# DEVELOPING STUDENTS' MATHEMATICAL THINKING AND ALGORITHMIC CULTURE IN IT CLASSES

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***Abstract:*** *National curriculum IT classes in Bulgarian schools put specific demands before students, teachers, and educators. The many facets of IT allow embedding some amusing mathematical problems as a subject of class projects and discussions. Dynamic visualization of such problems and varying their numerical data provide their "on click" simulation. When possible to be implemented, this approach creates inquiry-based learning environment which boosts students' mathematical thinking, inventiveness, and exploratory spirit. Recently mathematics and IT classes have become the major places to set up mathematical literacy and algorithmic culture of Bulgarian students. Modeling witty mathematical problems by dynamic geometry software supports that process both in an effective and attractive way.*

***Key words***: *Mathematical thinking, Algorithmic culture, Dynamic geometry, Snail problem, Inquiry-based learning*

## 1. Introduction

For centuries the general public has been regarding mental solving of tricky word problems as a sign of mathematical ingenuity. Cracking such problems has required more than doing arithmetic right: agile mind, logic, number sense, insight. According to Paulos [1. p.xiii], people with "a traditional computational view of mathematics" often do not recognize it behind logic and common sense. About 30 years ago Edward David pointed out in widely quoted reports [2], [3] that contemporary high technology is in its essence mathematical technology. Thus neither computing devices, nor information technologies can replace mathematical thinking in approaching modern theoretical and practical problems of our times. Therefore, students can only benefit from school activities implemented to help them mature mathematically. Usiskin [4], Confrey & Maloney [5], Ulm [6], and many others emphasize the necessity of building students' mathematical literacy. Here, some popular amusing word problems may just come in handy. In seemingly trivial situations they describe, there is still room for new perspectives, questions, and applications. As a result such problems alter mathematics and IT classes monotony and create an opening for inquiry-based learning.

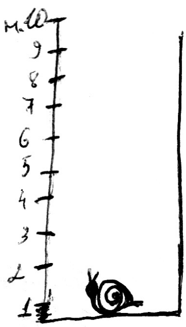
## 2. One old mathematical problem and the troubles caused to students

The problem for the snail crawling up and down in a well is fairly wide spread. Nevertheless, to solve it, for years different schoolchildren have sought professional mathematicians' help on the Internet. What I did not expect was that the same problem would cause troubles to adults. These were undergraduate students, enrolled in a university program for primary school teachers. The snail problem was given to the students by Gortcheva [7] and was formulated as follows:

***On a hot summer night in late June, the Tiny Snail found itself at the bottom of a 10-meter well. At dawn of July 1st it began its journey up. Every day it crawled 2 meters up and every night it slid 1 meter down. On which date did the Tiny Snail reach the edge of the well?***

According to Hughes-Hallett [8], "Learning mathematics generally involves two steps: learning mathematical principles and identifying mathematics in a context. Although students find the first step hard, they find the second step harder still." My students, who were prospective primary school teachers sought a way to represent the situation described in the problem both numerically and graphically. The success was not quite even: the chart in *Fig. 1-a* exhibits some artistic value, but shows how unsure its author felt.

**a)**




**b)**

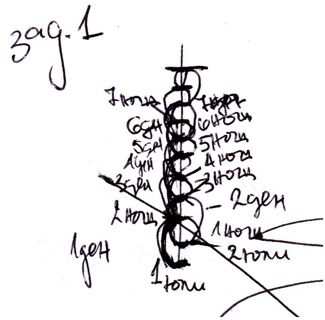



*Figure 1 - a, b. Charts to Problem 1, made by two pre-service primary school teachers*

*Figure 1-b* shows that the student began reasoning the problem successfully. Her achievement in finding the 1-meter-increment of the snail's day-and-night pattern of motion was undoubted. The student was very close to the final answer, but failed (or felt bored) in keeping track of the height reached by the snail each successive day.

Compared to the cases above, there were students, who did not obtain the right answer initially, but succeeded at a later stage. *Figure 2* demonstrates a higher level of speculations where the student made a conjecture about the date the snail reached the edge of the well. Apparently he doubted if his result made sense, checked it and crossed out his calculations to start again.

