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FEKETE-SZEGÖ INEQUALITY FOR UNIVERSALLY PRESTARLIKE FUNCTIONS

T. N. Shanmugam *, **J. Lourthu Mary** **

This paper is dedicated to the 70th anniversary of Professor Srivastava

Abstract

The universally prestarlike functions of order $\alpha \leq 1$ in the slit domain $\Lambda = \mathcal{C} \setminus [1, \infty)$ have been recently introduced by S. Ruscheweyh. This notion generalizes the corresponding one for functions in the unit disk Δ(and other circular domains in $\mathcal{C}$). In this paper, we obtain the coefficient inequalities and the Fekete-Szegö inequality for such functions.

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1. Introduction

Let $H(\Omega)$ denote the set of all analytic functions defined in a domain Ω. For domain Ω containing the origin $H_0(\Omega)$ stands for the set of all function $f \in H(\Omega)$ with $f(0) = 1$. We also use the notation $H_1(\Omega) = \{zf : f \in H_0(\Omega)\}$. In the special case when Ω is the open unit disk $\Delta = \{z \in \mathcal{C} : |z| < 1\}$, we use the abbreviation H, H_0 and H_1 respectively for $H(\Omega), H_0(\Omega)$ and $H_1(\Omega)$.

A function $f \in H_1$ is called starlike of order α with $(0 \leq \alpha < 1)$ satisfying the inequality

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha \qquad (z \in \Delta) \tag{1.1}$$

and the set of all such functions is denoted by S_α. The convolution or Hadamard Product of two functions $f(z) = \sum_{n=0}^{\infty} a_n z^n$ and $g(z) = \sum_{n=0}^{\infty} b_n z^n$ is defined as

$$(f * g)(z) = \sum_{n=0}^{\infty} a_n b_n z^n.$$

A function $f \in H_1$ is called prestarlike of order α if

$$\frac{z}{(1-z)^{2-2\alpha}} * f(z) \in S_\alpha. \tag{1.2}$$

The set of all such functions is denoted by $\mathcal{R}_\alpha$. The notion of prestarlike functions has been extended from the unit disk to other disk and half planes containing the origin. Let Ω be one such disk or half plane.Then there are two unique parameters $\gamma \in \mathcal{C} \setminus \{0\}$ and $\rho \in [0,1]$ such that

$$\Omega_{\gamma,\rho} = \{w_{\gamma,\rho}(z) : z \in \Delta\}, \tag{1.3}$$

where

$$w_{\gamma,\rho}(z) = \frac{\gamma z}{1-\rho z}.$$

Note that $1 \notin \Omega_{\gamma,\rho}$ iff $|\gamma + \rho| \leq 1$.

Definition 1.1. (see [2], [3], [4]) Let $\alpha \leq 1$, and $\Omega = \Omega_{\gamma,\rho}$ for some admissible pair (γ, ρ). A function $f \in H_1(\Omega_{\gamma,\rho})$ is called prestarlike of order α in $\Omega_{\gamma,\rho}$ if

$$f_{\gamma,\rho}(z) = \frac{1}{\gamma} f(w_{\gamma,\rho}(z)) \in \mathcal{R}_\alpha. \tag{1.4}$$

The set of all such functions f is denoted by $\mathcal{R}_\alpha(\Omega)$.

Let Λ be the slit domain $\mathcal{C} \setminus [1, \infty)$(the slit being along the positive real axis).

Definition 1.2. (see [2], [3], [4]) Let $\alpha \leq 1$. A function $f \in H_1(\Lambda)$ is called universally prestarlike of order α if and only if f is prestarlike of order α in all sets $\Omega_{\gamma,\rho}$ with $|\gamma + \rho| \leq 1$. The set of all such functions is denoted by $\mathcal{R}^u_\alpha$.

Example 1.1. A function $f(z) = \dfrac{z}{(1-z)^{1-2\alpha}}$ is prestarlike of order $0 \leq \alpha < 1$. When $\alpha = 0$ the function is universally prestarlike of order 0.When $\alpha = \frac{1}{2}$ the function $f(z) = z$ is the only entire function in $\mathcal{R}^u_\alpha$.

