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ИНСТИТУТ ПО МАТЕМАТИКА С ИЗЧИСЛИТЕЛЕН ЦЕНТЪР

INSTITUTE OF MATHEMATICS WITH COMPUTER CENTER

On the Black Body Oscillator Equation

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БЪЛГАРСКА АКАДЕМИЯ НА НАУКИТЕ

Preprint

January 1995

No 2

1869

BULGARIAN ACADEMY OF SCIENCES

Department of Operations Research

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It is known that at the end of the last century the classical physics failed in the explanation of heart radiation. In 1900 M. Planck devised the quantum theory endeavouring to deduce the law of black body radiation. From that time on the quantum mechanics worked out as an advanced theory on solving various physical problems. On the other hand the efforts to combine the relativistic theory with the quantum mechanics and then to build a unified field theory ran into great difficulties discussed in many textbooks (cf. [1] - [8]). The attempts to apply quantum mechanical methods to non-local field theories give rise to insurmountable problems exposed in [9]. All reasonings above mentioned suggest to us to consider once more the classical setting of the black body radiation.

In the present paper we try to analyze possible causes (from mathematical point of view) that generate the so-called by P. Ehrenfest "ultra-violet catastrophe". We establish that the derivation of the Lorentz radiation term is not quite correct from the point of view of the modern theory of functional differential equations. We propose a new form of the radiation term which corresponds to the original physical assumption due to P. A. M. Dirac [10]. This leads to a nonlinear functional differential equation of neutral type instead of the known linear oscillator equation.

Let us recall briefly the classical formulation of the problem in question following [11]. By a black body is meant a body with absorbtive power equal to unity, i.e. a body which absorbs the whole of the radiant energy falling upon it. To the black body is assigned the one-dimensional oscillator

$$\ddot{x}(t) + \omega^2 x(t) - \frac{2}{3}\frac{e^2}{m_0 c^3}\dddot{x}(t) = \frac{e}{m_0}E_x(t) \tag{1}$$

where by e is denoted the charge, by m_0 - the mass, by ω - the frequency of its proper oscillations, by $E_x(t)$ - electric field of the black body radiation. The summand $R^{rad} = -\frac{2}{3}\frac{e^2}{m_0 c^3}\dddot{x}(t)$ is the Lorentz radiation term. It represents the influence of the field generated by the moving charge on itself. Following H. Lorentz [12] (cf. also [13]) the radiation force can be calculated from $F = e\int \bar{E}\rho(\vec{r})\,d\vec{r}$, where $\bar{E}$ is the mean field generated by the moving charge, i.e. $\bar{E} = \int E\rho(\vec{r'})\,d\vec{r'}$. E can be found out using $E = -\frac{\partial\varphi}{\partial x} - \frac{1}{c}\frac{\partial A_x}{\partial t}$. It is known that the retarded potentials for the point charge are

$$\varphi = e\int \frac{\delta(t' - t + \frac{R'}{c})}{R'}\,dt' \;;\; A_x = \frac{e}{c}\int \frac{v(t')\delta(t' - t + \frac{R'}{c})}{R'}\,dt' \tag{2}$$

where $R' = |\vec{r} - \vec{r'}(t')|$ is the distance between the observer and the source and $\rho(\vec{r})$ is the density function. Assuming $\frac{R'}{c}$ to be a small parameter one has obtained

$$\delta(t'-t+\frac{R'}{c})=\delta(t'-t)+\frac{R'}{c}\dot{\delta}(t'-t)+\frac{R'^2}{2c^2}\ddot{\delta}(t-t')+\frac{R'^3}{6c^3}\dddot{\delta}(t-t')+\cdots \qquad (3)$$

The differention is with respect to t'. After replacing in (2) φ and A_x become

$$\varphi=\frac{e}{R}+\frac{e}{2c^2}\frac{\partial^2 R}{\partial t^2}-\frac{e}{6c^3}\frac{\partial^3 R^2}{\partial t^3}+\cdots\ ;\ A_x=\frac{e}{c}\frac{\dot{x}}{R}-\frac{e\ddot{x}}{c^2}+\cdots$$

where $R=|\vec{r}-\vec{r'}(t)|$. Omiting some transformations given in details in [12] and [13] it obtains for F: $F=-m^{el}\ddot{x}+\frac{2}{3}\frac{e^2}{c^3}\dddot{x}\ (t)+\cdots$, where m^{el} can be interpreted as a field electromagnetic mass, while $\frac{2}{3}\frac{e^2}{c^3}\dddot{x}\ (t)$ - as a radiation term.