The experience drawn on students' mistakes is essential from educational point of view. As a whole, future primary school teachers deal freely with problem's numerical data, which have been intentionally selected as simple as possible. Future primary school teachers are not required, as Usiskin [4] says, to "be able to duplicate the calculator or computer." But they are required to possess "the ability to understand what the results of the calculations mean and to verify whether the calculations are correct to within some bounds of correctness". The prospective teachers were expected to delve into problem context, make a relevant mathematical model, and apply it. Therefore, purposeful training of pre- and in-service primary school teachers in mathematical modeling is to be a major curricular activity, which is consistently supported by Pollak [9], Doerr [10], Manev et al. [11], Gortcheva [7], etc.

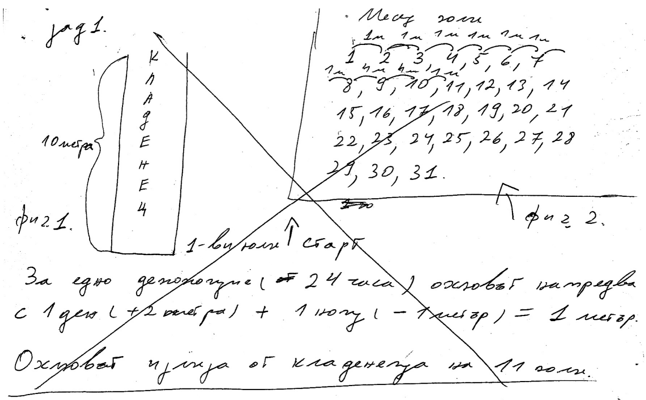



*Figure 2. A more sophisticated student reasoning: making a conjecture*

## 3. Dynamic geometry software and its impact on students

For students who have poorly imagined the snail problem situation, visualization through dynamic geometry software just comes in handy. Changing the numerical data and analyzing the results can positively influence their mathematical literacy. To illustrate the snail problem, I have written a GeoGebra [1] worksheet with a respective algorithm implemented. It is a ready-to-use visualization tool for teachers and students and to adjust it to a specific lesson plan special efforts are not needed.

Among the activities that would raise the effectiveness of mathematics and IT classes can be constructing lessons' plans and scenarios with the participation of the students. If the snail problem is to be discussed, selecting appropriate numerical examples may involve the whole class in exploratory work based on the worksheet. The search for interesting cases may be preceded by discussing Polya's four principles of problem solving [12], some ideas about testing algorithms, etc. Such activities can definitely help in building the students' number sense, mathematical thinking, and algorithmic culture.

## 4. Ideas for the snail problem scenario

By using worksheets of the snail problem, primary school teachers and students bridge the gap between content and context. To make sure that students control the sliders inserted in the worksheet and change numerical data successfully, the teacher asks them to explain the effect of each slider individually. Then the task to reproduce the problem on the worksheet comes quite naturally. One possible assignment is ***to make a sequence of images showing the position of the snail within several successive days***. This not only tests students' understanding, but also supports it. Such an activity trains students to be aware of stumbling blocks of inductive reasoning. It also helps them to report their results in an orderly and convincing manner (*Fig. 3*). A competition for best class work presentation sounds as an attractive social event at the end of the lesson.

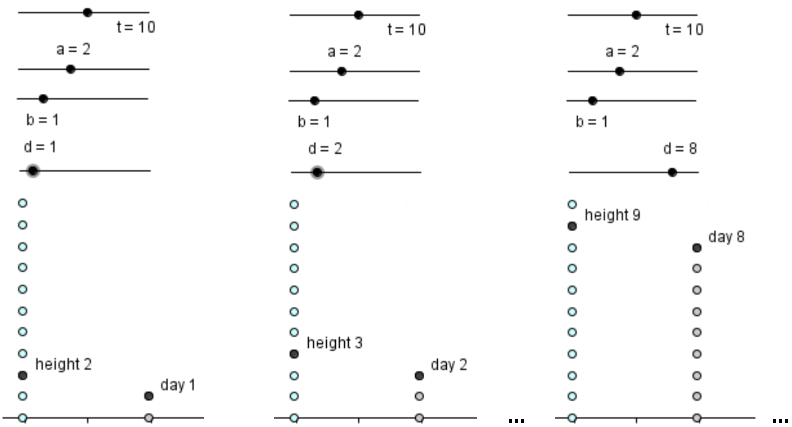



*Figure 3. A sample for reporting students' observations*

Solving the snail problem should not be an end in itself. Therefore further activities are advised to help students make sense of their work with the dynamic geometry worksheet so far. Here the thoughts of Polya [12. p. 5] written before the birth of contemporary IT, sound like just shared after a visit to a modern classroom: "The teacher who wishes to develop his students' ability to do problems must instill some interest for problems into their minds and give them plenty of opportunity for imitation and practice." Here are some questions for in-class research:

