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ON THE QUOTIENT FUNCTION EMPLOYED IN THE BLIND SOURCE SEPARATION PROBLEM

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Abstract

On the blind source separation problem, there is a method to use the quotient function of complex valued time-frequency informations of two observed signals. By studying the quotient function, we can estimate the number of sources under some assumptions. In our previous papers, we gave a mathematical formulation which is available for the sources without time delay. However, in general, we can not ignore the time delay. In this paper, we will reformulate our basic theorems related to the method of estimating the number of sources to be available for more general cases.

MSC 2010: 42C40, 94A12
Key Words and Phrases: quotient function, cumulative distribution function, blind source separation

1. Introduction

To treat blind source separation problem, in many cases, either statistical independence or statistical orthogonality of the sources has been assumed. Around 2000, papers as [4], [9], etc., considered the blind source separation problem under some independence of the windowed Fourier transforms of the sources in the time-frequency domain. To solve the blind source separation problem, as the first step, they try to detect the number of sources. The fundamental idea to detect the number of sources employed in [4], [9], [10], etc., is to consider the ratio of the windowed Fourier transforms of the two observed signals. In [1], [7] and [8], we gave a mathematical formulation for this estimation problem of the number of sources without time delay. In [5] and [6], we gave some remarks on our method for applications. Later, [2] and [3] treated the problem with time delay and gave an algorithm of the numerical experiment.

In this paper, we will reformulate our results in [1] so that we can apply to more general cases.

2. A method of blind source separation

Let $n \geq 2$ be an unknown integer and $x_k(t)$ be an observed signal of unknown sources $\{s_j\}_{j=1}^n$. We assume that observed signals $\{x_k\}$ are represented as

$$x_k(t) = \sum_{j=1}^{n} a_{jk} s_j(t - c_{jk}), \tag{1}$$

where a_{jk} are unknown real numbers and $c_{jk} > 0$.

A method of blind source separation is as follows:

1. Transfer x_1 and x_2 to complex valued continuous functions $X_1(t, \omega)$ and $X_2(t, \omega)$ in a time-frequency domain under suitable transformation.
2. Consider the quotient function $Q(t, \omega) = X_1(t, \omega)/X_2(t, \omega)$. By studying $Q(t, \omega)$, estimate the number of sources (say n) under some assumptions.
3. Take another $(n-2)$ observed signals and determine a_{jk}, c_{jk}, and the original sources s_j, $j, k = 1, \cdots, n$.

3. Mathematical formulation

Let $d, n \in \mathbf{N}$, $\mathbf{X} = \mathbf{R}^d, d \geq 2$, or $\mathbf{X} = \mathbf{C}^d, d \geq 1$, and $S_j(z), j = 1, 2, \cdots, n$, are linearly independent as complex valued continuous functions on $\mathbf{X}$. For $a_j, b_j \in \mathbf{R} \setminus \{0\}$ and $c_{jl} = (c_{jl,1}, c_{jl,2}, \cdots, c_{jl,d}) \in \mathbf{X}$, $l = 1, 2$, set

$$X_1(z; c_1) = \sum_{j=1}^{n} a_j S_j(z - c_{j1}), \quad X_2(z; c_2) = \sum_{j=1}^{n} b_j S_j(z - c_{j2}),$$

$$X_1(z;0) = \sum_{j=1}^{n} a_j S_j(z), \quad X_2(z;0) = \sum_{j=1}^{n} b_j S_j(z).$$

3.1. Complex valued quotient functions

Consider the quotient function

$$Q(z;c)=\frac{X_1(z;c_1)}{X_2(z;c_2)}=\frac{a_1S_1(z-c_{j1})+...+a_nS_n(z-c_{j1})}{b_1S_1(z-c_{j2})+...+b_nS_n(z-c_{j2})},\ Q(z;0)=\frac{X_1(z;0)}{X_2(z;0)}.$$

For $\eta > 0$, we define the following function $Q_\eta(z;c)$ which depends on the values of $\Im Q$, the imaginary part of Q:

$$Q_\eta(z;c) = \begin{cases} Q(z;c) & (|\Im Q(z;c)| < \eta), \\ 0 & (|\Im Q(z;c)| \geq \eta). \end{cases}$$