Example 1.2. A function $f(z) = \dfrac{z}{(1-z)^{\frac{1}{2}}}$ is universally prestarlike of order $\frac{1}{2}$.

Definition 1.3. (see [4]) Let $\phi(z)$ be an analytic function with positive real part on Δ, which satisfies $\phi(0) = 1$, $\phi'(0) > 0$ and which maps the unit disc Δ onto a region starlike with respect to 1 and symmetric with respect to the real axis. Then the class $\mathcal{R}^u_\alpha(\phi)$ consists of all analytic function $f \in H_1(\Lambda)$ satisfying

$$\frac{D^{3-2\alpha} f}{D^{2-2\alpha} f} \prec \phi(z), \tag{1.5}$$

where $\prec$ denotes the subordination,where $(D^\beta f)(z) = \frac{z}{(1-z)^\beta} \star f$, for $\beta \geq 0$. In particular, for $\beta = n \in$ N. We have $D^{n+1} f = \frac{z}{n!}(z^{n-1} f)^{(n)}$. We let $\mathcal{R}^u_\alpha(A, B)$ denote the class $\mathcal{R}^u_\alpha(\phi)$, where $\phi(z) = \dfrac{1 + Az}{1 + Bz}$ $(-1 \leq B < A \leq 1)$. For suitable choices of A,B,α the class $\mathcal{R}^u_\alpha(A, B)$ reduces to several well known classes of functions. $\mathcal{R}^u_{\frac{1}{2}}(1, -1)$ is the class S^* of starlike univalent functions.

Note 1.1. (see [4]) Let $F(z) = \displaystyle\sum_{k=0}^{\infty} a_k z^k = \int_0^1 \frac{d\mu(t)}{1 - tz}$ where, $a_k = \displaystyle\int_0^1 t^k d\mu(t)$, $\mu(t)$ is a probability measure on $[0, 1]$. Let T denote the set of all such functions F. They are analytic in the slit domain Λ.

Note 1.2. (see [3]) Let Ω be a circular domain containing the origin, $\alpha \leq 1$, and let $f \in \mathcal{R}_\alpha(\Omega)$, $F \in \mathcal{R}^u_\alpha$. Then $f * F \in \mathcal{R}_\alpha(\Omega)$.

To prove our result we need the following theorem.

Theorem 1.1. (see [2], [4]) *Let $0 \leq \alpha \leq 1$ and $f \in H_1(\Lambda)$. Then $f \in \mathcal{R}^u_\alpha$ if and only if*

$$\frac{D^{3-2\alpha} f}{D^{2-2\alpha} f} \in T. \tag{1.6}$$

This admits an explicit representation of the function in $\mathcal{R}^u_\alpha$. If $f \in H_0$ has all its Taylor coefficients at the origin different from zero we write $f^{(-1)}$ for the (possibly formal but) unique solution of $f * f^{(-1)} = \frac{1}{1-z}$.

Lemma 1.1. (see [1]) *If* $P_1(z) = 1 + c_1z + c_2z^2 + \dots$ *is an analytic function with positive real part in* Δ, *then*

$$|c_2 - vc_1^2| \leq \begin{cases} -4v + 2, & v \leq 0 \\ 2, & 0 \leq v \leq 1 \\ 4v + 2, & v \geq 1 \end{cases}$$

when $v < 0$, *or* $v > 1$, *the equality holds if and only if* $P_1(z)$ *is* $\dfrac{1+z}{1-z}$ *or one of its rotations.when* $0 < v < 1$, *then the equality holds if and only if* $P_1(z)$ *is* $\dfrac{1+z^2}{1-z^2}$ *or one of its rotations. If* $v = 0$, *the equality holds if and only if* $P_1(z) = \left(\dfrac{1}{2} + \dfrac{\lambda}{2}\right)\dfrac{1+z}{1-z} + \left(\dfrac{1}{2} - \dfrac{\lambda}{2}\right)\dfrac{1-z}{1+z}$. $0 \leq \lambda \leq 1$ *or one of its rotations. If* $v = 1$, *the equality holds if and only if* $P_1(z)$ *is the reciprocal of one of the function for which the equality holds in the case of* $v = 0$. *Also the above upper bound can be improved as follows when* $0 < v < 1$

$$|c_2 - vc_1^2| + v|c_1|^2 \leq 2 \qquad (0 < v \leq \frac{1}{2}),$$

$$|c_2 - vc_1^2| + (1 - v)|c_1|^2 \leq 2 \qquad (\frac{1}{2} < v \leq 1).$$

2. Series representation for universally prestarlike functions

Theorem 2.1. *Let* f *be an universally prestarlike function of order* $0 \leq \alpha \leq 1$, *then the function* $f(z)$ *has a representation of the form*