- ***When will the snail reach the edge of the 10-meter well provided that it crawls 5 meters up in daytime and 2 meters down at night? Answer the question first on paper and then check your solution using the worksheet;***
- ***By mistake, student A has given the same value of 2 to both worksheet sliders a and b. What does that mean? What do you expect to happen to “his” snail?***
- ***Student B has chosen the following values for the worksheet sliders: a = 5, b = 1, t = 9, d = 6. How would you interpret her worksheet?***
- ***If each day the snail moves 20 centimeters up and each night – 10 centimeters down, how the worksheet should be adapted?***

Since the algorithm implemented in the worksheet was built by calculating the height reached by the snail at the end of the current day, the teacher can show the sequence of these heights as a column of dots. This enables students to grasp the whole picture of the motion. It will be also useful if, working in groups they read each other’s worksheets and formulate a word problem, corresponding to the numerical data used by their peers. During such an activity students acquire data literacy. And unexpectedly, new characters and plots of mathematical problems may occur.

The idea of mathematical analogy can also be regarded, together with some touches of common sense:

- ***If the snail is up on a tree and is to reach the ground, how will the respective problem look like? What formulation would you suggest?***

These and many other questions put students and teachers on the road of open-ended mathematical problems and make them partners in inquiry-based learning. The capacity of such problems to boost creativity spreads not only to mathematics and IT classes, but possibly to other school subjects and life situations. For sixth and seventh graders I made ***a snail problem of a new type***, which allows multiple solutions, obtainable both experimentally and analytically:

***On a 10-meter stone well there is a critical water level mark. It is a circular brick ring on the inside of the well’s wall, built 4 meters above the bottom. Starting from the bottom, a snail moves 3 meters up daily and 2 meters down at night, until it reaches the well’s edge. How many times will the snail touch or cross the brick ring?***

## 5. Concluding remarks

To develop students’ mathematical thinking and algorithmic culture, any problem situation can help. Even the old and well-known snail problem, if used in parallel with dynamic geometry software, may revive its faded glory and help students:

– learn mathematics by doing it;
– use IT both to illustrate mathematical problems and to delve into them;
– perform elementary numerical experiments, systematize and report the results;
– ask questions; simulate situations and compose own mathematical problems;
– formulate and verify conjectures; develop mathematical thinking and intuition;
– construct and test algorithms of specific mathematical problems and develop algorithmic culture;
– share their experience with peers;
– adopt inquiry-based learning as an enduring approach to acquiring knowledge.

Classroom activities of the sort reflect modern trends in education, make it easier for students to like mathematics and use it in IT classes, and positively influence the process of building numeracy.

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## Notes

[1] GeoGebra is an application written in Java to be used for interactive geometry and is free for non-commercial users (http://www.geogebra.org, May 24, 2012).

# РАЗВИВАНЕ НА МАТЕМАТИЧЕСКО МИСЛЕНЕ И АЛГОРИТМИЧНА КУЛТУРА В ЧАСОВЕТЕ ПО ИТ

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***Резюме:*** *Програмата по информационни технологии (ИТ) за българското училище поставя специфични изисквания пред ученици, учители и експерти. Разностранните аспекти на ИТ позволяват включването и на занимателни задачи в урочните дейности. Динамичната визуализация на задачите и промяната на числовите данни са основа за компютърни симулации, осъществими „с едно кликване” на мишката. Това прави възможно реализирането на изследователски ориентирано обучение, което да подтиква към математическо мислене и откривателство. Напоследък часовете по математика и ИТ са сред немногобройните места в училище, където се гради математическо мислене и алгоритмична култура на подрастващите. Моделирането на занимателни математически задачи с динамичен софтуер може да подпомогне този процес по ефективен и привлекателен за учениците начин.*