We point out that expansion (3) means a disregarding of the delay generated by the finite velocity of propagation of the interaction. In this way the naturally arising neutral functional differential equation is replaced by an ordinary differential one, namely (1). Besides it is known from the theory of functional differential equations with a small parameter that a continuous dependence on the small parameter does not exist in general (cf. [9], [14]).

Equation (1) is obviously a third order linear equation. It can be solved by Fouriet transformation under assumption that $E_x(t)$ has a Fouriet expansion

$$E_x(t)=\sum_{-\infty}^{\infty}E_{xn}e^{in\omega_0 t}=\sum_{-\infty}^{\infty}E_{xn}e^{i\omega_n t}\ (\omega_n=n\omega_0).$$

Its solution is

$$x(t)=\sum_{-\infty}^{\infty}\frac{\frac{e}{m_0}E_{xn}e^{in\omega_0 t}}{\omega^2-(n\omega_0)^2+i\frac{2}{3}\frac{e^2(n\omega_0)^3}{m_0c^3}}$$

and then the mean energy is

$$\bar{E}=\overline{m_0\dot{x}^2}=2m_0\sum_{n=0}^{\infty}\frac{(n\omega_0)^2\frac{e^2}{m_0}|E_{xn}|^2}{[(n\omega_0)^2-\omega^2]^2+[\frac{2}{3}\frac{e^2(n\omega_0)^3}{m_0c^3}]^2}.$$

It has a maximum for those values of n for which $n\omega_0\approx\omega$. This means that it is assumed a relation between ω_0 and ω namely $\omega_0=\frac{\omega}{n_0}$. Then after not at all rigorous calculations (cf. for instance [15]) one can reached Rayleigh-Jeans radiation law

$$\rho(\omega)=\frac{\omega^2}{n^2c^3}\bar{E}=\frac{\omega^2}{n^2c^3}kT. \qquad (4)$$

Obviously the spectral intensity distribution increases as the square of the frequency and for great frequencies becomes infinite. This implies that the total energy of radiation is infinite too. P. Ehrenfest calls the divergent integral $u=\int_0^\infty\rho(\omega)\,d\omega=\frac{kT}{n^2c^3}\int_0^\infty\omega^2\,d\omega$ ultra-violet catastrophe. To remove this difficulties M. Planck postulates discrete, finite quanta of energy ε_0. The energy of the oscillators is to be $n\varepsilon_0, n\in\mathbb{N}$ and then he obtains his (Planck's) radiation law

$$\rho(\omega)=\frac{\hbar\omega^3}{n^2c^3(exp(\hbar\omega/kT)-1)} \qquad (5)$$

where $\hbar$ is the Planck constant.

The present paper consists of three sections. In the first one some fixed point results are given. The second section contains a derivation of a new form of the radiation term corresponding to the Dirac assumptions. In the third section an existance theorem for the oscillator equation is proved. It is shown that ultra-violet catastrophe does not exist.

1 Fixed point results

Let us recall some definitions and theorems from [16].

By $(X, \mathcal{A})$ we mean a sequentially complete T_2-separated uniform space with a uniformity generated by a saturated family of pseudometrics $\mathcal{A} = \{\rho_a(x,y) : a \in A\}$, A being an index set (cf. [17], [18]). Let $j : A \to A$ be a mapping of the index set into itself whose iterates are defined as follows $j^0(a) = a, j^k(a) = j(j^{k-1}(a)), k \in \mathbf{N}$. Consider a family of functions $(\Phi) = \{\Phi_a(t) : a \in A\}, \Phi_a(t) : \mathbf{R}^+ \to \mathbf{R}^+ \equiv [0,\infty)$ with the properties: $(\Phi 1) : \Phi_a(t)$ is non-decreasing, continuous from the right on $\mathbf{R}^+$ and $0 < \Phi_a(t) < t, \forall t > 0$; $(\Phi 2) : \forall a \in A \exists \bar{\Phi}_a(t) \in (\Phi)$ such that $sup\{\Phi_{j^k(a)} : k = 0,1,2,...\} \leq \bar{\Phi}_a(t)$ with $\bar{\Phi}_a(t)/t$ non-decreasing.

