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EPQ Model with Imperfect Quality Raw Material

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The classical economic production model (EPQ) has been extended in many directions to incorporate factors encountered in real-life situations. In this paper, an EPQ model that accounts for the cost of raw material needed for production is examined. It is assumed that the raw material acquired from the supplier contains a percentage of imperfect quality items. At the beginning of the inventory cycle, the raw material is received instantaneously, and a 100% screening process for detecting the imperfect quality items is conducted. Two different scenarios are considered. In the first, the imperfect quality items of the raw material are sold at a discounted price at the end of the screening period. In the second scenario, the imperfect quality items are kept in stock until the end of the inventory cycle and returned to the supplier when the next order is received.

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1. Introduction

The classical economic production model (EPQ) is based on simplifying assumptions that ignore many factors encountered in real life situations. Recently, this model has been extended in many directions by relaxing the underlying assumptions. Considerable research has been done on inventory models where the assumption of perfect quality items is lifted. Poretus [7] studied the effects of incorporating defective items into the classical economic order quantity (EOQ) model. Scwaller [9] proposed an EOQ model where a known proportion of defective items is present in incoming lots and the cost of finding and removing the defective items include both fixed and variable inspection costs. Zhang and Gerchak [10] considered the case in which the defective items are replaced by non-defective ones. Cheng [1] presented an EPQ model with imperfect production processes and having a demand dependent unit production cost. More recently, Salameh and Jaber [8] developed a model to determine the optimal lot size for an EOQ model where items received from the supplier contain a probabilistic percentage of imperfect items. The imperfect quality items are detected through a screening process and are sold in one batch at a discounted price. Hayek and Salameh [4] studied an EPQ model where the of imperfect quality items are reworked. Chiu [2] generalized the model in [4] by considering a production process with random defective rate where the defective items are reworked and unsatisfied demand is backlogged. Ozdemir [6] examined an EOQ model with defective items and shortages backordered. El-Kassar et al. [3] considered an EPQ model where the perfect and imperfect quality items produced have continuous demand. Khan et al. [5] presented an extensive review of EOQ/EPQ models with imperfect quality items.

In this paper, we examine an EPQ model that accounts for the cost of raw material needed for production. The raw material acquired from the supplier contains a percentage of imperfect quality items. It is assumed that at the beginning of the inventory cycle the raw material is ordered and received instantaneously. A 100% screening process for detecting the imperfect quality items of raw material is conducted at a rate greater than the production rate. Two different scenarios are considered. In the first, the imperfect quality items of the raw material are sold at a discounted price at the end of the screening period. In the second scenario, the imperfect quality items are kept in stock until the end of the inventory cycle and returned to the supplier when the next order of the raw material is received. The rest of this paper is organized as follows. The mathematical models for the two scenarios are developed in section 2. In section 3, a numerical example is given to illustrate the models and to compare the optimal policies of the two scenarios. Section 4 contains a conclusion and future research suggestions.

2. Mathematical models

The following notations will be used throughout this paper:

D = demand rate of finished product

P = production rate

y = raw material order size

q = percentage of imperfect quality items of raw material

$1-q$ = percentage of perfect quality items of raw material

x = screening rate for imperfect quality items of raw material ($x > P$)

y_{max}	=	maximum inventory level of finished product
K_p	=	setup cost for production
K_s	=	raw material ordering cost
h_r	=	raw material holding cost per unit per unit time
h_p	=	production holding cost per unit per unit time
C_r	=	unit cost of raw material
S	=	unit selling price of finished product
S_r	=	discounted unit selling price of imperfect quality items ($S_r < C_r$)
T	=	inventory cycle length
t_s	=	screening period
t_p	=	production period

