections for every pattern are represented by a set of “predecessors” and a set of “successors”.

The predecessors’ set of a concrete pattern contains all patterns from the lower layer which were used in the process of its generalization. Thus in cases of different intersections generating the same pattern in the final outcome all patterns appearing in the intersection would be united as predecessors of the resulting pattern. The predecessors’ sets for instances of layer one are empty.

The successors’ set of a concrete pattern contains the patterns from the upper layer which are created from it. The successors’ sets of patterns on the top of the pyramid are empty. These patterns are called “vertices” of the corresponded pyramids.

One pattern may be included in more than one pyramid, but the vertex pattern belongs to one pyramid only.

It is possible for any pyramid to contain only one instance.

Figure 1 illustrates the result of the generalization step of MPGN for the example dataset. For simplifying the texts in the figures here and later patterns are presented only with value attributes. The class label is omitted as it is known from the pyramids in which pattern belongs. Light points denote vertices of the created pyramids.

3.1.3. Pruning steps. The pruning steps combine two different processes:

- pre-pruning – in parallel with the generalization;
- post-pruning – pruning the contradictions.

Pre-pruning. During the generalization a huge amount of combinations arises in big datasets. To restrict the combinatorial explosion different techniques can be applied. We use two different mechanisms for deciding which of the created patterns are to be included in the process of generalization.

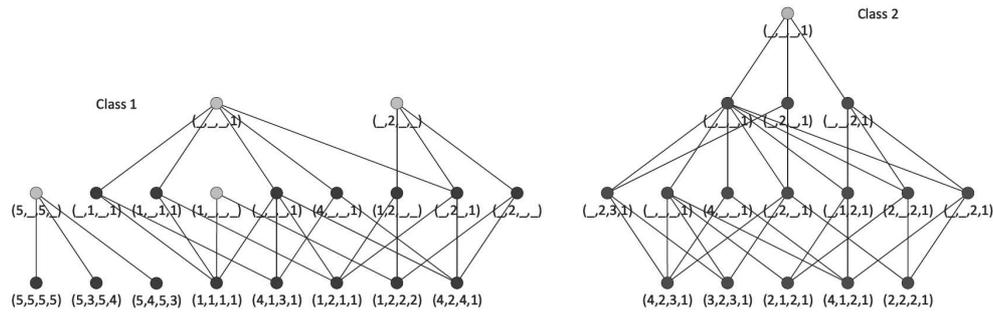


Fig. 1. MPGN – Result of the generalization step on the example dataset

The first mechanism allows excluding the patterns that are generated by few patterns from the lower layer. This is similar to support but here the direct predecessors rather than the primary instances are taken into account.

The other one tries to exclude the very general patterns from the beginning layers, using the presumption that these patterns will arise again in the upper layers. For this purpose, the ratio between the cardinality of the generated patterns and the cardinality of the predecessor patterns can be used as a restriction.

Post-pruning. Post-pruning is the process of iterative analysis of vertex patterns of all pyramids from different classes and removing all contradictory vertex patterns. As a result, some of the most general patterns are deleted, because the vertices with the same bodies were available in other classes (and they also are deleted). The primary pyramids are decomposed to several subpyramids with a lower number of layers. The vertices of such pyramids do not contradict vertices of pyramids of other classes.

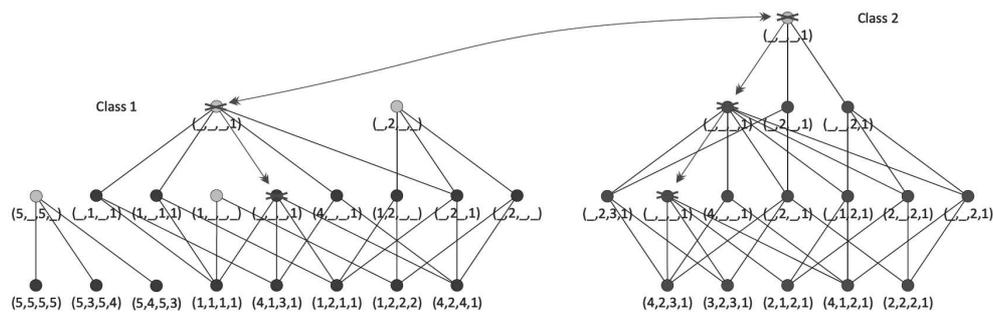


Fig. 2. Post-pruning – starting process

Figure 2 shows the start of the process, where vertices of pyramids of class 1 and class 2 are compared and contradictory vertices as well as all successor equal patterns are destroyed.

The process of destroying contradictory vertices causes the arising of new vertices from the patterns of corresponding pyramids. For new vertices the search and destruction of contradictory patterns are applied again.

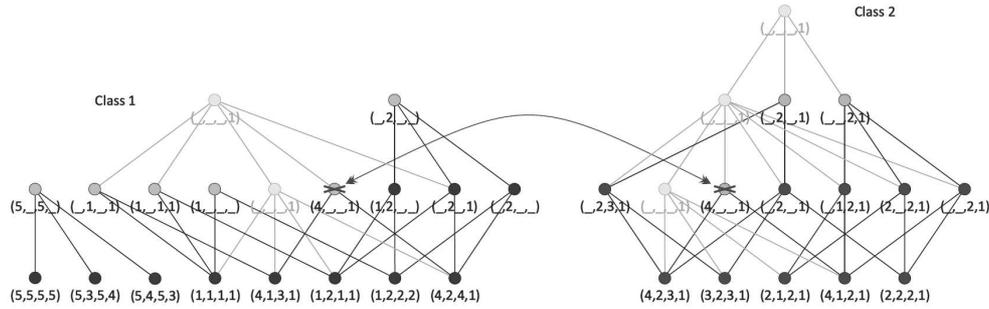


Fig. 3. Post-pruning – continuing the process

Figure 3 shows this next step on the example dataset. The process continues iteratively since no contradictions between vertices of pyramids are found. In our case after the second traversing no new contradictions were found and the process of destroying pyramids stops.