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

where

$$a_n = \left\{ \frac{\sum_{k=1}^{n-1} \mathcal{C}(\alpha, k) a_k b_{n-k}}{\mathcal{C}'(\alpha, n) - \mathcal{C}(\alpha, n)} \right\}, \quad n = 2, 3, \dots \tag{2.1}$$

$$\mathcal{C}(\alpha, n) = \frac{\prod_{k=2}^{n} (k - 2\alpha)}{(n-1)!}, \ \mathcal{C}(\alpha, k) = \frac{\prod_{m=2}^{k} (m - 2\alpha)}{(k-1)!}, \ \mathcal{C}(\alpha, 1) a_1 = 1$$

$\mathcal{C}'(\alpha,n) = \dfrac{\prod_{k=2}^{n}(k+1-2\alpha)}{(n-1)!}$, $b_n = \int_0^1 t^n d\mu(t)$ *and* $\mu(t)$ *is a probability measure on* $[0,1]$.

P r o o f. By Theorem 1.1, $f \in \mathcal{R}_\alpha^u$ if and only if $\dfrac{D^{3-2\alpha}f}{D^{2-2\alpha}f} \in T$. Hence, $\dfrac{D^{3-2\alpha}f}{D^{2-2\alpha}f} = \int_0^1 \dfrac{d\mu(t)}{1-tz}$, for some probability measure $\mu(t)$ on $[0,1]$,

$$\frac{D^{3-2\alpha}f}{D^{2-2\alpha}f} = \sum_{n=0}^{\infty} b_n z^n, \quad \text{where} \quad b_n = \int_0^1 t^n d\mu(t).$$

Therefore,

$$D^{3-2\alpha}f = z + \sum_{n=2}^{\infty} \mathcal{C}'(\alpha,n)a_n z^n,$$

where $\mathcal{C}'(\alpha,n) = \dfrac{\prod_{k=2}^{n}(k+1-2\alpha)}{(n-1)!}$, $\quad n = 2,3,\ldots$

Now,

$$D^{2-2\alpha}f = z + \sum_{n=2}^{\infty} \mathcal{C}(\alpha,n)a_n z^n,$$

where $\mathcal{C}(\alpha,n) = \dfrac{\prod_{k=2}^{n}(k-2\alpha)}{(n-1)!}$, $\quad n = 2,3,\ldots$

Therefore,

$$\frac{D^{3-2\alpha}f}{D^{2-2\alpha}f} = \frac{z + \sum_{n=2}^{\infty} \mathcal{C}'(\alpha,n)a_n z^n}{z + \sum_{n=2}^{\infty} \mathcal{C}(\alpha,n)a_n z^n} = \sum_{n=0}^{\infty} b_n z^n. \tag{2.2}$$

Equating the like of coefficients, we obtain for $n = 2,3,\ldots$:

$$a_n = \frac{\sum_{k=1}^{n-1} \mathcal{C}(\alpha,k)a_k b_{n-k}}{\mathcal{C}'(\alpha,n) - \mathcal{C}(\alpha,n)},$$

with $\mathcal{C}(\alpha,1)a_1 = 1$. ∎

3. Main result

We now establish the Fekete Szegö inequality.