In the case of $c_{j1} = c_{j2} = 0$, $j = 1, \cdots, n$, $Q(z;c)$ takes real value $q_j \equiv a_j/b_j$ on

$$E_j = \{z \in \mathbf{X}; S_j(z) \neq 0, S_k(z) = 0, (k \neq j)\}.$$

Let us introduce some notations. Put

$$D_j = \{z \in \mathbf{X};\ S_j(z) \neq 0\},$$

$$D_{c_j} = D_j \bigcup \{z \in \mathbf{X}; z - c_{j1} \in D_j\} \bigcup \{z \in \mathbf{X}; z - c_{j2} \in D_j\},$$

$$D = \bigcup_{j=1}^{n} D_{c_j}, \quad E = \bigcup_{j=1}^{n} E_j.$$

For $M > 0$, we set

$$B(M) = \{z = (z_1, \cdots, z_d);\ |z_l| < M, l = 1, \cdots, d\},\ D(M) = D \cap B(M),$$
$$E_j(M) = E_j \cap B(M),\ E(M) = \cup_{j=1}^{n} E_j(M),\ E^c(M) = D(M) \setminus E(M).$$

If $c_{j1} = c_{j2} = 0$, $j = 1, \cdots, n$, $D = E$ and $E_j \neq \emptyset$, we can easily detect the number of $\{S_j\}$ by counting the number of elements of the image $Q_\eta(D) = Q(D) = Q(E)$.

3.2. Cumulative distribution function on R

We denote the Lebesgue measure of a measurable set $A \in \mathbf{X}$ by

$$\mu(A) = \int_A dz,$$

where dz is the Lebesgue measure on A and use the following notation:

$$\nu_c(A) = \mu(\{z \in A;\ \Im Q(z;c) = 0,\ Q(z;c) \neq 0\}).$$

We consider a function $(G_\eta(M;c))(x)$ that describes the distribution of values of $\Re Q_\eta(z;c)$, the real part of $Q_\eta(z;c)$. For $(G_\eta(M;c))(x)$ to be well-defined, we assume the following condition throughout this paper.

$$\nu_c(D(M)) > 0.$$

For $\eta > 0, M > 0$, and $x \in \mathbf{R}$, $(G_\eta(M;c))(x)$ is defined as follows:

$$(G_\eta(M;c))(x) = \frac{\mu(\{z \in D(M);\ \Re Q_\eta(z;c) < x,\ Q_\eta(z;c) \neq 0\})}{\mu(\{z \in D(M);\ Q_\eta(z;c) \neq 0\})}. \quad (2)$$

By the definition $(G_\eta(M;c))(x)$ is a monotone increasing function. Further we define

$$(G_0(M;c))(x) = \frac{\mu(\{z \in D(M);\ \Re Q(z;c) < x,\ \Im Q(z;c) = 0,\ Q(z;c) \neq 0\})}{\nu_c(D(M))}.$$

Similar to Theorem 1 in [1], we can prove

$$\begin{aligned} |(G_\eta(M;c))(x) - (G_0(M;c)(x)| &\leq \frac{\mu(\{z \in D(M);\ 0 < |\Im Q_\eta(z;c)| < \eta\})}{\mu(\{z \in D(M);\ Q_\eta(z;c) \neq 0\})} \\ &\equiv \beta_\eta(M;c) \quad (3) \end{aligned}$$

and $\lim_{\eta \to 0}(G_\eta(M;c))(x) = (G_0(M;c))(x)$.

3.3. In the case of $D = E$

First we consider the case that $c_{j1} = c_{j2} = 0$, $E_j(M) \neq \emptyset$, $j = 1, \cdots, n$ and $D = E$. In this case, $(G_\eta(M;0))(x)$ is the step function (see Fig. 1):

$$H_0(x) \equiv (G_\eta(M;0))(x) = \sum_{j=1}^{n} \frac{\nu_0(E_j(M))}{\nu_0(E(M))} Y(x - q_j),\ \ Y(x) = \begin{cases} 1 & (x > 0), \\ 0 & (x \leq 0). \end{cases}$$

Thus we can detect the number of $\{S_j\}$ by counting the number of the steps which appear on the graph of $(G_\eta(M;c))(x)$. In numerical analysis, the graph of $G'_\eta(M;c) = d(G_\eta(M;c))(x)/dx$, where the derivative is taken in the sense of distribution, is often used. Since the derivative of the Heaviside function $Y(x)$ is the delta function, we may count the number of the peaks which appear on the graph of $G'_\eta(M;c)$. In the following we assume $q_1 < q_2 < \cdots < q_n$.