The operator $T : X \to X$ is said to be Φ-contractive if $\rho_a(Tx,Ty) \leq \Phi_a(\rho_{j(a)}(x,y))$ for every $x, y \in X$ and $a \in A$.

Theorem 1 *[16] If $T : X \to X$ is Φ-contractive and there is $x_0 \in X$ for which exists $Q = Q(a, x_0, Tx_0)$ such that $\rho_{j^k(a)}(x_0, Tx_0) \leq Q(k = 0,1,2,...)$, then T has at least one fixed point in X. If X is j-bounded (that is, for every $x, y \in X$ and $a \in A$ there is a constant $Q = Q(a,x,y)$ such that $\rho_{j^k(a)}(x,y) \leq Q; k = 1,2,...)$ then the fixed point of T is unique.*

Let $(2^X)_b$ be the family of all bounded subsets of X. Let F be a family of functions $\alpha : A \to \mathbf{R}^+$ with a uniformity generated by point-wise convergence and partial order defined in a usual way: $\alpha_1 \preceq \alpha_2$ if $\alpha_1(a) \leq \alpha_2(a) \forall a \in A$.

By $\gamma_a : (2^X)_b \to F$ we mean Kuratowski or Hausdorff measure of noncompactness (cf. [19]). If X is a locally convex topological vector space we suppose that X possesses a property denoted by (C); the convex closure of every compact is a compact in X. Property (C) is satisfied provided X to be complete or even quasicomplete [20].

An operator $T : X \to X$ is said to be Φ-densifying if for every bounded set $\Omega \subset X$ the following inequality holds true $\gamma_a(T(\Omega)) \leq \Phi_a(\gamma_{j(a)}(\Omega)), a \in A$.

Let $M \subset X$ be bounded closed and convex where X is a locally convex topological vector space. The following theorem guarantees an existence of a general class Φ-densifying operators.

Theorem 2 *[21] If $T : M \times X \to X$ has the properties: 1) for each fixed $y \in X$ the set $T(M,y)$ is totally bounded; 2) for each fixed $x \in M$ and every $y, \bar{y} \in X, a \in A$ is satisfied $\|T(x,y) - T(x,\bar{y})\|_a \leq \Phi_a(\|y - \bar{y}\|_{j(a)})$.*
Then the operator $\widetilde{T}(x) = T(x,x) : M \to X$ is Φ-densifying.

Theorem 3 *[21] Let $T : M \to M$ be continuous Φ-densifying mapping. If there exists a constant $Q = Q(a, M) > 0$ such that*

$$\gamma_{j^n(a)}(M) \leq Q < \infty (n = 0,1,2,...),$$

then T has at least one fixed point in M.

2 A new form of the radiation term

As we have already mentioned in the introduction the main purpose is to derive a new form of the generally accepted Lorentz radiation term $R^{rad} = -\frac{2}{3}\frac{e^2}{m_0c^3}\dddot{x}(t)$. We would like to point out that the radiation term here proposed is obtained on the base of the original physical assumptions due to Dirac in [10]. In the next Section we investigate the question of how this new radiation term influences over the ultra-violet catastrophe.