We begin by developing a model that describes the first scenario. In this case, a product is produced at a rate P, where P is greater than the demand rate D for the finished product. Since the production process requires the use of raw material, it is assumed that an order of size y of raw material is placed and received at the beginning of production cycle. Also, the raw material received is assumed to contain a percentage q of imperfect quality items. The number of imperfect quality items of raw material received is yq, and the remaning perfect quality items, $y(1-q)$, are used to produce the finished product. Therefore, the length of the production period is

$$t_p = \frac{y(1-q)}{P}, \tag{1}$$

and the length of inventory cycle is

$$T = \frac{y(1-q)}{D}. \tag{2}$$

At the beginning of the production cycle, a 100% screening process for detecting the imperfect quality items is conducted at a screening rate x, where $x > P$. The length of the production period is

$$t_s = \frac{y}{x}. \tag{3}$$

Throughout the screening period, the perfect quality items of raw material are used in the production process. Hence, the inventory level of raw material is depleted at a rate P until the end of screening period. When the screening process stops, the number of perfect and imperfect quality items of raw material reaches a level of

$$y - Pt_s = y\left(1 - \frac{P}{x}\right). \tag{4}$$

At this time, the imperfect quality items are sold at a discounted price S_r, which is less than the unit purchasing cost of raw material C_r. Thus, the inventory level of raw material drops from $y - Pt_s$ by an amount qy. The total number of items of raw material left is

$$y - Pt_s - qy = y\left(1 - \frac{P}{x} - q\right). \tag{5}$$

The inventory level of raw material decreases at a rate P until the end of production period where it reaches zero. The inventory level of raw material is shown in Figure 1.

During the production period, finished items are produced at a rate P and part of these items are sold at a rate D. Therefore, an inventory of finished items is accumulating throughout the production period at a rate $P - D$ until a maximum level of $y_{\max}$ is reached. The inventory level of the finished items is shown in Figure 2. From (1), the maximum level of finished items is

$$y_{\max} = t_P(P - D) = y(1-q)\left(1 - \frac{D}{P}\right). \tag{6}$$

To find the optimal order quantity, we first calculate the total cost per inventory cycle which is the sum of purchasing cost, production cost, ordering cost of raw material, setup cost of production, raw material holding cost and finished product holding cost. Except for the holding costs, the components are as follows:

$$\text{Purchasing cost of raw material} = C_r y, \tag{7}$$

$$\text{Production cost} = C_p y(1-q), \tag{8}$$

$$\text{Ordering cost of raw material} = K_s, \tag{9}$$

$$\text{Set up cost} = K_p. \tag{10}$$

The holding cost of raw material is the product of the average inventory, cycle length, and the holding cost per unit time h_r. To find the average inventory, the area under the curve is divided by the cycle length. Using (4) and (5), the area under the curve representing the inventory level of raw material, see Figure 1, is given by

$$\text{Area} = \frac{(y + y - Pt_s)t_s}{2} + \frac{(y - Pt_s - qy)(t_p - t_s)}{2}. \tag{11}$$

From (1) and (3), (11) becomes

$$\text{Area} = y^2\left(\frac{(1-q)^2}{2P} + \frac{q}{x}\right). \tag{12}$$

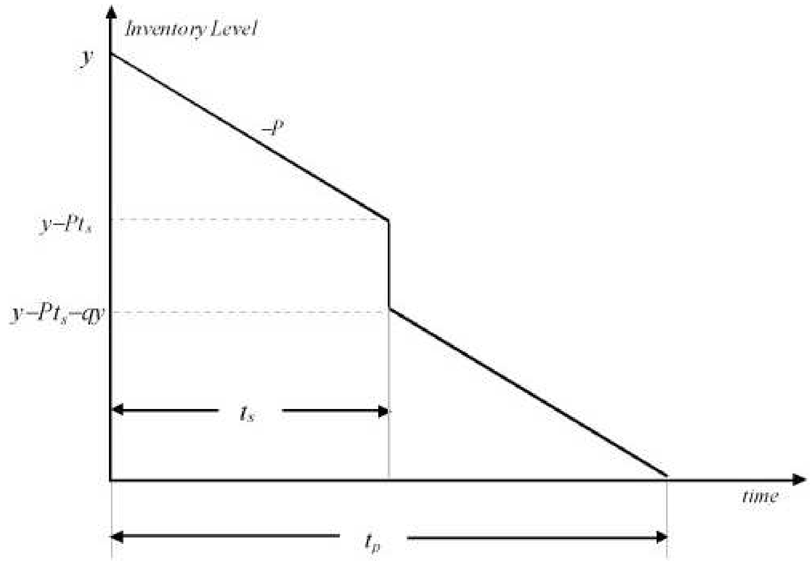

Fig. 1: Raw Material Inventory Level, Imperfect Items Sold at a Discount

The average inventory over the entire cycle is obtained by dividing (12) by the inventory cycle length so that