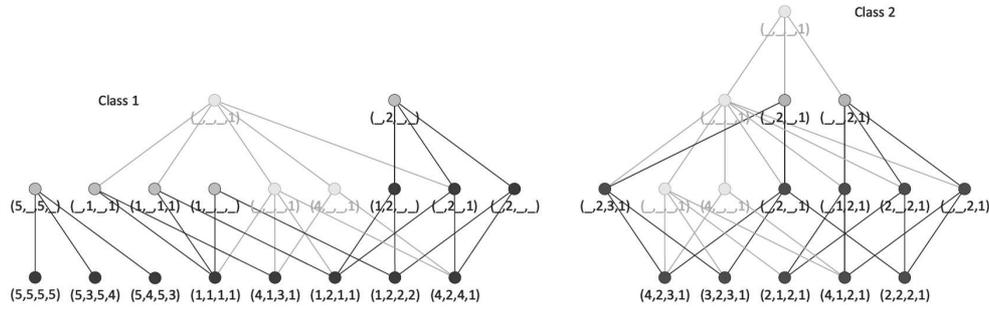


Fig. 4. Final result of post-pruning

Figure 4 shows the final result of the post-pruning. In grey we show the destroyed parts of pyramids.

3.2. Recognition process. The instance to recognize is given by the values of its attributes $Q = (?|b_1, \dots, b_n)$. Some of the values may be omitted. If

some attributes are numerical, the values of these attributes are replaced with the number of the corresponding discretized interval where the value belongs. The categorical attributes are also recoded with the corresponding number values.

Initially the set of classes that represent potential answers CS includes all classes: $CS = \{c | c = 1, \dots, M_c\}$.

The recognition process consists of two main steps:

- creating a recognition set for every class separately;
- analyzing the resulting recognition sets from all classes and making a decision on which class is to be given as an answer.

3.2.1. Creating recognition set for each class. At this stage each class is processed separately.

The goal is to create for each class the recognition set which contains all patterns with maximal cardinality that have 100% intersection with the query.

The process starts from the vertices of all pyramids that belong to the examined class. Using the predecessor sets of the patterns in the recognition set each pattern is replaced with the set of their predecessors that have 100% intersection with the query, if this set is not empty. After reducing the recognition set keeping only patterns with maximal coverage the process is iteratively repeated down to the layers until no new patterns appear in the recognition set.

The process of creating recognition set for each class can also be implemented on parallel computers.

3.2.2. Analyzing results and making a final decision. Figure 5 shows the general schema of this step.

The result of the first step of the processing are the recognition sets for all classes RSc , $c \in [1, \dots, M_c]$, which contain the patterns with maximal cardinality for this class $MaxCard(RSc)$, $c \in [1, \dots, M_c]$ that have 100% intersection with Q .

The goal is to find the class which contains the patterns with highest cardinality in its recognition set.

For this purpose, first the maximum of all maximal cardinalities of the recognition sets of classes from CS is discovered.

$$MaxCr = \max_{c \in CS} MaxCard(RSc)$$

All classes that have not such maximal cardinality are excluded from the set CS.

$$CS = \{c : MaxCard(RSc) = MaxCr\}$$

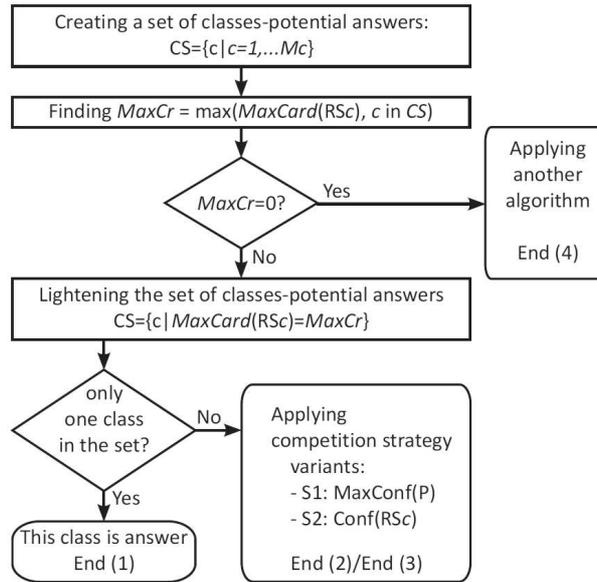


Fig. 5. MPGN – comparative analysis between classes

After this step, if only one class remains in CS, then this class is given as an answer (End 1).

Let us see the recognition process over the example dataset for query $Q = (?|5, 2, 3, 1)$, shown in Figure 6.

Each class is examined separately.

Only vertex $(1|-, 2, -, -)$ of class 1 matches the query. From its predecessors the pattern $(1|-, 2, -, 1)$ match the query and has a greater cardinality, equal to 2. No match is found in its predecessors and process for class 1 stops.

In the case of class 2 the vertex $(2|-, 2, -, 1)$ matches the query and its predecessor $(2|-, 2, 3, 1)$ also matches and has a greater cardinality, equal to 3.

As a result of comparison of maximal cardinalities between classes class 2 is given as the answer.

In the case when several classes exist with maximal cardinality $MaxCr$ (i.e. $Card(CS) > 1$), different strategies can be used to choose the best competitor. Here we will discuss two basic options:

- S1: choose from each class a single rule with maximal confidence within the class and compare it with others;
- S2: find the “confidence of recognition set”, i.e. the number of instances

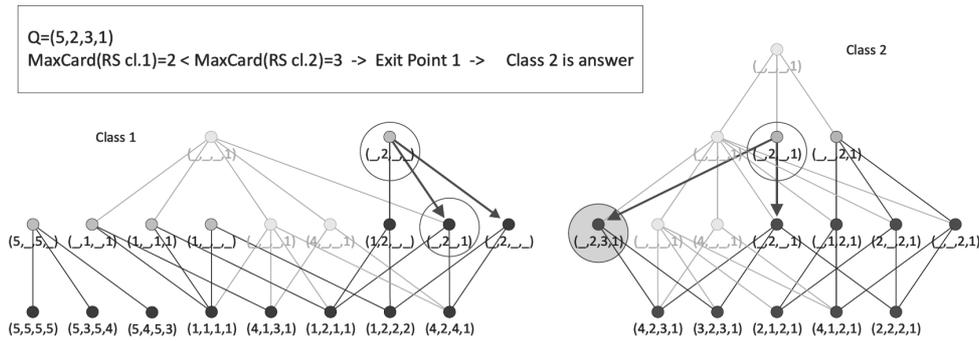


Fig. 6. Example of recognition in MPGN – Exit Point 1

that are covered by patterns from the recognition set of this class over the number of all instances of this class, and compare results.