As we have already remarked we use the physical model from [10]. The derivation is valid for 4-dimensional case nevertheless we shall apply the results obtained to the 1-dimensional case. By $\check{x}_k (k = 1, 2, 3, 4)$ we denote the components of the "incoming" field (cf. [10]), by $\hat{x}_k (k = 1, 2, 3, 4)$ - the components of the "outgoing" field. In accordance with Dirac assumptions the radiation term can be defined as a half of the difference between both retarded and advanced potentials, that is,

$$F_{kn}^{rad} = \frac{1}{2}\left[\left(\frac{\partial A_n^{ret}}{\partial \check{x}_k} - \frac{\partial A_k^{ret}}{\partial \check{x}_n}\right) - \left(\frac{\partial A_n^{adv}}{\partial \hat{x}_k} - \frac{\partial A_k^{adv}}{\partial \hat{x}_n}\right)\right]$$

where

$$A_n^{ret} = -\frac{e\check{\lambda}_n}{\langle \check{\lambda}, \xi_{ret}\rangle}, A_n^{adv} = -\frac{e\hat{\lambda}_n}{\langle \hat{\lambda}, \xi_{adv}\rangle},$$

and ξ_{ret}, ξ_{adv} are isotrope vectors lying on the light cone

$$\xi_{ret}\left[\check{x}_1(t) - x_1(\check{t}), \check{x}_2(t) - x_2(\check{t}), \check{x}_3(t) - x_3(\check{t}), ic(t - \check{t})\right], t > \check{t};$$

$$\xi_{adv}\left[\hat{x}_1(t) - x_1(\hat{t}), \hat{x}_2(t) - x_2(\hat{t}), \hat{x}_3(t) - x_3(\hat{t}), ic(t - \hat{t})\right], \hat{t} > t;$$

c is the speed of light;

$$\check{\lambda}(\check{u}_1(t), \check{u}_2(t), \check{u}_3(t), ic), \hat{\lambda}(\hat{u}_1(t), \hat{u}_2(t), \hat{u}_3(t), ic),$$

$$\check{u}_\alpha(t) = \frac{d\check{x}_\alpha(t)}{dt}, \hat{u}_\alpha(t) = \frac{d\hat{x}_\alpha(t)}{dt} \quad (\alpha = 1, 2, 3);$$

$\langle \cdot, \cdot \rangle$ is the scalar product in the Minkowski space.

The explicite form of F_{kn}^{rad} can be calculated following the technics from [22] (cf. also [21]). Before to write down the radiation term we have to adjust the times of incoming and outgoing fields setting $t - \check{t} = \tau^{ret}(t)$ and $\hat{t} = t + \tau^{adv}(t)$. Then the isotrope vectors ξ_{ret} and ξ_{adv} become

$$\xi_{ret}\left(\check{x}(t) - x(t - \tau^{ret}(t)), ic\tau^{ret}(t)\right); \xi_{adv}\left(\hat{x}(t) - x(t + \tau^{adv}(t)), ic\tau^{adv}(t)\right)$$

(throughout the remainder of the present paper we need the one-dimensional case) where the functions $\tau^{ret}(t)$ and $\tau^{adv}(t)$ can be defined from

$$\tau^{ret}(t) = \frac{1}{c}\left|\check{x}(t) - x(t - \tau^{ret}(t))\right|; \tau^{adv}(t) = \frac{1}{c}\left|\hat{x}(t) - x(t + \tau^{adv}(t))\right| \tag{6}$$

Having in mind that the components of the velosity vectors $\check{u}_\alpha = \frac{d\check{x}_\alpha}{dt}, \hat{u}_\alpha = \frac{d\hat{x}_\alpha}{dt}$ reduce to $\check{u}, \hat{u}$, we are able to give the new form of the radiation term R^{rad} denoting from now on by:

$$R_*^{rad} \equiv F_{kn}^{rad} \text{ (1-dimensional)} =$$

$$= \frac{c^2\Delta^2}{2m_0c^2}\left\{\frac{[\xi_{ret} - \tau^{ret}\breve{u}(t)][\breve{u}(t)u(t-\tau^{ret}) - c^2] - [u(t-\tau^{ret}) - \breve{u}(t)][\xi_{ret}\breve{u}(t) - c^2\tau^{ret}]}{[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^3}\cdot\right.$$

$$\cdot\left[(\Delta^{ret})^2 + D^{ret}\left(\xi_{ret} + \frac{\xi_{ret}u^2(t-\tau^{ret}) - c^2\tau^{ret}\Delta^{ret}u(t-\tau^{ret})}{(\Delta^{ret})^2}\right)\frac{du(t-\tau^{ret})}{dt}\right] +$$