$$\text{Average inventory} = y^2\left(\frac{(1-q)^2}{2P}+\frac{q}{x}\right)\frac{1}{T}. \tag{13}$$

Using (2) and (13), the holding cost of raw material becomes

$$\text{Holding cost of raw material} = \; y^2\left(\frac{(1-q)^2}{2P}+\frac{q}{x}\right)h_r. \tag{14}$$

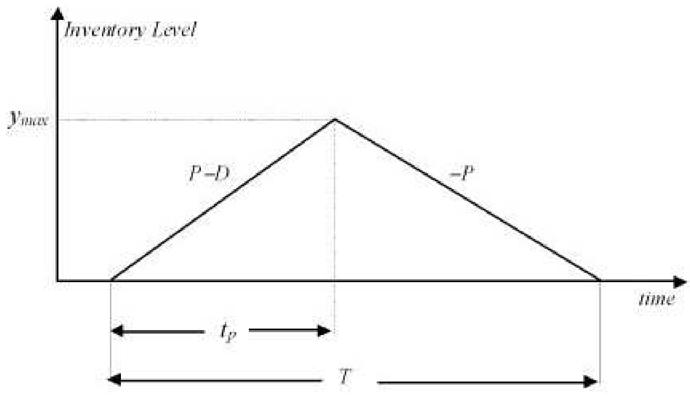

Fig. 2: Finished Product Inventory Level

The holding cost for finished items is the product of the average inventory, cycle length and the sum of the two holding cost per unit per unit time, h_P and h_r. Since the area under the curve representing the inventory level of finished items is $\frac{1}{2}Ty_{\max}$, we have that the average inventory is $\frac{1}{2}y_{\max}$. Using (6), we obtain

$$\text{Holding cost for finished items} = \frac{y}{2}(1-q)\left(1-\frac{D}{P}\right)(h_p+h_r)T. \tag{15}$$

Now, the total inventory cost per cycle function $TC(y)$ is the sum of the expressions in (7)-(10), (14) and (15). That is,

$$\begin{aligned} TC(y) &= C_r y + C_p y(1-q) + K_s + K_p + y^2\left(\frac{(1-q)^2}{2P}+\frac{q}{x}\right)h_r \\ &\quad +\frac{y}{2}(1-q)\left(1-\frac{D}{P}\right)(h_p+h_r)T. \end{aligned} \tag{16}$$

The total inventory cost per unit time function $TCU(y)$ is obtained by dividing (16) by the inventory cycle length $T = y(1-q)/D$. Hence,

$$\begin{aligned} TCU(y) &= C_r\frac{D}{1-q} + C_pD + (K_s+K_p)\frac{D}{y(1-q)} \\ &\quad +yD\left(\frac{(1-q)}{2P}+\frac{q}{(1-q)x}\right)h_r + \frac{y}{2}(1-q)\left(1-\frac{D}{P}\right)(h_p+h_r). \end{aligned} \tag{17}$$

Now, the total revenue function $TR(y)$ is the sum of sales revenue of the finished product and the discounted sales of the imperfect quality items. That is,

$$TR(y) = Sy\ (1-q) + S_r q y. \tag{18}$$

Dividing (18) by T, we have that the total revenue per unit time is

$$TRU(y) = SD + S_r q D\frac{1}{(1-q)}. \tag{19}$$

Theorem 1. *The optimal order quantity is*

$$y^* = \sqrt{\frac{2(K_s+K_p)D}{(h_p+h_r)\left(1-\frac{D}{P}\right)(1-q)^2 + Dh_r\left(\frac{(1-q)^2}{P}+\frac{2q}{x}\right)}}\ . \tag{20}$$

Moreover, the EOQ is unique.