Variant multiple classes-candidates: S1. The algorithm for the S1-option can be summarized as follows:

- find the pattern with maximal confidence of each of the recognition sets of the classes in CS: $MaxConf(P \in RSc) = \max_{P \in RSc} (Conf(P))$;
- find the maximum of received numbers from all classes:
 $MaxCn = \max_{c \in CS} MaxConf(P \in RSc)$;
- reduce CS retaining only classes that have such maximum: $CS = \{c : MaxCard(RSc) = MaxCr \text{ and } MaxConf(P \in RSc) = MaxCn\}$;
- if only one class has such a maximum (End 2), this class is given as an answer; otherwise the class with maximal support from CS is given as answer. (End 3).

Let us see the behaviour of MPGN recognition: S1 strategy (Figure 7) on the case of query $Q = (?|5, 2, 5, 1)$.

For class 1 the patterns that matched the query with maximal cardinality 2 are $(1|5, _ , 5, _)$ and $(1| _ , 2, _ , 1)$. For class 2 the pattern that matched the query with maximal cardinality again 2 is $(2| _ , 2, _ , 1)$.

Because of the equality of the maximal cardinality we continue with finding the rule with maximal confidence within each class.

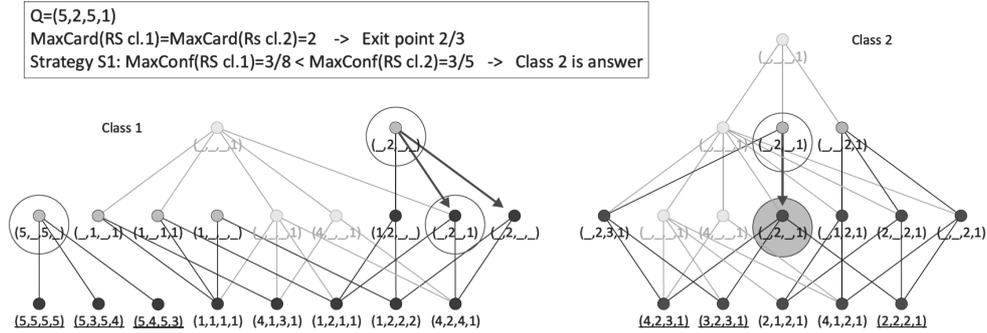


Fig. 7. Example of recognition in MPGN – Exit Point 2: Strategy S1

For class 1 the confidence of the pattern $(1|5, _, 5, _)$ is $3/8$ (maximal for this class). For class 2 $(2|_, 2, _, 1)$ has confidence $3/5$. Following strategy S1 class 2 is given as answer.

Variant multiple classes-candidates: S2. This algorithm is similar to the previous. The main difference is that instead of finding the pattern with maximal confidence of each of the recognition sets, here the “confidence of recognition set” is evaluated, i.e. the number of instances that are covered by patterns from the recognition set of this class over the number of all instances of this class and then results are compared. The rationale behind this is to take into account how many instances had been covered by all patterns from the recognition set. In practice this is the disjunction of the predecessors’ sets of Layer 1 (not direct predecessors) of the patterns that belong to RSc.

Let us see the behaviour of MPGN recognition: S2 strategy (Figure 8) on the case of the same query $Q = (?|5, 2, 5, 1)$.

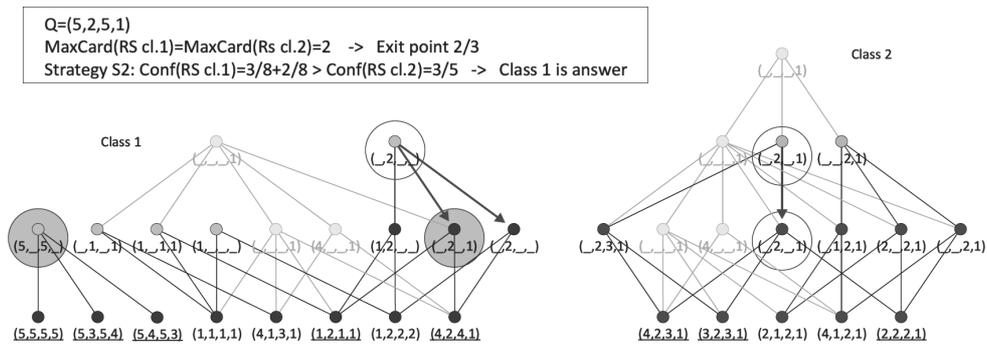


Fig. 8. Example of recognition in MPGN – Exit Point 2: Strategy S2

Following strategy S2 we found the confidences of the set of patterns that matches the query with maximal cardinality. They are correspondingly: $3/8+2/8=0.625$ for class 1 and $3/5=0.600$ for class 2. As a result class 1 is given as answer.

Variant: empty recognition sets. The worst case is when all recognition sets were empty.

Here we create new recognition sets, including instances with maximal intersection percentage with the query.

$$MaxIPerc(RSc, Q) = \max_{R \in RSc} (IntersectPerc(R, Q));$$

$$RSc = \{R : IntersectPerc(R, Q) = MaxIPerc(RSc)\}.$$

$$Find\ Conf(RSc) = \frac{|RSc|}{|\{\text{instances of class } c\}|}$$

The class that contains the set with maximal confidence is given as an answer. The reason is that a higher confidence is received because the rules are more inherent to this class. If two or more classes have equal confidence, than the class with maximal support is given as answer (End 4).

4. Program implementation. PGN and MPGN are implemented in the frame of the data mining environment PaGaNe. It uses the advantages of multi-dimensional numbered information spaces, provided by the access method ArM 32, such as the possibilities:

- to change searching with direct addressing in well-structured tasks;
- to build growing space hierarchies of information elements;
- to build interconnections between information elements stored in the information base.

An important feature of the approaches used in PaGaNe is the replacement of the symbolic values of the objects' features with integer numbers of the elements of corresponding ordered sets. All instances or patterns can be represented by a vector of integer values, which may be used as a co-ordinate address in the corresponding multi-dimensional information space.

4.1. Preprocessing step. The preprocessing step is intended to convert the learning set to a standard form for further steps. It consists of: (1) discretization of numerical attributes; (2) numbering the values of attributes; and (3) attribute subset selection.

4.1.1. Discretization. Because MPGN deal with categorical attributes, different discretization methods are implemented as a preprocessing step in PaGaNe. Special attention is paid to this process in [25].