$$+D^{ret}\frac{[\xi_{ret}\breve{u}(t) - c^2\tau^{ret}]\left[\frac{du(t-\tau^{ret})}{dt} + \frac{u(t-\tau^{ret})-\breve{u}(t)}{(\Delta^{ret})^2}u(t-\tau^{ret})\frac{du(t-\tau^{ret})}{dt}\right]}{[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^2} -$$

$$\left.-\frac{[\xi_{ret} - \breve{u}(t)\tau^{ret}]\left[\breve{u}(t) + \frac{\breve{u}(t)u^2(t-\tau^{ret})}{(\Delta^{ret})^2} - \frac{c^2u(t-\tau^{ret})}{\Delta^{ret}}\right]\frac{du(t-\tau^{ret})}{dt}}{[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^2}\right\} -$$

$$-\frac{e^2\Delta^2}{2m_0c^2}\left\{\frac{[\xi_{adv} - \hat{u}(t)\tau^{adv}][\hat{u}(t)u(t+\tau^{adv}) - c^2] - [u(t+\tau^{adv}) - \hat{u}(t)][\xi_{adv}\hat{u}(t) - c^2\tau^{adv}]}{[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^3}\cdot\right.$$

$$\cdot\left[(\Delta^{adv})^2 + D^{adv}\left(\xi_{adv} + \frac{\xi_{adv}u^2(t+\tau^{adv})}{(\Delta^{adv})^2} - \frac{c^2\tau^{adv}u(t+\tau^{adv})}{\Delta^{adv}}\right)\right]\frac{du(t+\tau^{adv})}{dt} +$$

$$+D^{adv}\frac{[\xi_{adv}\hat{u}(t) - c^2\tau^{adv}]\left[\frac{du(t+\tau^{adv})}{dt} + \frac{u(t+\tau^{adv})-\hat{u}(t)}{(\Delta^{adv})^2}u(t+\tau^{adv})\frac{du(t+\tau^{adv})}{dt}\right]}{[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^2} -$$

$$\left.-\frac{[\xi_{adv} - \hat{u}(t)\tau^{adv}]\left[\hat{u}(t) + \frac{\hat{u}(t)u^2(t+\tau^{adv})}{(\Delta^{adv})^2} - \frac{c^2u(t+\tau^{adv})}{\Delta^{adv}}\right]\frac{du(t+\tau^{adv})}{dt}}{[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^2}\right\},$$

where $\Delta = [c^2 - u^2(t)]^{\frac{1}{2}}, \Delta^{ret} = [c^2 - u^2(t-\tau^{ret})]^{\frac{1}{2}}, \Delta^{adv} = [c^2 - u^2(t+\tau^{adv})]^{\frac{1}{2}}$,

$$D^{ret} = 1 - \frac{\xi_{ret}u(t-\tau^{ret})}{c|\xi_{ret}|}, D^{adv} = 1 - \frac{\xi_{adv}u(t+\tau^{adv})}{c|\xi_{adv}|}.$$

Instead of (1) we propose the following equation which can be assigned to the black body:

$$\ddot{x}(t) + \omega^2x(t) - R_*^{rad} = \frac{e}{m}E_x(t) \tag{7}$$

We put $w(t) = \dfrac{du(t)}{dt} = \dfrac{d^2x(t)}{dt^2}$ and then obtain

$$w(t) + \omega^2\left[x_0 + u_0t + \int_0^t\int_0^s w(\theta)\,d\theta\,ds\right] - R_*^{rad} = \frac{e}{m}E_x(t) \tag{8}$$

where x_0 and u_0 are the initial values.