Proof. The economic order quantity is determined by maximizing the total profit. The total profit function per cycle is $TP(y) = TR(y) - TC(y)$. From (17) and (19), we get

$$TPU(y) = SD + \frac{S_r q D}{(1-q)} - C_r\frac{D}{1-q} - C_pD - \frac{(K_s+K_p)D}{y(1-q)}$$

$$-yD\left(\frac{(1-q)}{2P}+\frac{q}{(1-q)x}\right)h_r-\frac{y}{2}(1-q)(1-\frac{D}{P})(h_p+h_r). \tag{21}$$

Differentiating the expression of $TPU(y)$ in (21) with respect to y, we get

$$\frac{d}{dy}(TPU(y))=\frac{(K_s+K_p)D}{y^2(1-q)}-D\left(\frac{1-q}{2P}+\frac{q}{(1-q)x}\right)h_r$$
$$-\frac{1}{2}(1-q)\left(1-\frac{D}{P}\right)(h_p+h_r). \tag{22}$$

Setting the derivative in (22) equal to zero and solving for y, we obtain the economic order quantity

$$y^*=\sqrt{\frac{2(K_s+K_p)D}{(h_p+h_r)\left(1-\frac{D}{P}\right)(1-q)^2+Dh_r\left(\frac{(1-q)^2}{P}+\frac{2q}{x}\right)}}. \tag{23}$$

The second derivative of $TPU(y)$ is

$$\frac{d^2}{dy}(TPU(y))=-\frac{2(K_s+K_p)}{y^3(1-q)}. \tag{24}$$

The right hand side of (24) is always negative so that y^* in (23) is the unique maximizer of the $TPU(y)$ function. ■

Now we develop the mathematical model for the second scenario. In this case, the screened imperfect quality items of raw material are kept until the end of the inventory cycle and returned to the supplier when the next order is received. The behavior of the inventory level of raw material is shown in Fig. 3.

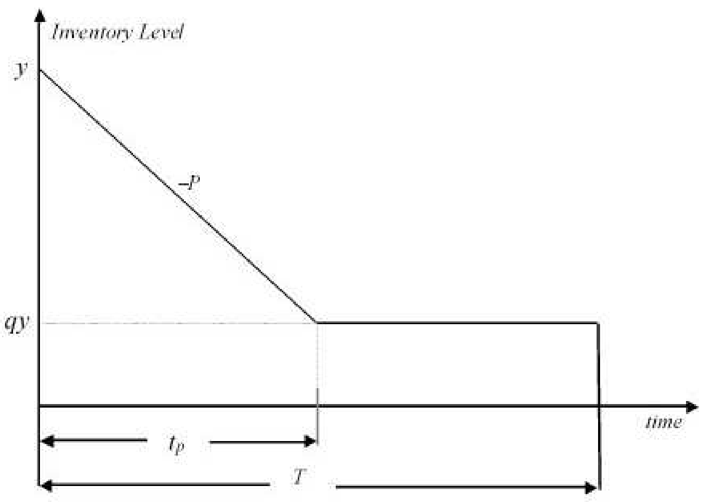

Fig. 3: Raw Material Inventory Level, Imperfect Items Returned to Supplier

To find the optimal order quantity, we calculate the total cost per cycle which is identical to that of the first scenario except for the holding cost of the raw material. The area under the curve representing the inventory level in Figure 3 is:

$$\text{Area} = qyT + \frac{1}{2}y(1-q)t_p. \tag{25}$$

Dividing (25) by T, we have

$$\text{Average inventory} = qy + \frac{D}{2P}y(1-q). \tag{26}$$

Multiplying (26) by h_r and T, we obtain

$$\text{Holding cost of raw material} = y\left(q + \frac{D}{2P}(1-q)\right)h_rT. \tag{27}$$