4.1.2. Converting primary instances into numerical vectors. At this stage, the bijective functions between primary values and positive integers are generated. After that, the input instances are coded into numerical vectors, replacing attribute values with their corresponding numbers. The idea is to prepare data for direct use as addresses in the multi-dimensional numbered information spaces.

4.1.3. Attribute subset selection. The statistical observations on PaGaNe's performance over the dataset can show that some attributes give no important information and the environment allows withdrawing such attributes from participation in further processing. The automatic subset selection is an open part of the implementation of PaGaNe and it is at the front of current and near-future investigation [26].

4.2. Training process. A very important aspect is that for each class there exists a separate class space, which has a multilayer structure called "pyramids". All layers have equal structure and consist of "pattern-set" and "link-space". For each class space a "vertex-set" also is maintained, which is used at the recognition stage.

4.2.1. Construction elements. Here we will define the structure of "pattern-set", "link-space", and "vertex-set".

Pattern-set. Each pattern belongs to a definite class c and layer l . The full denotation of pattern should be $P(c, l)$ in order to make clear which class this pattern belongs to (note that c is the class value of the pattern). We omit c whenever it is clear from the context and denote $P(l)$. When l is also clear from the context we will denote only P .

All patterns of the class c which belongs to layer l form their pattern-set: $PS(c, l) = \{P^i(c, l) | i = 1, \dots, n_{c,l}\}$. Each pattern $P(c, l)$ from $PS(c, l)$ have identifiers $pid(P, c, l)$, or $pid(P)$ for short when no ambiguity arises, which are natural numbers. The identifiers are created in increasing order of input of the patterns into pattern-set.

The process of generalization creates "vertical" interconnections between patterns from different (neighbourhood) layers. These interconnections for every pattern are represented by two sets of "predecessors" and "successors".

The predecessors' set $PredS(P^i)$ contains the identifiers of patterns from the lower layer, which were used in the process of obtaining this pattern. The predecessors sets for instances of layer one are empty.

The successors' set $SuccS(P^i)$ contains the identifiers of patterns from the upper layer, which are created by this pattern. The successors' sets of patterns on the top of the pyramid are empty. These patterns are called “vertices” of the corresponding pyramids.

One pattern may be included in more than one pyramid. The vertex pattern belongs only to one pyramid (they become top of the pyramids).

Link-space. The goal of the Link-space is to describe all regularities between attributes which are available in the classes. Links to the patterns which contain it are created for every value of each attribute, thus allowing to create a hierarchy of sets. The structure of this hierarchy is as follows:

- attribute value set: a set of patterns for given value of given attribute;
- attribute set: a set of attribute value sets for given attribute;
- link-space (one): a set of all possible attribute sets.

The creation of the link-space uses the advantages of multi-dimensional numbered information spaces, especially the possibility to overcome searching by using direct pointing via coordinate addresses.

Below we will focus our attention on the link-space, which becomes a key element of accelerating the creation of new patterns as well as searching patterns satisfying the queries.

Let c be one number of the examined class and l be the number of given layer of c :

- attribute value set $VS(c, l, t, v)$, $v = 1, \dots, n_t$ is a set of all identifiers of instances/patterns for class c , layer l , which have value v for the attribute t : $VS(c, l, t, v) = \{pid(P^i, c, l), i = 0, \dots, x \mid a_t^i = v\}$;
- attribute set $AS(c, l, a)$ for a concrete attribute $a = 1, \dots, n$ is a set of attribute value sets for class c , layer l and attribute t : $AS(c, l, a) = \{VS(c, l, t, 1), \dots, VS(c, l, t, n_t)\}$, where n_t is the number of values of attribute t ;
- link-space $LS(c, l)$ is a set of all possible attribute sets for class c and layer l : $LS(c, l) = \{AS(c, l, 1), \dots, AS(c, l, n)\}$.

In Figure 9, the visualization of class 3 of the “Lenses” dataset [27] during the creation of the patterns is shown.

Such information is stored in ArM-structures by a very simple convention

- the attribute value sets $VS(c, l, t, v)$ is stored in the points of the ArM-archive

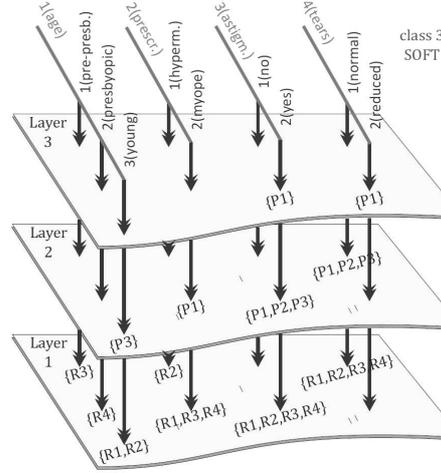


Fig. 9. Visualization of link-spaces

using the corresponding address $(4, c, l, t, v)$, where 4 is the dimension of the ArM space, c is the number of the class, l is the number of the layer, t is the number of the attribute and v is the number of the corresponding values of the given attribute. The disposition of link-spaces in ArM-structures allows very fast extraction of available patterns in the corresponding layer and class.

Vertex-set. The vertex-set contains information about the patterns that have no successors in the pyramids of the corresponding class.

$$VrS(c) = \{pid(P^i, c, l) \mid i = 1, \dots, n_{c,l}; l = 1, \dots, l_{\max} : SuccS(P^i) = \emptyset\}.$$

4.2.2. Generation of the rules. The process of rule generation is a chain of creating the patterns of the upper layer as an intersection between patterns from the lower layer until new patterns are generated.

Each instance $R = (c|a_1, \dots, a_n)$ from the learning set is included into the pattern-set of the first layer of its class c : $R \in PS(c, 1)$.