In order to shorten the further calculations we introduce the following notations:

$$A_{ret} = \frac{[\xi_{ret} - \tau^{ret}\breve{u}(t)][\breve{u}(t)u(t-\tau^{ret}) - c^2] - [u(t-\tau^{ret}) - \breve{u}(t)][\xi_{ret}\breve{u}(t) - c^2\tau^{ret}]}{[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^3},$$

$$B_{ret} = \xi_{ret} + \frac{\xi_{ret}u^2(t-\tau^{ret})}{(\Delta^{ret})^2} - \frac{c^2\tau^{ret}u(t-\tau^{ret})}{\Delta^{ret}}, E_{ret} = \frac{\xi_{ret}\breve{u}(t) - c^2\tau^{ret}}{[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^2},$$

$$G_{ret} = \frac{[\xi_{ret}\breve{u}(t) - c^2\tau^{ret}][u(t-\tau^{ret}) - \breve{u}(t)]}{(\Delta^{ret})^2[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^2}, H_{ret} = \frac{\xi_{ret} - \breve{u}(t)\tau^{ret}}{[\xi_{ret}u(t-\tau^{ret}) - c^2\tau^{ret}]^2},$$

$$C_{ret} = \check{u}(t) + \frac{\check{u}(t)u^2(t-\tau^{ret})}{(\Delta^{ret})^2} - \frac{c^2 u(t-\tau^{ret})}{\Delta^{ret}},$$

$$A_{adv} = \frac{[\xi_{adv} - \tau^{adv}\hat{u}(t)][\hat{u}(t)u(t+\tau^{adv}) - c^2] - [u(t+\tau^{adv}) - \hat{u}(t)][\xi_{adv}\hat{u}(t) - c^2\tau^{adv}]}{[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^3},$$

$$B_{adv} = \xi_{adv} + \frac{\xi_{adv}u^2(t+\tau^{adv})}{(\Delta^{adv})^2} - \frac{c^2\tau^{adv}u(t+\tau^{adv})}{\Delta^{adv}}, E_{adv} = \frac{\xi_{adv}\hat{u}(t) - c^2\tau^{adv}}{[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^2},$$

$$G_{adv} = \frac{[\xi_{adv}\hat{u}(t) - c^2\tau^{adv}][u(t+\tau^{adv}) - \hat{u}(t)]}{(\Delta^{adv})^2[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^2}, H_{adv} = \frac{\xi_{adv} - \hat{u}(t)\tau^{adv}}{[\xi_{adv}u(t+\tau^{adv}) - c^2\tau^{adv}]^2},$$

$$C_{adv} = \hat{u}(t) + \frac{\hat{u}(t)u^2(t+\tau^{adv})}{(\Delta^{adv})^2} - \frac{c^2 u(t+\tau^{adv})}{\Delta^{adv}}.$$

3 Existence results

In this section we formulate an initial value problem for (7): for prescribing initial acceleration $w_0^{(0)}(t)$ defined on $(-\infty, 0]$, to find a solution on $[0, \infty)$ of

$$\begin{aligned} w(t) &= \frac{e}{m_0}E_x(t) - \omega^2\left[x_0 + u_0 t + \int_0^t\int_0^s w(\theta)\,d\theta\,ds\right] + \\ &+ \frac{c^2\Delta^2}{2m_0c^2}\{(\Delta^{ret})^2 A_{ret} + D^{ret}\left[a^{ret} + E_{ret}\right] w(t-\tau^{ret})\} - \\ &- \frac{e^2\Delta^2}{2m_0c^2}\left\{(\Delta^{adv})^2 A_{adv} + D^{adv}\left[a^{adv} + E_{adv}\right] w(t+\tau^{adv})\right\} \quad ,t > 0 \\ w(t) &= w_0^{(0)}(t), t \leq 0, \end{aligned} \tag{9}$$

where

$$a^{ret} = A_{ret}B_{ret} + G_{ret}u(t-\tau^{ret}) - H_{ret}C_{ret},\ a^{adv} = A_{adv}B_{adv} + G_{adv}u(t+\tau^{adv}) - H_{adv}C_{adv},$$

$$x(t) = x_0 + u_0 t + \int_0^t\int_0^s w(\theta)\,d\theta\,ds,$$

x_0 is the initial position, u_0 - the initial velocity.

Recall that the functions $\tau^{ret}(t)$ and $\tau^{adv}(t)$ are defined as solutions of equations (6).

In fact, (9) and (6) should be considered jointly like a system. In what follows, however, we show that (6) has a unique continuous solution (each of equations).