THEOREM 3.1. *Let* $\phi(z) = 1 + B_1 z + B_2 z^2 + \ldots$. *If* $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$ *is a universally prestarlike function of order* α, *then*

$$|a_3 - \mu a_2^2| \le \begin{cases} \dfrac{B_2 + (2-2\alpha)B_1^2 - (3-2\alpha)B_1^2\mu}{3-2\alpha}, & \mu \le \sigma_1 \\ \dfrac{B_1}{3-2\alpha}, & \sigma_1 \le \mu \le \sigma_2 \\ \dfrac{-B_2 - (2-2\alpha)B_1^2 + (3-2\alpha)B_1^2\mu}{3-2\alpha}, & \mu \ge \sigma_2, \end{cases}$$

where $\sigma_1 = \dfrac{(B_2 - B_1) + (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2}$, $\sigma_2 = \dfrac{(B_2 + B_1) + (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2}$. *The result is sharp.*

P r o o f. If $f \in \mathcal{R}_\alpha^u$, then there is a Schwartz function $w(z)$, analytic in Δ with $w(0) = 0$ and $|w(z)| < 1$ in Δ such that $\dfrac{D^{3-2\alpha}f}{D^{2-2\alpha}f} = \phi(w(z))$. Define the function $P_1(z)$ by $P_1(z) = \dfrac{1+w(z)}{1-w(z)} = 1 + c_1 z + c_2 z^2 + \ldots$. Since $w(z)$ is a Schwartz function, we see that $Re P_1(z) > 0$ and $P_1(0) = 1$. Define the function $P(z) = \dfrac{D^{3-2\alpha}f}{D^{2-2\alpha}f} = 1 + b_1 z + b_2 z^2 + \ldots$ Now, $P(z) = \phi\left(\dfrac{P_1(z)-1}{P_1(z)+1}\right)$, where

$$\frac{P_1(z)-1}{P_1(z)+1} = \frac{c_1 z + c_2 z^2 + \ldots}{2 + c_1 z + c_2 z^2 + \ldots} = \frac{1}{2}\left[c_1 z + z^2[c_2 - \frac{c_1^2}{2}] + z^3[c_3 - c_1 c_2 + \frac{c_1^3}{4}] + \ldots\right].$$

Hence, on simplification, we get

$$P(z) = 1 + \frac{B_1 c_1 z}{2} + \left[\frac{B_1}{2}\left(c_2 - \frac{c_1^2}{2}\right) + \frac{B_2 c_1^2}{4}\right] z^2 + \ldots$$

Therefore,

$$1 + b_1 z + b_2 z^2 + \ldots = 1 + \frac{B_1 c_1 z}{2} + \left[\frac{B_1}{2}\left(c_2 - \frac{c_1^2}{2}\right) + \frac{B_2 c_1^2}{4}\right] z^2 + \ldots$$

Equating the like coefficients, we get

$$b_1 = \frac{B_1 c_1}{2}, \tag{3.1}$$

$$b_2 = \frac{B_1}{2}\left(c_2 - \frac{c_1^2}{2}\right) + \frac{B_2 c_1^2}{4}. \tag{3.2}$$

Now, $\dfrac{D^{3-2\alpha} f}{D^{2-2\alpha} f} = 1 + b_1 z + b_2 z^2 + \dots$ From equation (2.2), we have

$$1 + \left[\mathcal{C}'(\alpha,2)a_2 - \mathcal{C}(\alpha,2)a_2\right] z + \Big[\mathcal{C}'(\alpha,3)a_3 - \mathcal{C}(\alpha,2)\mathcal{C}'(\alpha,2)a_2^2 - \mathcal{C}(\alpha,3)a_3$$

$$+ \left(\mathcal{C}(\alpha,2)a_2\right)^2\Big] z^2 + \dots = 1 + b_1 z + b_2 z^2 + \dots$$

Equating the coefficients of z and z^2 respectively and simplifying, we get

$$a_2 = b_1 \quad , \quad a_3 = \frac{b_2 + (2-2\alpha)b_1^2}{3-2\alpha}. \tag{3.3}$$

Applying equations (3.1) and (3.2) in (3.3), we get

$$a_2 = \frac{B_1 c_1}{2} \quad , \quad a_3 = \frac{1}{3-2\alpha}\left[\frac{B_1}{2}\left(c_2 - \frac{c_1^2}{2}\right) + \frac{B_2 c_1^2}{4} + (2-2\alpha)\frac{B_1^2 c_1^2}{4}\right].$$

