FINANCIAL EDUCATION THROUGH MATHEMATICS AND IT CURRICULA: POCKET MONEY MANAGEMENT

Iordanka Gortcheva

Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences
gorcheva@math.bas.bg

Abstract: *Mathematics and IT classes in the Bulgarian school provide various opportunities for developing students' logical, mathematical, and technological thinking. Being an important part of mathematical literacy, financial literacy can be systematically built in the frame of national mathematics and IT curricula. Following that objective, exemplary word problems have been designed to make middle and high school students discover elementary truths in finances. Not only is their topic of managing pocket money attractive to both students and parents, but their very content impels the students to reason out real life financial situations. Using spreadsheets for calculations and representations helps the students to experience how compound interest works. Such practice improves their planning and decision-making skills in finances and is highly proactive towards their financial responsibility. Work on word problems of the kind is also beneficial to the students from the vocational schools of banking and finances. It provides them with valuable experience for their future internship and jobs.*

Key words*: Financial literacy, Mathematics curriculum, IT curriculum, Compound interest, Spreadsheets, Pocket money, Word problems*

An allowance resembles a salary that teaches children to plan their purchases.
M. Ruckenstein

1. Introduction

Achieving financial security and economic independence from both a personal and a social perspective is strongly related to mathematical knowledge, intuition, logical thinking, and decision-making skills. For many adults financial literacy and responsibility have been acquired through a painful process of late payments on their credit cards and loans. However, K-12 students may start their eye-opening journey into financial education when still in school. Ruckenstein, who studies children's consumption attitude [1], grounds her research in Maurer's concept that money provides “a universal yardstick against which to measure and evaluate the universe of objects, relations, services, and persons” [2]. The management of personal money is probably “the most common context in which ordinary people are faced with sophisticated quantitative issues” [3]. Therefore, in order to meet the needs of society, the teachers, educators, and parents ought to bring up financially literate and responsible generations.

Mathematics and IT curricula provide various opportunities which are to be creatively used for the purposes of financial education. Ruckenstein shows, that small children are vividly interested in their personal pocket money budgeting. In Bulgarian mathematics curriculum, pocket money has already become a topic in some Grade 1 word problems [4]. In Grade 6, Bulgarian middle school students are introduced to more substantial financial ideas. Since they already operate with percent and exponents of rational numbers, they can solve some word problems on compound interest [5, pp. 84-88]. Further, based on their knowledge of geometric sequence and exponential function [6, pp. 25-29], [7, p. 9-10], in Grade 11 the students are able to elaborate on real financial situations.

To make such problem solving activities more frequent, spreadsheets could be used in Grade 6 mathematics classes. Also, financial problems could be modeled in Grade 6 IT classes: IT curriculum allows using spreadsheets from Grade 5 onwards [8]-[10]. As a result, the students can gain practical experience in compound interest and other financial topics and improve their IT skills.

Collecting money obviously makes spending it less desirable.
M. Ruckenstein

2. From collecting to saving

From both financial and educative perspectives, children's pocket money regularly persists in students', parents', and experts' discussions. This is a reason to intertwine it in problems accessible for students' mathematical and IT knowledge. The word problem that follows is designed for sixthgraders, but it can appeal to many high school students as well:

Problem 1. During her school years, every day from September 15th, 1999 up to September 14th, 2011 Peggy's parents gave her $1 pocket money, except on the leap years' February 29th, when they took her to a restaurant.

Instead of spending her daily dollars, the first grader Peggy collected them in a piggy bank. A year later she heard that money in banks brings money, emptied her piggy bank and asked her parents to put the collected $365 in a real bank. Thus on September 15th, 2000 they opened for her a 5% APR one-year deposit, automatically compounded and renewed on maturity on September 15th. Each year on that date Peggy added a next sum of $365.

(1.A) How much money did grown-up Peggy have in her deposit on September 15th, 2011?

(1.B) Let us assume that Peggy's bank will keep the same 5% APR on one-year deposits long enough. From September 15th, 2011 on Peggy does not receive any more pocket money from her parents. Nevertheless, she continues to save $1 per day, making her daily cup of coffee at home instead of buying it. Thus each year on September 15th she continues to put $365 in

her one-year deposit. How much money will she have in that deposit on September 15th, 2060?