Replacing the raw material holding cost term in (16) by (27) and dividing by T, we obtain the TCU function for the 2nd scenario as

$$TCU(y) = C_r\frac{D}{1-q} + C_pD + (K_s+K_p)\frac{D}{y(1-q)}$$
$$+y\left(q + \frac{D}{2P}(1-q)\right)h_r + \frac{y}{2}(1-q)\left(1-\frac{D}{P}\right)(h_p+h_r). \tag{28}$$

Since the imperfect quality items of raw material are returned to the supplier, the total revenue function is

$$TR(y) = Sy(1-q) + C_rqy. \tag{29}$$

Dividing (29) by T and subtracting (28), we have

$$TPU(y) = SD - C_rD - C_pD - (K_s+K_p)\frac{D}{y(1-q)}$$
$$-y\left(q + \frac{D}{2P}(1-q)\right)h_r - \frac{y}{2}(1-q)\left(1-\frac{D}{P}\right)(h_p+h_r). \tag{30}$$

Maximizing (30) in a manner similar to that of Theorem 1, we obtain the economic order quantity

$$y^* = \sqrt{\frac{2(K_s+K_p)D}{(h_p+h_r)(1-\frac{D}{P})(1-q)^2 + \left(2q(1-q) + \frac{D}{P}(1-q)^2\right)h_r}}. \tag{31}$$

3. Numerical examples

Consider a production process where the daily demand rate for an item is 5 units and the production rate is 10 units per day. The raw material used in production is ordered from the supplier where 30% of the items received are found defective. Screening for imperfect quality items of the raw material is conducted at a rate of 20 items per day. The ordering cost for the raw material is \$100 and the setup cost is \$183. The holding cost of raw material is \$0.01 per unit per day while the holding cost due to production is \$0.02 per unit per day. Hence, the holding cost of one unit of the finished product \$0.03 per day. The purchasing cost of one item of raw material is \$5 and the unit production cost is \$10. The selling price is \$25 per unit. The imperfect quality items screened may be sold at the end of screening period at a discounted price of \$3, or may be kept in stock and returned to the supplier when the next order arrives. To determine the optimal order policy, the two scenarios are compared.

The parameters of the problem are $D = 5$, $P = 10$, $q = 0.3$, $x = 20$, $K_s = 100$, $K_P = 183$, $h_p = 0.02$, $h_r = 0.01$, $C_r = 5$, $C_p = 10$, $S = 25$, and $S_r = 3$. Evaluating the policy where the imperfect quality items are returned to supplier, the optimal order quantity is obtained from (20) as $y^* = 500.4 \approx 500$. The optimal number of items produced during a production cycle is $y^*(1-q) = 350$ units. The length of inventory cycle is $T^* = \frac{y^*(1-q)}{D} = 70$ days, the production period is $t_p^* = \frac{y^*(1-q)}{P} = 35$ days, and the screening period is $t_s^* = \frac{y^*}{x} = 25$ days. The total inventory cost per day is \$93.79, the total revenue per day is \$131.43, and the maximum total profit per day is \$37.64.

As for the case where the imperfect quality items are returned, the optimal order quantity $y^* = 449.6 \approx 500$ is calculated using (31). The optimal number of items produced is 315. The production period 31.5 days and the inventory period length is 63 days. The total cost per day is \$94.71, the total revenue per day is \$134.71 and the total profit is \$40.00.

From the above analysis, the optimal operating policy is to return the imperfect items to the supplier when the next order arrives.

4. Conclusion

The model presented in this paper extends the classic economic production quantity (EPQ) model to the case where raw material with imperfect quality items are used in the production process. Two scenarios for this model were considered. The optimal operating policy was derived by maximizing the total profit per unit time. Explicit expressions for the optimal order quantity for the two scenarios were obtained. The uniqueness of the optimal solutions was demonstrated. A numerical example was given to illustrate how the optimal policy can be determined by evaluating the two scenarios.

For future work, the effect of time value of money and credit facility for this model may be investigated. The model can be extended by considering probabilistic percentage of imperfect quality items. In another direction, the possible use of imperfect quality items in the production process may be examined. In this case, the production process yields two types of finished product with perfect and imperfect quality.

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