Starting from layer $l = 2$ the following steps are made:

(1) Creating the link-space of the lower layer $l - 1$ of class c by adding the identifiers of patterns $P^i \in PS(c, l - 1)$, $i = 1, \dots, n_{c,l-1}$ to the attribute value sets of the values: $pid(P^i) \in VS(c, l - 1, k^i, a_k^i)$, $k = 1, \dots, n$. Note that these sets became ordered during creation;

(2) The set of patterns which are intersections of the patterns of the lower layer $P^i \cap P^j$, $P^i, P^j \in PS(c, l - 1)$, $i, j = 1, \dots, n_{c,l-1}$, $i \neq j$ is created. The algorithm for creating this set is given below;

(3) If patterns are not generated (i.e. $PS(c, l) = \{ \}$), then the process of generating the rules for this class stops;

(4) On the basis of obtaining a set of patterns, the pattern-set $PS(c, l)$ of layer l is created as follows:

- Each pattern from the receiving set of patterns is checked for existence in the $PS(c, l)$;
- If this pattern does not exist in $PS(c, l)$, it receives an identifier which is equal to the next number of identifiers of the patterns in the pattern-set; the pattern is added at the end of the pattern-set; and its predecessor-set is created with two pairs $\{(pid(P^i), l - 1), (pid(P^j), l - 1)\}$;
- If this pattern already exists in $PS(c, l)$, its predecessor-set is formed as a union of the existent predecessor-set and $\{(pid(P^i), l - 1), (pid(P^j), l - 1)\}$.

(5) For each pattern from layer l check for existence the same pattern in lower layers (from layer $l - 1$ to layer 2). If such a pattern exists, then it is removed from the pattern-set and corresponding link-space of the lower layer and the predecessor-set of the current pattern is enriched with the predecessor-set of the removed pattern (by union).

(6) Incrementing layer l and repeating the process.

As a result, for every class a separate pyramidal network structure is built. Each pyramid is described by a predecessor-set and a successor-set of patterns in neighbour layers.

4.2.3. Generating the set of patterns which are intersections of patterns from the lower level. For restricting the exponential growth of intersections two parameters are included in program implementation:

- L1 (from 0 to 100) – percentage of reduced patterns per layer;
- L2 (from 0 to 100) – minimal ratio in percent of the cardinality of the generated pattern to the maximal cardinality of patterns in lower layer.

The process of generating the intersections of the patterns from a given pattern-set $PS(c, l)$ loops through each pattern $P^i \in PS(c, l)$.

For this pattern $P^i = (c|a_1^i, \dots, a_n^i)$ the generation of possible patterns is:

- (1) An empty set of resulting patterns is created;
- (2) For all attribute values a_1^i, \dots, a_n^i different from “-” of P^i we take the corresponding attribute-value-sets $VS(c, l, k^i, a_k^i)$, $i = 1, \dots, n$. The numbers of identifiers of the patterns in these sets are ordered.

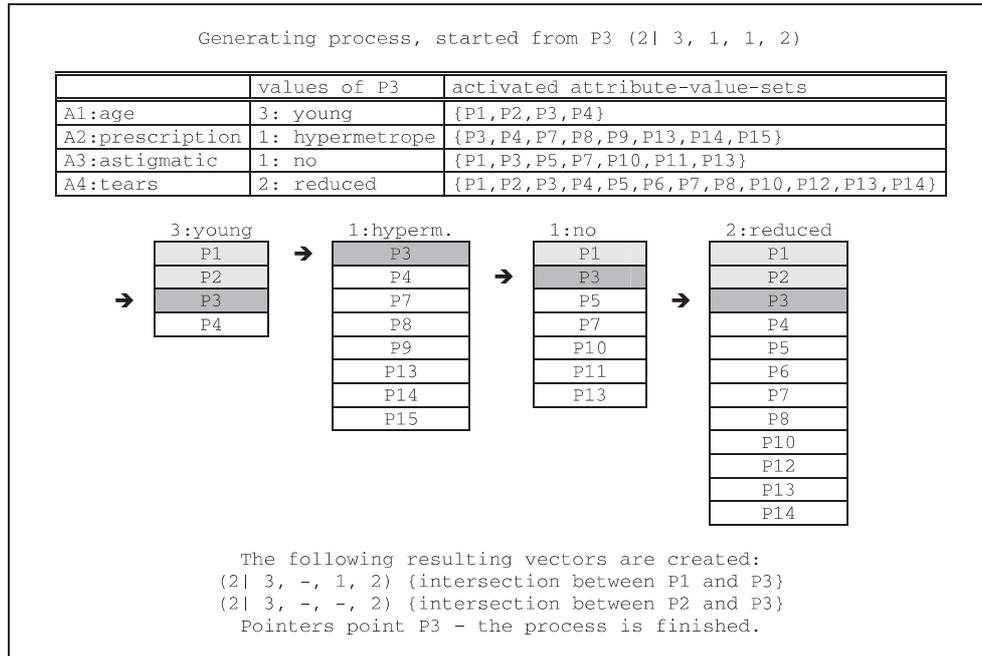


Fig. 10. Visualization of the process of generating a set of patterns

(3) All extracted attribute-value-sets are activated.
 (4) From each of them the first identifier is given.
 (5) While at least one attribute-value-set is active, the following steps are made:

- Assign the initial values of the resulting pattern: $V = (c|-, -, \dots, -)$;
- Locate the minimal identifier $pid(P^j)$ from all active attribute-value-sets;
- If $pid(P^j) = pid(P^i)$, then this attribute-value-set is deactivated;
- All active attribute-value-sets $VS(c, l, k^j, a_k^j)$, $j = 1, \dots, k_{c,l}$, for which $pid(P^j)$ is the current identifier, cause filling of the corresponding attribute value a_k^i of k^{th} attribute in V . For these sets the next identifier is given;
- If $|V| > 0$ and $\min\left(\frac{|V|}{|P^i|}, \frac{|V|}{|P^j|}\right) > L2$, this pattern is included into the set of resulting patterns with additional information containing $pid(P^i)$ and $pid(P^j)$.

This process is illustrated in Figure 10.

At the end the patterns in the created pattern set are sorted by the number of their predecessors and L1% of them with fewer predecessors are removed.

4.3. Recognition process. The record to be recognized is represented by the values of its attributes $Q = (?|b_1, \dots, b_n)$. Some of the features may be omitted. The classification stage consists of several steps:

(1) Using the service attribute-values-space, the system takes corresponding attribute value sets for all attributes b_1, \dots, b_n as well as the attribute value sets for “-” as a value of each attribute.

(2) The union of these sets gives a set of possible classes $\{c_1, \dots, c_y\}$, which the record may belong to. This approach decreases the amount of information needed for pattern recognition.

(3) All classes which are presented in this union $\{c_1, \dots, c_y\}$ are scanned in parallel. For each class c_x and for each layer of class space of c_x the following steps are done:

- For all attribute values b_1, \dots, b_n different from “-” of Q we obtain the corresponding attribute value sets from the link-space of class c_x of current layer;
- The intersection between all these sets is made. As a result a recognition set of candidate patterns is created. If this set is empty, the target class is a class with maximal support;
- For each pattern P which is a member of the recognition set calculate $IntersectPerc(P, Q)$.