Now,

$$\begin{aligned} a_3 - \mu a_2^2 &= \frac{1}{3-2\alpha}\left[\frac{B_1}{2}\left(c_2 - \frac{c_1^2}{2}\right) + \frac{B_2 c_1^2}{4} + (2-2\alpha)\frac{B_1^2 c_1^2}{4}\right] - \mu\frac{B_1^2 c_1^2}{4} \\ &= \frac{1}{3-2\alpha}\frac{B_1}{2}\left[c_2 - c_1^2\left[\frac{1}{2} - \frac{B_2}{2B_1} - (2-2\alpha)\frac{B_1}{2} + (3-2\alpha)\mu\frac{B_1}{2}\right]\right] \\ &= \frac{B_1}{2(3-2\alpha)}\left[c_2 - c_1^2 v\right], \end{aligned}$$

where

$$v = \left[\frac{1}{2} - \frac{B_2}{2B_1} - (2-2\alpha)\frac{B_1}{2} + (3-2\alpha)\mu\frac{B_1}{2}\right].$$

Now by an application of Lemma 1.1, if $\mu \le \sigma_1$,

$$|a_3 - \mu a_2^2| \le \frac{B_2 + (2-2\alpha)B_1^2 - (3-2\alpha)B_1^2\mu}{3-2\alpha},$$

where

$$\sigma_1 = \frac{(B_2 - B_1) + (2-2\alpha)B_1^2\mu}{(3-2\alpha)B_1^2}.$$

Now, if $\sigma_1 \leq \mu \leq \sigma_2$,

$$|a_3 - \mu a_2^2| \leq \frac{B_1}{3-2\alpha}.$$

Now, if $\mu \geq \sigma_2$,

$$|a_3 - \mu a_2^2| \leq \frac{-B_2 - (2-2\alpha)B_1^2 + (3-2\alpha)B_1^2\mu}{3-2\alpha},$$

where

$$\sigma_2 = \frac{(B_2+B_1) + (2-2\alpha)B_1^2\mu}{(3-2\alpha)B_1^2}.$$

If $\mu = \sigma_1$, then the equality holds in Lemma 1.1, if and only if

$$P_1(z) = \left(\frac{1}{2} + \frac{\lambda}{2}\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{\lambda}{2}\right)\frac{1-z}{1+z},\ 0 \leq \lambda \leq 1,$$

or one of its rotations. If $\mu = \sigma_2$, then

$$\frac{1}{P_1(z)} = \frac{1}{\left(\frac{1}{2} + \frac{\lambda}{2}\right)\frac{1+z}{1-z} + \left(\frac{1}{2} - \frac{\lambda}{2}\right)\frac{1-z}{1+z}}.$$

If $\sigma_1 < \mu < \sigma_2$, $P_1(z) = \dfrac{1+\lambda z^2}{1-\lambda z^2}$. To show that the bounds are sharp, we define the function $K_\alpha^{\phi_n}$ $(n = 2, 3, \ldots)$ by

$$\frac{D^{3-2\alpha}K_\alpha^{\phi_n}}{D^{3-2\alpha}K_\alpha^{\phi_n}} = \phi(z^{n-1}),$$

$K_\alpha^{\phi_n}(0) = 0$, $(K_\alpha^{\phi_n})'(0) = 1$ and function F_α^λ and G_α^λ $(0 \leq \lambda \leq 1)$ by

$$\frac{\left(D^{3-2\alpha}F_\alpha^\lambda\right)(z)}{\left(D^{2-2\alpha}F_\alpha^\lambda\right)(z)} = \phi\left(\frac{z(z+\lambda)}{1+\lambda z}\right),$$

$F_\alpha^\lambda(0) = 0$, $(F_\alpha^\lambda)'(0) = 1$ and similarly,

$$\frac{\left(D^{3-2\alpha}G_\alpha^\lambda\right)(z)}{\left(D^{2-2\alpha}G_\alpha^\lambda\right)(z)} = \phi\left(\frac{z(z+\lambda)}{1+\lambda z}\right)$$