(1.C) According to the tax legislation, since January 1st, 2013 from each deposit on maturity the banks withhold a tax, equal to 10% of the interest gained. Suggest a way to calculate the amount of money Peggy will have in her deposit on maturity dates after the tax deduction.

To answer the questions above, spreadsheets are not by all means needed. However, they allow the students to visualize the calculations of compound interest year by year and display other information as well. Using a spreadsheet to answer Question **(1.A)** takes off the burden of calculating percent and makes the problem accessible for the sixth graders (Table 1):

Table 1. Question **(1.A)**: Detailed visualization of compound interest

End of Grade	Year of Deposit	Sept 15th,	Principal ($)	Interest ($)	Interest Sum ($)	Installments Sum ($)
1	0	2000	365.00	0.00	0.00	365.00
2	1	2001	748.25	18.25	18.25	730.00
3	2	2002	1 150.66	37.41	55.66	1 095.00
4	3	2003	1 573.20	57.53	113.20	1 460.00
5	4	2004	2 016.86	78.66	191.86	1 825.00
6	5	2005	2 482.70	100.84	292.70	2 190.00
7	6	2006	2 971.83	124.13	416.83	2 555.00
8	7	2007	3 485.42	148.59	565.42	2 920.00
9	8	2008	4 024.70	174.27	739.70	3 285.00
10	9	2009	4 590.93	201.23	940.93	3 650.00
11	10	2010	5 185.48	229.55	1 170.48	4 015.00
12	11	2011	5 809.75	259.27	1 429.75	4 380.00

When Question **(1.B)** is to be answered, creating a rather long spreadsheet and scrolling it down may be boring for the students. The purpose of the spreadsheet is to visualize the effects of compound interest and allow the students to analyze the results. It will be more intriguing, if answering Question **(1.B)**, the students take into account the information Question **(1.C)** contains. What they have to do is to use APR 4.5% from year 2013 on (Table 2). Since Table 1 and Table 2 are excerpts from the same spreadsheet, to distinguish the results obtained for the 4.5% APR, italics are used.

Unlike Table 1, in Table 2 the leftmost column shows Peggy's age. The last two columns are meant to make the students compare the amount of money deposited with the interest earned. The students can observe that 28 years after the deposit has been opened, the accumulated interest is already greater than the deposited sum; in 43 years it is doubled; in 61 years – tripled, etc. Going further, they make conjectures and seek theoretical explanations. Such activities support inquiry-based learning and are highly proactive towards students' mathematical and financial literacy.

Table 2. A combined answer to Questions **(1.B)-(1.C)** via spreadsheet

Peggy's Age	Year of Deposit	Sept 15th,	Principal ($)	Interest ($)	Interest Sum ($)	Installments Sum ($)
20	12	2012	6 465.24	290.49	1 720.24	4 745.00
21	*13*	*2013*	*7 121.17*	*290.94*	*2 011.17*	*5 110.00*
22	*14*	*2014*	*7 806.63*	*320.45*	*2 331.63*	*5 475.00*
23	*15*	*2015*	*8 522.93*	*351.30*	*2 682.93*	*5 840.00*
24	*16*	*2016*	*9 271.46*	*383.53*	*3 066.46*	*6 205.00*
25	*17*	*2017*	*10 053.67*	*417.22*	*3 483.67*	*6 570.00*
…						
36	*28*	*2028*	*21 367.66*	*904.42*	*10 782.66*	*10 585.00*
…						
51	*43*	*2043*	*48 938.64*	*2 091.69*	*32 878.64*	*16 060.00*
…						
61	*53*	*2053*	*80 485.41*	*3 450.16*	*60 775.41*	*19 710.00*
…						
68	*60*	*2060*	*112 456.52*	*4 826.91*	*90 191.52*	*22 265.00*

Children gradually become adults and they need to be prepared for economic realities.
M. Ruckenstein

3. Loans as facts of life

As Braunstein and Welch point out, "financial literacy deficiencies can affect an individual's or family's day-to-day money management and ability to save for long-term goals" [11]. The authors emphasize how important "the timing and format of training, as well as human traits" are and warn that "ineffective money management can result in behaviors that make consumers vulnerable to severe financial crises". Therefore, it is worth mathematics and IT teachers' efforts to adapt the traditional textbook problems to realities of life.