(4) From all recognition sets of the classes and layers the patterns with maximal cardinality are found.

(5) These recognition sets are reduced by excluding the patterns whose cardinality is less than the maximal cardinality. The new set of classes-potential answers $\{c_1, \dots, c_{y'}\}$ contains only classes whose recognition sets are not empty.

(6) If only one class is in the set of classes-potential answers, then this is the target class and the process stops.

(7) Otherwise, if this set is empty, we produce again a primary set of classes-potential answers $\{c_1, \dots, c_{y'}\} = \{c_1, \dots, c_y\}$ and the process continues with examining this set.

(8) Examine $\{c_1, \dots, c_{y'}\}$:

- for each class which is a member of this set, the number of instances with maximal intersection percentage with the query is found and the ratio between these numbers and all instances in the class is calculated;

- the maximum of intersection percentages from all classes is determined and in the set $\{c_1, \dots, c_{y'}\}$ only classes with this maximal percentage and maximal ratio is left;
- if $\{c_1, \dots, c_{y'}\}$ contains only one class, this class is given as answer. Otherwise the class of $\{c_1, \dots, c_{y'}\}$ with maximal instances is given as an answer. And the process stops.

We can assume that the possibilities for keeping numerous created patterns in a manageable way allow using this environment to test different kinds of recognition models.

Here the algorithm of recognition strategy S1 was described. The algorithm for strategy S2 differs only in points 4 and 5, where the criterion for selection of classes-candidates is the so-called “confidence of recognition set”.

5. Sensitivity analysis. We made different experiments for studying the specific behaviour of the proposed algorithm and for comparing the results of MPGN and other classifiers that use similar classification models. For the experiments we used 25 datasets from UCI Machine Learning Repository [27], which contain different numbers of classes and different types of attributes: audiology, balance_scale, blood_transfusion, breast_cancer_wo, car, cmc, credit, ecoli, forestfires, glass, haberman, hayes-roth, hepatitis, iris, lenses, monks1, monks2, monks3, post-operative, soybean, tae, tic_tac_toe, wine, winequality-red, and zoo.

In order to receive more stable results we applied a 5-fold cross-validation technique. For achieving equality of data used for different classifiers we exported learning sets and examining sets from PaGaNe as input files in other programs.

We compared the proposed classifiers PGN and MPGN with some of those implemented in Weka [28] and LUCS-KDD Repository [29]. In the experiments we made a comparison with CMAR (thresholds: 1% support and 50% confidence) [5] as representative of other associative classifiers (implemented in the LUCS-KDD Repository), OneR [30] and JRip (Weka implementation of RIPPER [31]) as representatives of decision rules classifiers, and J48 (Weka implementation of C 4.5 [32]) and REPTree [28] as representatives of decision trees. The comparisons are two-fold measuring: (1) overall accuracy; and (2) F-measure results.

5.1. Examining the Exit Points of MPGN. During the design of MPGN we believe that the pyramidal network contains enough information to classify. So we expect that most unseen cases can be classified by using the

structure. In other words we expect that in the recognition phase only few cases will have an empty recognition set. We are also interested in how many cases have a recognition set with one class and how many cases have multiple conflicting classes. The latter cases can be classified on the basis of confidence or support. At the second step of the recognition phase MPGN can fail in different situations:

- only one class is a class-candidate – we mark this case as Exit point 1;
- several classes are class-candidates. In this case two strategies are suggested in order to choose the best competitor: S1: from each class choose a single rule with the maximal confidence within the class and compare with others; and S2: find “confidence of recognition set”, i.e. the number of instances that are covered by patterns from the recognition set of this class over the number of all instances of this class and compare results. In both strategies the cases are classified based on maximal confidence (Exit point 2) or maximal support (Exit point 3);
- empty recognition sets – in this case another algorithm is used – the Exit point 4.

Figure 11 illustrates the percentage of different kinds of exits. In most cases the recognition leads to exit 1, which means that applying MPGN is worthwhile.

We examine in more detail the two strategies for multiple classes (Exit points 2 and 3). Figure 12 presents the scatter plot of the obtained accuracies for

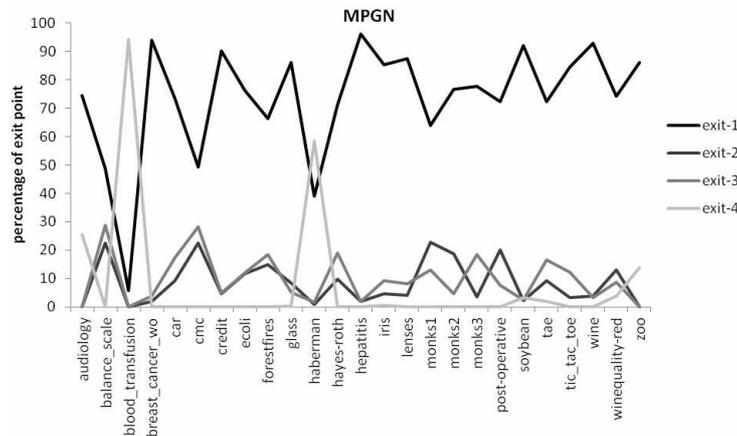


Fig. 11. The exit points for MPGN

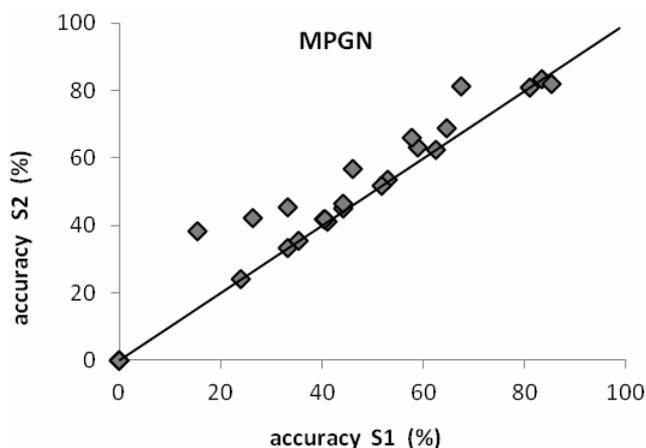


Fig. 12. Scatter plot of the obtained accuracies for MPGN-S1 and MPGN-S2

Table 1. Percentage of overall accuracy of examined datasets for PGN, MPGN-S1, MPGN-S2, CMAR, OneR, JRip, J48, and REPTree