We make the following assumptions:

(C) the velocity of the oscillator does not exeed a function $\bar{c}(t) : \mathbf{R} \to \mathbf{R}^+$, that is $|u(t)| \leq \bar{c}(t) < c$ and $sup\{\bar{c}(t) : t \in I\} = \bar{c}_I < c$ for every compact $I \subset \mathbf{R}$.

(R) $\check{r}(t) \geq (c+\bar{c})\tau^{ret}$, $\hat{r}(t) \geq (c+\bar{c})\tau^{adv}$, where $\check{r}(t) = \frac{1}{c}|\check{x}(t) - x(t)|$, $\hat{r}(t) = \frac{1}{c}|\hat{x}(t) - x(t)|$.

(T) the functions $\tau^{ret}(t)$ and $\tau^{adv}(t)$ have a strictly positive lower bound, i.e. $\tau^{ret}(t) \geq \tau_0, \tau^{adv}(t) \geq \tau_0$.

Assumption (R) does not contradict the inequalities $\check{r}(t) \leq 2c\tau^{ret}$ and $\hat{r}(t) \leq 2c\tau^{adv}$. (It is easy to check that every solution of (6) satisfies the inequalities above mentioned). Assumption (T) is quite natural because otherwise the incoming and outgoing fields should be act instantaneously. Assumption (C) is introduced for sake of technical convenient.

Theorem 4 *Under assumptions* $(C),(R)$ *and* (T) *each of equations (6) has a unique solution which is continuous and satisfies the inequalities* $\frac{t}{\tau^{ret}(t)} \leq a$, $\frac{t}{\tau^{adv}(t)} \leq a$, *where* $a = const. > 0$.

Proof: Consider $C(\mathbf{R})$ with a locally convex topology generated by a saturated family of seminorms $\mathcal{A} = \{\|\tau\|_I : I \in \mathcal{B}\}$, where B consists of all compact intervals $I \subset \mathbf{R}$; $\|\tau\|_I = sup\{|\tau(t)| : t \in I\}$. We prove an existence theorem for the first equation of (6). For the second one the proof can be accomplished in the same way.

Define the operator $T : C(\mathbf{R}) \to C(\mathbf{R})$

$$(T\tau)(t) = \frac{1}{c}|\check{x}(t) - x(t - \tau(t))|.$$

Proceed as in [21] we establish the inequality $\|T\tau - T\bar{\tau}\|_I \leq \frac{\bar{c}_I}{c}\|\tau - \bar{\tau}\|_I$, where $\bar{c}_I = sup\{\bar{c}(t) : t \in I\}$.

Define the set $M \subset C(\mathbf{R})$: $M = \{\tau(t) \in C(\mathbf{R}) : \tau(t) \geq \tau_0 \; and \; \frac{t}{\tau(t)} \leq a\}$.

One can be checked that M is closed. The following inequalities imply that T maps M into itself:

$(T\tau)(t) = \frac{1}{c}|\check{x}(t) - x(t - \tau(t))| \geq \frac{1}{c}|\check{x}(t) - x(t)| - \frac{1}{c}|x(t) - x(t - \tau(t))| =$

$= \frac{1}{c}(c + \bar{c})\tau(t) - \frac{1}{c}\bar{c}\tau(t) = \tau(t).$

Then Theorem 1 garantees that T has a unique fixed point which is a solution of the first equation from (6). Theorem 4 is thus proved.

Further on we assume

(W) there exists a function $w_0(t) : \mathbf{R} \to \mathbf{R}^+$ (whose properties will be prescribed below) such that $|w(t)| \leq w_0(t)$.