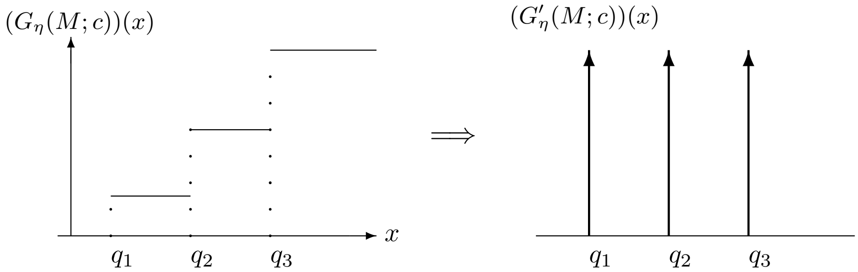

Fig. 1

Next we consider the case that $\sum_j (c_{j1}^2 + c_{j2}^2) \neq 0$; (that is, at least one c_{jk} does not vanish), $E_j(M) \neq \emptyset$, $j = 1, \cdots, n$, and $D = E$. Since we assume that $S_j(z)$ are continuous, $S_j(z) \approx S_j(z - c_{jk})$ for sufficiently small c_{jk}. Put $S_j(z - c_{jk}) - S_j(z) = \xi_{jk} e^{i\theta_{jk}}$, $\xi_{jk} > 0$, $\theta_{jk} \in [-\pi, \pi]$, $\xi_1 = \sum_{k=1}^n a_k \xi_{k1} e^{i\theta_{k1}}$ and $\xi_2 = \sum_{k=1}^n b_k \xi_{k2} e^{i\theta_{k2}}$. Then on the set $E_j(M)$,

$$\Re Q_\eta(z;c) - q_j = \Re\frac{\xi_1 + a_j S_j(z)}{\xi_2 + b_j S_j(z)} - \frac{a_j}{b_j} = \Re\frac{b_j\xi_1 - a_j\xi_2}{b_j(\xi_2 + b_j S_j)} = \Re\frac{\xi_1 - q_j\xi_2}{(\xi_2 + b_j S_j)}.$$

Define

$$H_c(x) = \sum_{j=1}^n \frac{\nu_c(E_j(M))}{\nu_c(E(M))} Y(x - q_j),$$

$$(g(M;c))(x) = \frac{\mu(\{z \in E(M); \Re Q(z;c) < x, \Im Q(z;c) = 0,\ Q(z;c) \neq 0\})}{\nu_c(E(M))}.$$

Then similar to Theorem 2 in [1], by using Lemma 2 in [1] we have

$$|G_0(M;c)(x) - g(M;c)(x)| \leq \frac{\nu(E^c(M))}{\nu_c(D(M))} \equiv \beta_0(M;c). \tag{4}$$

Since we assume that $D = E$, $\beta_0(M;c) = 0$. Thus by (3) and (4) we have

$$\begin{aligned} &|(G_\eta(M;c))(x) - (g(M;c))(x)| \\ &= |(G_\eta(M;c))(x) - (G_0(M;c))(x) + (G_0(M;c))(x) - (g(M;c))(x)| \\ &\leq \beta_\eta(M;c) + \beta_0(M;c) = \beta_\eta(M;c). \end{aligned}$$

At last, we consider the difference between $(g(M;c))(x)$ and $H_c(x)$. Put

$$\max_{z\in E_j} |Q_\eta(z;c) - q_j| = e_j,\ q_j^+ = q_j + e_j,\ q_j^- = q_j - e_j.$$