Preparing financially literate students does not mean just teaching them to calculate compound interest. Their understanding that compound interest makes both deposits and loans grow is already a sign for financial maturity. The students also learn that money transmission processes are essential for economic development and social wealth, as well as for individuals' well-being: deposits and loans are the two sides of the same coin and interest is the price paid for using them. Reversing the plot of **Problem 1** reveals to the students what actions their parents often take to secure the family:

<u>Problem 2.</u> While Peggy was at school, every day her parents gave her \$1 pocket money. This lasted from September 15th, 1999 up to September 14th, 2011. The only exceptions occurred in leap years: each February 29th they took her to a restaurant.

To provide the \$1daily allowance, Peggy's parents intended to take a 12-year loan of \$4 380 at 5% interest rate. They agreed with their credit institution to make payments once a year, starting from September 1st, 2000 and ending on September 1st, 2011. Before signing the loan contract, they examined the following options of loan amortization and made the loan payments charts:

(2.A) Each year to make equal payments, i.e. to sign a *constant-payment* loan. What did the chart of such loan payments look like?

(2.B) Each year to pay equal amounts of the loan principal, i.e. to sign a *constant-principal* loan. What did the chart of such loan payments look like?

(2.C) Compare the two charts and explain which one you would choose.

(2.D) Eventually Peggy's parents signed a 12-month constant-payment loan of \$4 380 at 5% interest rate. What did the chart of their loan amortization look like? What pros and cons of their decision can you point out?

The above models of loan amortization are strongly idealized – for example, no bank fees are included. However, they give the students an idea of the interest accumulation on loans.

Constructing the constant-payment loan chart for Questions **(2.A)** and **(2.D)** requires yearly payments *V* to be calculated in advance. Grade 11 students are prepared to do that: since the loan principal *K*, the number *n* of the periods, and the interest rate are given, they only apply their textbook formula $V = K\dfrac{q^n(q-1)}{q^n-1}$ [6], where (q -1) is the APR in decimal representation.

The purpose of **Problem 2** is not merely to construct possible payment charts but to make a decision which one is more profitable for the borrower. Therefore the students are to figure out themselves what auxiliary information from the spreadsheet can help. The total interest paid on the loan seems to be a good criterion for comparison (Tables 3 – 5).

The constant-principal loan payments (Table 4) in **(2.B)** are a well-known financial practice [12]-[14], not yet described in the Bulgarian middle and high school mathematics textbooks. Constructing such payments charts is simple enough to be accessible for the sixth graders: the constant-principal component is just the quotient $\dfrac{K}{n}$. Further, as Table 4 illustrates, the size of the payments is the sum of the interest owed on the principal and the above mentioned component. Moreover, the total interest paid on the loan in **(2.B)** is less than in **(2.A)**.

Table 3. Question **(2.A)**: $4 380, 12-year *constant-payment* loan amortization

End of Grade	Year of Loan	Sept 1st,	Principal ($)	Interest ($)	Due Sum ($)	Payment ($)
1	1	2000	4380.00	219.00	4 599.00	494.18
2	2	2001	4 104.82	205.24	4 310.06	494.18
3	3	2002	3 815.88	190.79	4 006.68	494.18
4	4	2003	3 512.50	175.62	3 688.12	494.18
5	5	2004	3 193.94	159.70	3 353.64	494.18
6	6	2005	2 859.46	142.97	3 002.43	494.18
7	7	2006	2 508.25	125.41	2 633.66	494.18
8	8	2007	2 139.48	106.97	2 246.46	494.18
9	9	2008	1 752.28	87.61	1 839.89	494.18
10	10	2009	1 345.71	67.29	1 413.00	494.18
11	11	2010	918.82	45.94	964.76	494.18
12	12	2011	470.58	23.53	494.11	494.11
Loan:	$ 4 380.00				Total Interest Paid:	$ 1 550.09

The analysis of the loan amortization charts **(2.A)** and **(2.B)** shows that if Peggy's parents had chosen **(2.B)**, being attracted by the lesser interest, they would have taken into account the burden of higher payments during the first several years (Tables 3 – 4). This situation teaches the students to consider all details whenever a financial decision is to be made.