$G_\alpha^\lambda(0) = 0$, $(G_\alpha^\lambda)'(0) = 1$. Clearly, the functions $K_\alpha^{\phi_n}, F_\alpha^\lambda, G_\alpha^\lambda \in \mathcal{R}_\alpha^u$. Also we write $K_\alpha^\phi := K_\alpha^{\phi_2}$. If $\mu < \sigma_1$ or $\mu < \sigma_2$, then the equality holds if and only if f is K_α^ϕ or one of its rotations. When $\sigma_1 < \mu < \sigma_2$, then the equality holds if and only if f is $K_\alpha^{\phi_3}$ or one of its rotations. If $\mu = \sigma_1$, then the equality holds if and only if f is F_α^λ or one of its rotations If $\mu = \sigma_2$ then the equality holds if and only if f is G_α^λ or one of its rotations. Hence the result follows. ∎

REMARK 3.1. If $\sigma_1 \leq \mu \leq \sigma_2$, then in view of Lemma 1.1, Theorem 3.1 can be improved. Let σ_3 be given by

$$\sigma_3 = \frac{B_2 + (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2}.$$

If $\sigma_1 \leq \mu \leq \sigma_3$, then

$$|a_3 - \mu a_2^2| + \left(\frac{(3-2\alpha)\mu B_1^2 - [(B_2 - B_1) + (2-2\alpha)B_1^2]}{(3-2\alpha)B_1^2}\right)|a_2^2| \leq \frac{B_1}{3-2\alpha}.$$

If $\sigma_2 \leq \mu \leq \sigma_3$, then

$$|a_3 - \mu a_2^2| + \left(\frac{-(3-2\alpha)\mu B_1^2 B_2 + B_1 + (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2}\right)|a_2^2| \leq \frac{B_1}{3-2\alpha}.$$

P r o o f. For $\sigma_1 \leq \mu \leq \sigma_3$, we have

$$\begin{aligned}
&|a_3 - \mu a_2^2| + (\mu - \sigma_1)|a_2^2| \\
&= \frac{B_1}{2(3-2\alpha)}|c_2 - vc_1^2| + \left(\mu - \frac{[(B_2 - B_1) + (2-2\alpha)B_1^2]}{(3-2\alpha)B_1^2}\right)\frac{B_1^2|c_1|^2}{4} \\
&= \frac{B_1}{2(3-2\alpha)}\left(\frac{(3-2\alpha)\mu B_1^2 - B_2 - B_1 - (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2}\right)\frac{B_1^2|c_1|^2}{4} \\
&= \frac{B_1}{(3-2\alpha)}\left[\frac{1}{2}|c_2 - vc_1^2| + \frac{1}{2}v|c_1|^2\right] \\
&= \frac{B_1}{(3-2\alpha)}\left[\frac{1}{2}\left[|c_2 - vc_1^2| + v|c_1|^2\right]\right].
\end{aligned}$$

Now, by using Lemma 1.1, we get $|a_3 - \mu a_2^2| + (\mu - \sigma_1)|a_2^2| \leq \dfrac{B_1}{(3-2\alpha)}$. Now, for $\sigma_2 \leq \mu \leq \sigma_3$, we have

$$\begin{aligned}
&|a_3 - \mu a_2^2| + (\sigma_2 - \mu)|a_2^2| \\
&= \frac{B_1}{2(3-2\alpha)}|c_2 - vc_1^2| + \left(\frac{B_2 + B_1 + (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2} - \mu\right)\frac{B_1^2|c_1|^2}{4} \\
&= \frac{B_1}{2(3-2\alpha)}\left(\frac{-(3-2\alpha)\mu B_1^2 + B_2 + B_1 + (2-2\alpha)B_1^2}{(3-2\alpha)B_1^2}\right)\frac{B_1^2|c_1|^2}{4} \\
&= \frac{B_1}{(3-2\alpha)}\left(\frac{1}{2}\left[|c_2 - vc_1^2| + (1-v)|c_1|^2\right]\right).
\end{aligned}$$

Now, by using Lemma 1.1, we get $|a_3 - \mu a_2^2| + (\sigma_2 - \mu)|a_2^2| \leq \dfrac{B_1}{(3-2\alpha)}$.
Hence the result follows.

■

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Department of Mathematics
Anna University Chennai, Chennai – 600025, INDIA

* *e-mail: shan@annauniv.edu*

** *e-mail: j_lourthumary@annauniv.edu*

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