Assume $\gamma \equiv \max_j e_j < \min_{j\neq k} |q_j - q_k|/2$. Then similar to Proposition 2 in [1] we can prove that

$$(g(M;c))(x) = H_c(x), \quad x \in \mathbf{R} \setminus \bigcup_j [q_j^-, q_j^+],$$

$$H_c(x-\gamma) \leq (g(M;c))(x) \leq H_c(x+\gamma), \quad x \in \bigcup_j [q_j^-, q_j^+].$$

Thus putting

$$H_\pm(x) = \left(\sum_{j=1}^{n} \frac{\nu_c(E_j(M))}{\nu_c(E(M))} \pm \beta_\eta(M;c) \right) Y\left(x - q_j^\mp\right),$$

we have the following Theorem:

Theorem 1. *Let $D = E, \nu_c(E(M)) > 0$ and $E_j(M) \neq \emptyset,\ j = 1, \cdots, n$. Assume $\gamma < \min_{j\neq k} |q_j - q_k|/2$. Then the graph of the monotone increasing function $(G_\eta(M;c))(x)$ is contained in the closed domain $E = \{(x,y) \in \mathbf{R}^2;\ H_-(x) \leq y \leq H_+(x), 0 \leq y \leq 1\}$. (See the following stair like domain (Fig2̇, left)).*

A graph of $(G_\eta(M;c))(x)$ will be as in the right Fig. 2 and is in the stair like domain E on the left Fig. 2.

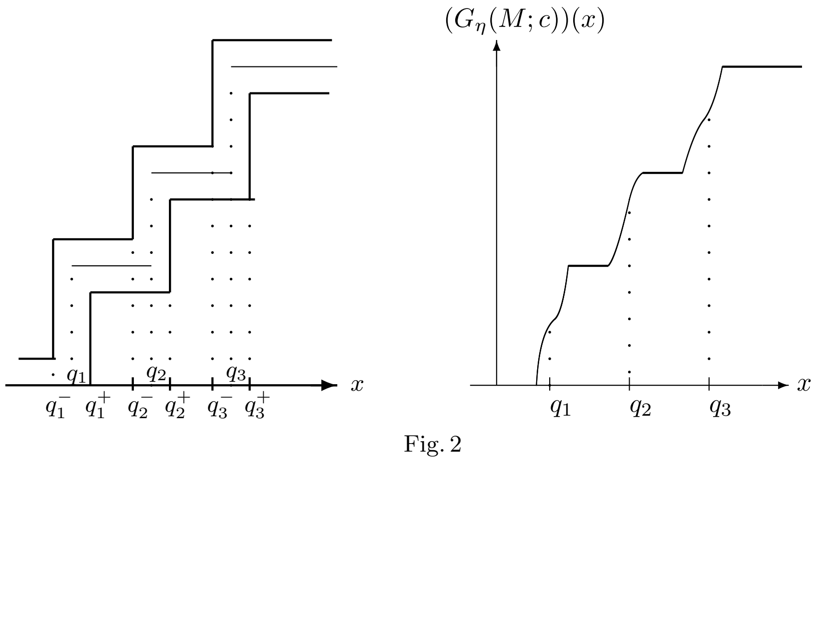

Fig. 2

REMARK. If $|\min_{j\neq k}|q_j - q_k| - 2\gamma|$ is so small, then it will be hard to see the steps on the graph of $(G_\eta(M;c))(x)$.

3.4. In the case of $D \neq E$

When $D \neq E$, then $\beta_0(M;c) \neq 0$ and e_j will be bigger. Thus the stair like domain in left Fig. 2 becomes larger (see Fig. 3). Put

$$\rho_\eta(M;c) = \beta_\eta(M;c) + \beta_0(M;c),$$

$$\tilde{H}_\pm(x) = \left(\sum_{j=1}^{n} \frac{\nu_c(E_j(M))}{\nu_c(E(M))} \pm \rho_\eta(M;c)\right) Y\left(x - q_j^\mp\right).$$

Then Theorem 1 will be restated as follows:

THEOREM 2. *Let $\nu_c(E(M)) > 0$ and $E_j(M) \neq \emptyset$, $j = 1, \cdots, n$. Assume $\gamma < \min_{j\neq k}|q_j - q_k|/2$. Then the graph of the monotone increasing function $(G_\eta(M;c))(x)$ is contained in the closed domain $E = \{(x,y) \in \mathbf{R}^2;\ \tilde{H}_-(x) \leq y \leq \tilde{H}_+(x), 0 \leq y \leq 1\}$.*