As **Problem 2** illustrates, sometimes knowledgeable and attentive prospective borrowers create amortization plans which work better for them than the ones the credit institutions offer. The next task is both a practice for students' mathematical knowledge and a challenge for their communication and presentation skills:

Suppose that a family are going to take the same loan as Problem 2 describes. The banker offers them conditions from (2.A) only. The family who have just found out that (2.B) is better for them, are to convince the banker to accept (2.B) instead of (2.A). How can they justify their choice?

Comparison between **(2.A)** and **(2.B)** undoubtedly shows that the bank profits from choosing condition **(2.A)**. However, the better the loan contract conditions for the borrower, the smaller the risk for the bank. Speculations of the sort sound reasonable and may help the family to negotiate their condition.

Table 4. Question **(2.B)**: $4 380, 12-year *constant-principal* loan amortization

End of Grade	Year of Loan	Sept 1st ,	Principal ($)	Interest ($)	Payment on Principal ($)	Payment ($)
1	1	2000	4 380.00	219.00	365.00	584.00
2	2	2001	4 015.00	200.75	365.00	565.75
3	3	2002	3 650.00	182.50	365.00	547.50
4	4	2003	3 285.00	164.25	365.00	529.25
5	5	2004	2 920.00	146.00	365.00	511.00
6	6	2005	2 555.00	127.75	365.00	492.75
7	7	2006	2 190.00	109.50	365.00	474.50
8	8	2007	1 825.00	91.25	365.00	456.25
9	9	2008	1 460.00	73.00	365.00	438.00
10	10	2009	1 095.00	54.75	365.00	419.75
11	11	2010	730.00	36.50	365.00	401.50
12	12	2011	365.00	18.25	365.00	383.25
Loan:	$ 4380.00				Total Interest Paid:	$1 423.50

Question **(2.D)** is designed to shed light on how contrary to the deposits, shorter terms of keeping loans favor the owners (Table 5):

Table 5. Question **(2.D)**: $4 380, 12-month *constant-payment* loan amortization

Begins Grade	Month of the Loan	Payments Date	Principal ($)	Interest ($)	Due Sum ($)	Payment ($)
1	1	1999-10-01	4380.00	18.25	4 398.25	374.96
	2	1999-11-01	4 023.29	16.77	4 040.06	374.96
	3	1999-12-01	3 665.10	15.27	3 680.37	374.96
	4	2000-01-01	3 305.41	13.77	3 319.18	374.96
	5	2000-02-01	2 944.22	12.27	2 956.49	374.96
	6	2000-03-01	2 581.53	10.76	2 592.29	374.96
	7	2000-04-01	2 217.33	9.24	2 226.57	374.96
	8	2000-05-01	1 851.61	7.72	1 859.32	374.96
	9	2000-06-01	1 484.36	6.19	1 490.55	374.96
	10	2000-07-01	1 115.59	4.65	1 120.24	374.96
	11	2000-08-01	745.28	3.11	748.38	374.96
	12	2000-09-01	373.42	1.56	374.98	374.98
Loan:	$ 4380.00				Total Interest Paid:	$ 119.54

The issues **Problem 2** raises motivate the students to actively learn and apply mathematics. The results of such activities can spread further, when surprisingly the parents start listening to their children's financial opinions.

Children do recognize the opportunities for social transformations that money opens up.
M. Ruckenstein

4. Concluding remarks

The word problems herein discussed offer the middle and high school students opportunities to experience financial situations, which are close to their and their families' needs. They give the students a taste for inquiry in the financial world with the help of spreadsheets. The problems are particularly beneficial to the students from the vocational schools of banking and finances. Mathematical modeling and problem solving of the kind prepares them for a successful start of their internship and real financial careers.

As Thomas and Brown suggest, teaching is not necessary for learning to occur. Immersed in modern digital and social networks environments, present day students "need interest and passion to explore and learn" [15]. With basic mathematical concepts in mind, even after graduating from school, they will still be capable of exploring and learning about finances. Thus long after the textbook mathematical formulae have been forgotten, they will not be afraid to make decisions and take financial responsibilities far beyond the scope of their students' pocket money.