Datasets	PGN	MPGN-S1	MPGN-S2	CMAR	OneR	JRip	J48	REPTree
audiology	75.50	69.00	69.00	59.18	47.00	69.50	72.00	62.50
balance_scale	77.89	81.41	83.49	86.70	60.10	71.95	66.18	67.15
breast_cancer_wo	96.43	92.85	93.56	93.85	91.85	93.28	94.28	93.99
car	92.59	82.87	85.71	81.77	70.03	86.75	90.80	88.20
cmc	49.90	46.03	46.64	53.16	47.25	50.38	51.60	50.17
credit	87.54	85.65	86.09	87.10	85.51	85.07	85.36	85.07
haberman	55.27	73.21	73.21	71.90	72.88	73.21	73.21	74.20
hayes-roth	81.94	65.22	67.49	83.42	50.77	78.12	68.23	73.53
hepatitis	80.65	81.94	81.94	84.52	81.94	77.42	79.36	79.36
lenses	74.00	83.00	83.00	88.00	62.00	83.00	83.00	80.00
monks1	100.00	82.9	85.92	100.00	74.98	87.53	94.68	88.91
monks2	73.06	80.52	81.02	59.74	65.73	58.73	59.90	63.90
monks3	98.56	90.43	93.50	98.92	79.97	98.92	98.92	98.92
post-operative	66.67	52.22	52.22	51.11	68.89	70.00	71.11	71.11
soybean	93.15	84.00	84.00	78.48	37.44	85.35	87.64	78.18
tae	52.94	52.88	52.88	35.74	45.76	34.43	46.97	40.43
tic_tac_toe	88.93	96.13	95.62	98.75	69.93	98.02	84.23	80.37
wine	96.09	92.19	93.87	91.70	78.63	90.45	87.03	88.16
winequality-red	64.98	59.35	59.60	56.29	55.54	53.72	58.22	57.03
zoo	98.10	89.24	89.24	94.19	73.29	88.19	94.14	82.19

both strategies. We can conclude that Strategy 2 has a mean accuracy of 49% and Strategy 1 has a mean accuracy of 46%. The Friedman test shows $\chi_F^2 = 2.56$, $\alpha_{0.05} = 1.960$, which means that the MPGN-S2 statistically outperforms MPGN-S1.

In conclusion, the analysis of different recognition parts taught us that the initial idea of the PaGaNe algorithms works well and that for many cases the recognition set contains only one and the correct class.

5.2. Comparison with overall accuracy. In Table 1 the obtained results for the overall accuracy are shown.

We use the Friedman test to detect statistically significant differences between the classifiers in terms of average accuracy [33]. The Friedman test is a non-parametric test, based on the ranking of the algorithms on each dataset instead of the true accuracy estimates. We use Average Ranks ranking method, which is a simple ranking method, inspired by Friedman’s statistic [34].

In order to apply the Friedman test to measure the statistical dissimilarity of different classifiers we ranked the results (Table 2).

Table 2. Ranking by accuracy of examined classifierscentering

Datasets	PGN	MPGN-S1	MPGN-S2	CMAR	OneR	JRip	J48	REPTree
audiology	1	4.5	4.5	7	8	3	2	6
balance_scale	4	3	2	1	8	5	7	6
breast_cancer_wo	1	7	5	4	8	6	2	3
car	1	6	5	7	8	4	2	3
cmc	5	8	7	1	6	3	2	4
credit	1	4	3	2	5	7.5	6	7.5
haberman	8	3.5	3.5	7	6	3.5	3.5	1
hayes-roth	2	7	6	1	8	3	5	4
hepatitis	5	3	3	1	3	8	6.5	6.5
lenses	7	3.5	3.5	1	8	3.5	3.5	6
monks1	1.5	7	6	1.5	8	5	3	4
monks2	3	2	1	7	4	8	6	5
monks3	5	7	6	2.5	8	2.5	2.5	2.5
post-operative	5	6.5	6.5	8	4	3	1.5	1.5
soybean	1	4.5	4.5	6	8	3	2	7
tae	1	2.5	2.5	7	5	8	4	6
tic_tac_toe	5	3	4	1	8	2	6	7
wine	1	3	2	4	8	5	7	6
winequality-red	1	3	2	6	7	8	4	5
zoo	1	4.5	4.5	2	8	6	3	7
average	2.975	4.625	4.075	3.85	6.8	4.85	3.925	4.9

The null hypothesis critical value for the Friedman test is $\alpha_{0.05} = 14.067$; in our case $\chi_F^2 = 29.492$, which means that the null-hypothesis is rejected, i.e. the classifiers are statistically different. We see that PGN has the best overall performance among the examined classifiers. MPGN is competitive with J48, JRip and REPTree.

5.3. Analyzing F-measures on some multi-class datasets. The overall accuracy of a particular accuracy may be good in some cases, and yet it might not be able to recognize some of the class labels for different reasons – small percentage of presence of given class label, confusion with another, etc. We make the experiments over datasets with two particular characteristics – too many class labels, and unbalanced support of different class labels – such are the Glass and Soybean datasets. We choose the F-measure as the harmonic mean of recall (measure of completeness) and precision (measure of exactness).

Glass dataset. The Glass dataset has 6 class labels with very uneven distribution between them: 2#: 35.51%; 1#: 32.71%; 7#: 13.55%; 3#: 7.94%;

Table 3. Percentage of F-measure from tested classifiers for the Glass dataset

Class labels	PGN	MPGN-S1	MPGN-S2	OneR	CMAR	Jrip	J48	REPTree
2# (35.5%)	80.3	81.6	82.2	57.5	79.7	67.9	78.1	75.2
1# (32.7%)	80.3	81.1	81.1	64.4	80.5	69.0	76.5	70.1
7# (13.6%)	89.7	89.7	89.7	53.8	90.3	85.7	84.2	67.8
3# (7.9%)	51.9	59.5	57.9	0.0	38.5	16.0	29.6	34.5
5# (6.1%)	58.1	69.2	69.2	0.0	64.0	53.8	64.5	21.1
6# (4.2%)	88.9	94.1	94.1	0.0	77.8	60.0	55.6	80.0

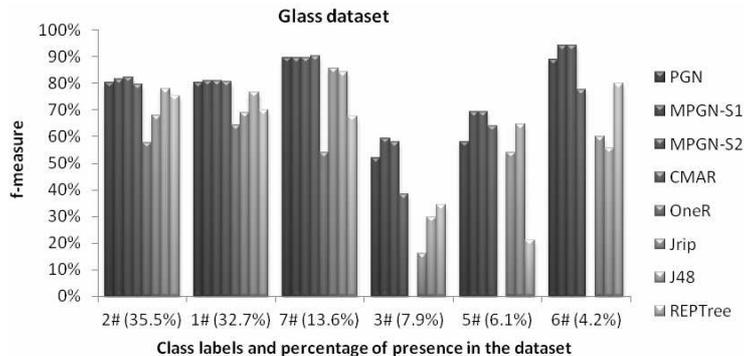


Fig. 13. F-measure for examined classifiers for class-labels of Glass dataset

5#: 6.07%; 6#: 4.21%.