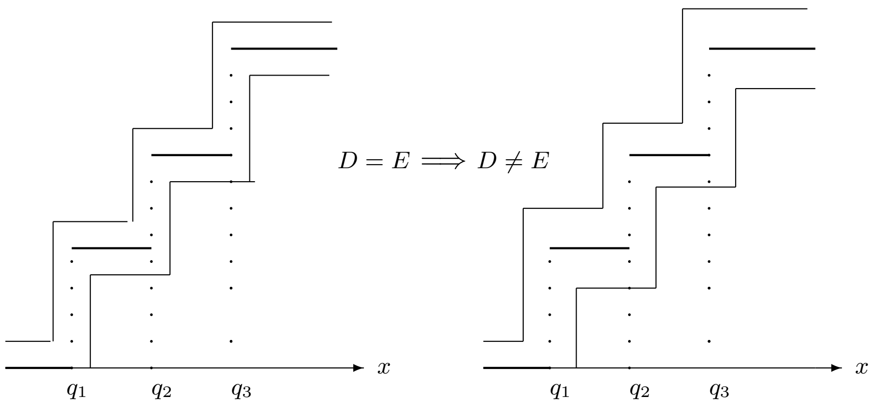

Fig. 3

4. A generalization

The assumption $\nu_c(E(M)) > 0$ in Theorem 1 and Theorem 2 is restrictive, in our previous papers we introduced the following space:

$$E_j(\delta;M) = \{z \in D(M);\ |b_k S_k(z)| \leq \delta |b_j S_j(z)|\ (k \neq j),\ \ S_j(z) \neq 0\}.$$

Roughly speaking, $E_j(\delta;M)$ describes the set of points where $S_j(z)$ is dominant over the other $S_k(z)$'s $(k \neq j)$. In this paper we introduce the following spaces:

$$\begin{aligned} E_{c_{j1}}(\delta) = \{z \in D(M); |b_k S_k(z-c_{k1})| &\leq \delta |b_j S_j(z)| \quad (k \neq j), \\ &\leq \delta_1 |b_j S_j(z-c_{j1})|, \quad S_j(z) \neq 0\}, \\ E_{c_{j2}}(\delta) = \{z \in D(M); |b_k S_k(z-c_{k2})| &\leq \delta |b_j S_j(z)| \quad (k \neq j), \\ &\leq \delta_2 |b_j S_j(z-c_{j2})|, \quad S_j(z) \neq 0\}, \end{aligned}$$

$$E_{c_j}(\delta;M) = E_{c_{j1}}(\delta) \bigcap E_{c_{j2}}(\delta),$$

$$E(\delta,c;M) = \cup_j E_{c_j}(\delta;M), \quad E^c(\delta,c;M) = D(M) \setminus E(\delta,c;M).$$

If $\delta_2 < 1/(n-1)$, then on the set $E_{c_j}(\delta;M)$, we have

$$\begin{aligned} &|Q_\eta(z) - q_j| = \left| \frac{\sum_{k=1}^n a_k S_k(z-c_{k1})}{\sum_{k=1}^n b_k S_k(z-c_{k2})} - q_j \right| \\ &\leq \frac{|\sum_{k=1}^n b_k(q_k S_k(z-c_{k1}) - q_j S_k(z-c_{k2}))|}{|b_j S_j(z-c_{j2})| - \sum_{k\neq j} |b_k S_k(z-c_{k2})|} \\ &\leq \frac{|\sum_{k=1}^n b_k(q_k S_k(z-c_{k1}) - q_j S_k(z-c_{k2}))|}{\delta/\delta_2 |b_j S_j(z)| - (n-1)\delta |b_j S_j(z)|} \\ &= \frac{|\sum_{k=1}^n b_k\left((q_k - q_j) S_k(z-c_{k1}) + q_j(S_k(z-c_{k1}) - S_k(z-c_{k2}))\right)|}{\delta/\delta_2 |b_j S_j(z)| - (n-1)\delta |b_j S_j(z)|} \\ &\leq \frac{(n-1)\delta_2 \Delta}{1-(n-1)\delta_2} + \Delta' \frac{\delta_2}{\delta} \frac{|\sum_{k=0}^n b_k(S_k(z-c_{k1}) - S_k(z-c_{k2}))|}{(1-(n-1)\delta_2)|b_j S_j(z)|}, \end{aligned}$$