Any society strongly benefits from financially literate and accountable people. Framed in mathematics and IT curricula financial education can do even more: convince the students that investments in knowledge lead to social and personal prosperity.

References

1. Ruckenstein, M. Time Scales of Consumption: Children, Money and Transactional Orders. *Journal of Consumer Culture*. 2010, 10(3), 383-404.
2. Maurer, B. The Anthropology of Money. *Annual Review of A*nthropology. 2006, 35, 15-36.
3. Steen, L. A. (ed.) Mathematics and Democracy: A Case for Quantitative Literacy. Princeton, NJ: NCED. 2001.
4. Bogdanova, M. A., Nikova, K. S. & Georgieva, N. D. *Mathematics for Grade 1*. Sofia, Bulgaria: Bulvest 2000 Publishing House. 2002. (in Bulgarian)
5. Lozanov, C., Vitanov, T. & Kalcheva, A. *Mathematics for Grade 6*. Sofia, Bulgaria: Anubis Publishing House. 2008. (in Bulgarian)
6. Lozanov, C., Vitanov, T. & Nedevski, P. *Mathematics for Eleventh Grade: Advanced Course*. Sofia, Bulgaria: Anubis. 2001. (in Bulgarian)
7. Kyurkchiev, N. *Selected Topics in Applied Mathematics of Finance.* Sofia, Bulgaria: Prof. Marin Drinov Academic Publishing House. 2012. (in Bulgarian)

8. Dobreva, M. et al. *The Computer in My World: Information Technologies for Grade 5.* Sofia, Bulgaria: Anubis. 2006. (in Bulgarian)
9. Dobreva, M. et al. *I Learn and Create with a Computer: Information Technologies for Grade 6.* Sofia, Bulgaria: Anubis. 2007. (in Bulgarian)
10. Stefanova, E. et al. *You and the ICT: Information Technologies for Grade 7.* Sofia, Bulgaria: Anubis. 2007. (in Bulgarian)
11. Braunstein, S. & Welch, C. Financial Literacy: An Overview of Practice, Research and Policy. *Federal Reserve Bulletin*. 2002, 88(11), pp. 445-457.
12. Yovkova, Y. & Petkov, B. *Financial Mathematics.* 2nd ed. Sofia, Bulgaria: Nova Zvezda. 2005. (in Bulgarian)
13. Ainsworth, P. et al. *Introduction to Accounting: An Integrated Approach*. Boston, MA: McGraw-Hill. 2000.
14. Brigham, E. F., Gapenski, L. C. & Daves, P. R. *Intermediate Financial Management.* 6th ed. Orlando, FL: The Dryden Press. 1999.
15. Thomas, D. & Brown, J. S. *A New Culture of Learning: Cultivating the Imagination for a World of Constant Change.* CSIPP. 2011.

ФИНАНСОВО ОБУЧЕНИЕ ЧРЕЗ УЧЕБНОТО СЪДЪРЖАНИЕ ПО МАТЕМАТИКА И ИТ: УПРАВЛЕНИЕ НА ДЖОБНИТЕ ПАРИ

Йорданка Горчева

Институт по математика и информатика към БАН
gorcheva@math.bas.bg

Резюме: *Часовете по математика и ИТ в българското училище предоставят разнообразни възможности за развиване на логическото, математическото и технологичното мислене на учениците. Финансовата грамотност, разглеждана като част от математическата, може да бъде систематически изграждана в рамките на учебното съдържание по математика и ИТ. Следвайки тази цел, предлагам примерни задачи, които позволяват на учениците от средния и горния училищен курс да откриват самостоятелно азбучни за финансовата област истини. Интерес представляват както темата за джобните пари, така и съдържанието на задачите, тъй като подтикват учениците да разсъждават върху реални финансови ситуации. Използването на електронни таблици за получаване и представяне на резултатите спомага те да добият реална представа за ефекта на сложната лихва. Това подобрява уменията им за финансово моделиране, планиране и вземане на решения и се отразява благотворно на финансовата им отговорност. Предложените задачи са полезни и за професионалната подготовка на учениците от финансово-стопанските гимназии и могат да допринесат за успешния старт на стажовете и кариерата на бъдещите специалисти.*