Table 3 and Figure 13 present the F-measure for each class label from different classifiers.

As we can see PGN and MPGN have good performance for each of the class labels. For instance, the low-presented class label “3#” is not well performed by CMAR, OneR, JRip, J48 and REPTree; “5#” – by OneR and REPTree; “6#” – by OneR (F-measures are less than 50%).

Soybean dataset. The Soybean dataset has 19 class labels with different distributions – from 13.029% to 0.326%.

Table 4 and Figure 14 show the F-measure for each class label from different classifiers. Here the class with lowest support “2-4-d-injury” is not recognized by any classifier because of the very low presence (1 instance) – it falls either into the learning set or into the examining set.

As we can see PGN recognizes successfully all other class labels instead of differences of their support. J48 also recognizes well but fails in low presented classes. MPGN has a relatively good behaviour.

Looking at the results of the overall accuracy as well as the refined analysis of the behaviour of classifiers over multi-classes non-uniform distributed datasets we can conclude that associative classifiers as a whole and especially PGN and MPGN have a potential to create vital classification algorithms.

6. Conclusion and future work. The main contributions can be summarized as:

- a method for effective building and storing of pattern sets in the multi-layer structure MPGN during the process of associative rule mining using the possibilities of multidimensional numbered information spaces has been developed;
- software for the proposed algorithms and structures has been implemented in the frame of the data mining environment system PaGaNe;
- the conducted experiments prove the vitality of proposed approaches showing the good performance of MPGN in comparison with other classifiers with similar recognition models, and especially in the case of multi-class datasets with uneven distribution of the class labels.

The work highlighted some possible directions for further research which could tackle areas such as:

Table 4. Percentage of F-measure from tested classifiers for the Soybean dataset

Class labels	PGN	MPGN-S1	MPGN-S2	CMAR	OneR	Jrip	J48	REPTree
alternarialeaf-spot	86.7	77.4	77.4	93.0	35.8	62.3	89.2	50.7
brown-spot	91.8	74.7	74.7	73.4	38.2	71.9	87.5	58.2
frog-eye-leaf-spot	83.8	75.7	75.7	51.6	69.6	61.7	87.8	52.3
phytophthora-rot	100.0	100.0	100.0	91.9	71.2	76.8	95.2	73.3
anthracnose	100.0	93.0	93.0	94.7	76.9	81.1	89.5	75.0
brown-stem-rot	100.0	97.4	97.4	70.3	0.0	100.0	88.4	54.9
bacterial-blight	90.0	75.0	75.0	73.3	0.0	0.0	90.0	30.8
bacterial-pustule	90.0	85.7	85.7	80.0	0.0	75.0	85.7	42.9
charcoal-rot	100.0	100.0	100.0	100.0	28.6	94.7	100.0	57.1
diaporthe-stem-canker	100.0	88.9	88.9	100.0	41.7	84.2	88.9	42.9
downy-mildew	100.0	100.0	100.0	95.2	0.0	82.4	100.0	0.0
phyllosticta-leaf-spot	70.6	42.9	42.9	100.0	0.0	33.3	82.4	0.0
powdery-mildew	100.0	100.0	100.0	16.7	0.0	75.0	90.0	0.0
purple-seed-stain	95.2	100.0	100.0	100.0	0.0	88.9	82.4	46.2
rhizoctonia-root-rot	100.0	75.0	75.0	100.0	0.0	75.0	100.0	46.2
cyst-nematode	100.0	100.0	100.0	100.0	0.0	50.0	92.3	0.0
diap.-pod-&-stem-blight	100.0	90.9	90.9	57.1	50.0	66.7	66.7	54.5
herbicide-injury	88.9	40.0	40.0	82.4	0.0	57.1	75.0	40.0
2-4-d-injury	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

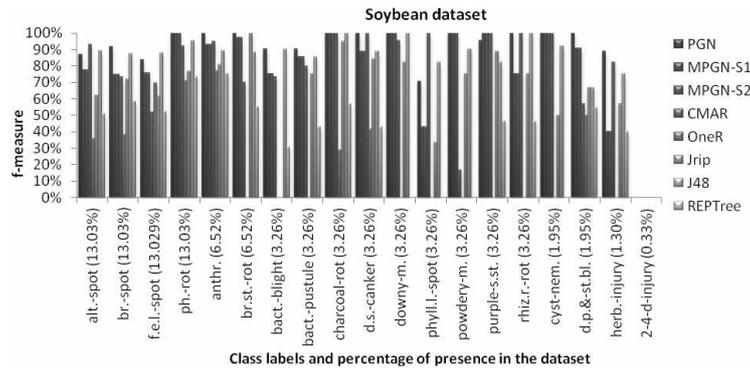


Fig. 14. F-measure for examined classifiers for Soybean dataset

- implementing PGN pruning and recognition ideas over pyramidal structures of MPGN;
- analyzing different variants of pruning and recognition algorithms based on statistical evidence, structured over already created pyramidal structures

of patterns in order to achieve better recognition results;

- proposing different techniques for rule quality measure taking into account the confidence of the rule in order to overcome the process of rejecting one rule in favour of the other one, rarely observed in the dataset;
- testing the possibilities of MPGN using exit-1 recognition in the field of campaign management;
- applying the established algorithms PGN and MPGN in different application areas such as business intelligence or global monitoring.

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Iliya Mitov
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Acad G. Bonchev Str., Bl. 8
1113 Sofia, Bulgaria
e-mail: mitov@mail.bg

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