where we put $\Delta = \max_{j,k} |q_j - q_k|$ and $\Delta' = \max_j |q_j|$. Further we put

$$C_j = \max_{z \in E_{c_j}(\delta;M)} \frac{|\sum_{k=0}^n b_k(S_k(z-c_{k1}) - S_k(z-c_{k2}))|}{|b_j S_j(z)|},$$

$$\gamma(\delta) = \frac{(n-1)\delta_2 \Delta}{1-(n-1)\delta_2}, \ \gamma(c) = \frac{\delta_2}{\delta} \frac{\Delta' \max_j C_j}{1-(n-1)\delta_2}, \quad \gamma(\delta,c) = \gamma(\delta) + \gamma(c).$$

If $S_j(z-c_{j1}) = S_j(z-c_{j2}), j = 1, \cdots, n$, then $\gamma(c) = 0$. Further if $c_{j1} = c_{j2} = 0, j = 1, \cdots, n$, then we can take $\delta = \delta_1 = \delta_2$ besides $\gamma(c) = 0$. Define

$$(g_\delta(M;c))(x) = \frac{\mu(\{z \in E(\delta,c;M); \Re Q(z;c) < x, \Im Q(z;c) = 0, \ Q(z;c) \neq 0\})}{\nu_c(E(\delta,c;M))},$$

$$H_{c,\delta}(x) = \sum_{j=1}^{n} \frac{\nu_c(E_{c_j}(\delta; M))}{\nu_c(E(\delta, c; M))} Y\left(x - q_j\right)).$$

Then similarly to Theorems 1 and 2 in [1], we can prove

$$\begin{aligned} &|G_\eta(M;c)(x) - (g_\delta(M;c))(x)| \\ &= |G_\eta(M;c)(x) - (G_0(M;c))(x) + (G_0(M;c))(x) - (g_\delta(M;c))(x)| \\ &\leq \beta_\eta(M;c) + \frac{\nu_c(E^c_{c_j}(\delta; M))}{\nu_c(D(M))} \equiv \rho_\eta(\delta, c; M), \end{aligned}$$

where $\beta_\eta(M;c)$ is defined by (3). Further similar to Proposition 2 in [1], under the assumption of $\gamma(\delta, c) < \min_{j \neq k} |q_j - q_k|/2$, we can prove that

$$(g_\delta(M;c))(x) = H_{c,\delta}(x), \quad x \in \mathbf{R} \setminus U[q; \gamma(\delta, c)],$$

$$H_{c,\delta}(x - \gamma(\delta, c)) \leq (g_\delta(M;c))(x) \leq H_{c,\delta}(x + \gamma(\delta, c)), \quad x \in U[q; \gamma(\delta, c)],$$

where we put $U[q;\gamma(\delta,c)] = \bigcup_j [q_j - \gamma(\delta,c), q_j + \gamma(\delta,c)]$. Define the two step functions by

$$(H^{\pm}_{c,\eta}(\delta; M))(x) = \left(\sum_{j=1}^{n} \frac{\nu_c(E_{c_j}(\delta; M))}{\nu_c(E(\delta, c; M))} \pm \rho_\eta(\delta, c; M) \right) Y\left(x - (q_j \mp \gamma(\delta, c))\right).$$

From the above-mentioned consideration, the following theorem is a generalization of Theorem 3 in [1]:

THEOREM 3. *Let $\eta > 0$, $1/(n-1) > \delta_2 > 0$ and let $(G_\eta(M;c))(x)$ be the function defined by* (2). *We assume that δ and δ_2 are chosen so small that $\gamma(\delta, c) < \min_{j \neq k} |q_j - q_k|/2$ is satisfied. Further, we assume $\nu_c(E(\delta, c; M)) > 0$. Then the graph of the monotone increasing function $(G_\eta(M;c))(x)$ is contained in the closed domain:*

$$\{(x,y) \in \mathbf{R}^2;\ (H^-_{c,\eta}(\delta, c; M))(x) \leq y \leq (H^+_{c,\eta}(\delta, c; M))(x),\ 0 \leq y \leq 1\}.$$

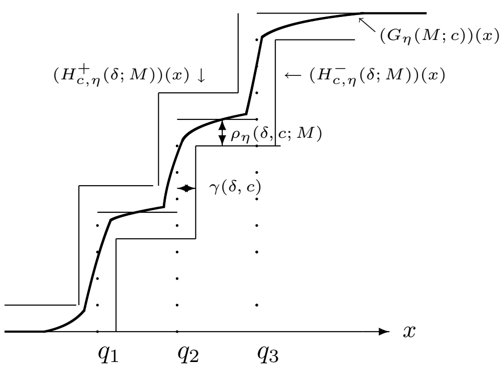

Fig. 4

When $X = \mathbf{R}^2$ and $c_{j1} = c_{j2} = 0, j = 1, \cdots, n$, then this theorem is just Theorem 3 in [1].

3.4. Remarks on Theorem 3

1. $\gamma(\delta)$ is an increasing function in δ.
2. For fixed δ, put $F(n) = \gamma(\delta)$. Since $F'(n) = \delta\Delta/(1-(n-1)\delta)^2 > 0$, $\gamma(\delta)$, $\gamma(c)$ and $\gamma(\delta; c)$ are increasing functions in n for fixed δ.
3. If $S_j(z - c_{j1}) \approx S_j(z - c_{j2}), j = 1, \cdots, n$, then $\gamma(c) \approx 0$. Further if $c_{jk} \approx 0, j = 1, \cdots, n, k = 1, 2$, then we can take $\delta_1 \approx \delta \approx \delta_2$ besides $\gamma(c) \approx 0$.
4. $\nu_c(E^c(\delta, c; M))$ is a decreasing function in δ.
5. When q_j is close to the neighbor ones, it will be difficult to find a step in the graph of $(G_\eta(M; c))(x)$ or we may take two peaks for one peak in the graph of $(G'_\eta(M; c))(x)$. Thus it is preferable that Δ is not small.
6. In general, Δ' is not small. Therefore, $S_j(z - c_{j1}) \approx S_j(z - c_{j2}), j = 1, \cdots, n$, are necessary to $\gamma(c) \approx 0$.

4. Remark for applications

To estimate the number of sources, both $\gamma(\delta, c)$ and $\rho_n(\delta, c; M)$ are expected to be small enough. By remarks 1, 2 and 5 in §3.4, since Δ is expected to be suitabley large, we have to take δ so small if we have many sources (i.e. n is large). This means, for large n, we can estimate the number of sources only the case that there exist domains such that each S_j dominates over the other sources. Except the trivial cases, it will be hard to estimate the number of sources when we have many sources. Further, in general, we can not ignore the time delay; that is, at leaset one c_{jk} does not vanish. But by remark 6 in §3.4, $S_j(z - c_{j1}) \approx S_j(z - c_{j2}), j = 1, \cdots, n$, are expected.

As an example, we will consider the case that two observed signals are given by (1). To transform the sources in the time-frequency domain, we will take an wavelet transformation. The continuous wavelet transform $W_\psi s(t, \omega)$ of $s \in L^2(\mathbf{R})$ is defined by

$$W_\psi s(t, \omega) = |\omega|^{-1/2} \int_{\mathbf{R}} s(x) \overline{\psi(\frac{x-t}{\omega})} dx,$$

where ψ is a wavelet function (see [1], for example). Thus, when we consider wavelet transforms of sources with time delay, we have to study the quotient function such as

$$Q(t,\omega) = \frac{\sum_{j=1}^{n} a_j S_j(t - c_{j1,1}, \omega)}{\sum_{j=1}^{n} b_j S_j(t - c_{j2,1}, \omega)}.$$

This is the case that $X = \mathbf{R}^2_{(t,\omega)}$ and $c_{jk} = (c_{jk,1}, 0)$.

For a numerical experiment, see [2] and [3] for example.

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