Let us denote by F the right hand side of (9) and present it as a sum $F = \bar{F} + \check{F} - \hat{F}$, where

$$\bar{F} = \frac{e}{m_0}E_x(t) - \omega^2 x(t),$$

$$\check{F} = \frac{e^2\Delta^2}{2m_0c^2}\{(\Delta^{ret})^2 A_{ret} + D^{ret}[a^{ret} + E_{ret}]w(t - \tau^{ret})\},$$

$$\hat{F} = \frac{e^2\Delta^2}{2m_0c^2}\{(\Delta^{adv})^2 A_{adv} + D^{adv}[a^{adv} + E_{adv}]w(t + \tau^{adv})\}.$$

We need the following estimates:

$$|F| \leq |\bar{F}| + |\check{F}| + |\hat{F}| \leq$$

$$\leq \frac{|e|}{m_0}|E_x(t)| + \omega^2|x(t)| + \frac{e^2}{2m_0}\left[\frac{V_1}{(\tau^{ret})^2} + \frac{V_2 w_0(t)}{\tau^{ret}} + \frac{V_1}{(\tau^{adv})^2} + \frac{V_2 w_0(t)}{\tau^{adv}}\right],$$

$$V_1 = \frac{4(1 + \nu + \nu^2 + \nu^3)\sqrt{1 + \nu^2}}{(1 - \nu)^3}\frac{1}{c^2};\; \nu = \frac{\bar{c}}{c};$$

$$V_2 = \frac{(15 + 12\nu - 9\nu^2 - 6\nu^3)\sqrt{1 + \nu^2}}{(1 - \nu)^4(1 + \nu)}\frac{1}{c^3} + \frac{2 + 2\nu - 4\nu^2 + 2\nu^4 + (6\nu + 6\nu^2)\sqrt{1 - \nu^4}}{(1 - \nu)^4(1 + \nu)}\frac{1}{c^2};$$

$$|\check{F}(\xi_{ret}, u(t), u(t - \tau^{ret}), w(t - \tau^{ret})) - \check{F}(\bar{\xi}_{ret}, \bar{u}(t), \bar{u}(t - \tau^{ret}), \bar{w}(t - \tau^{ret}))| \leq$$

$$\leq \frac{e^2}{2m_0}\{\frac{w_0(t)\Gamma_1}{\tau^{ret}(t)}|u(t) - \bar{u}(t)| + \frac{w_0(t)\Gamma_2}{[\tau^{ret}(t)]^2}|\xi_{ret} - \bar{\xi}_{ret}| +$$

$$+ \frac{w_0(t)\Gamma_3}{\tau^{ret}(t)}|u(t - \tau^{ret}) - \bar{u}(t - \tau^{ret})| + \frac{w_0(t)\Gamma_4}{\tau^{ret}(t)}|w(t - \tau^{ret}) - \bar{w}(t - \tau^{ret})|\},$$

$$|\hat{F}(\xi_{adv},u(t),u(t+\tau^{adv}),w(t+\tau^{adv}))-\hat{F}(\bar{\xi}_{adv},\bar{u}(t),\bar{u}(t+\tau^{adv}),\bar{w}(t+\tau^{adv}))|\leq$$

$$\leq\frac{e^2}{2m_0}\{\frac{w_0(t)\Gamma_1}{\tau^{adv}(t)}|u(t)-\bar{u}(t)|+\frac{w_0(t)\Gamma_2}{[\tau^{adv}(t)]^2}|\xi_{adv}-\bar{\xi}_{adv}|+$$

$$+\frac{w_0(t)\Gamma_3}{\tau^{adv}(t)}|u(t+\tau^{adv})-\bar{u}(t+\tau^{adv})|+\frac{w_0(t)\Gamma_4}{\tau^{adv}(t)}|w(t+\tau^{adv})-\bar{w}(t+\tau^{adv})|\},$$

$$\Gamma_1=\frac{1+11\nu+15\nu^2+7\nu^3-16\nu^4}{(1-\nu)^4}\frac{1}{c^4}+\frac{12\nu^2+6\nu^3+6\nu^4\sqrt{1-\nu^2}}{(1-\nu)^4}\frac{1}{c^3},$$

$$\Gamma_2=\frac{(12\nu^2+18\nu^3+12\nu^4+6\nu^5)\sqrt{1-\nu^2}}{(1-\nu)^4}\frac{1}{c^2}+\frac{8\nu+24\nu^2+26\nu^3-10\nu^5}{(1-\nu)^4}\frac{1}{c^2}+\frac{1+\nu}{(1-\nu)^4}\frac{1}{c^4},$$

$$\Gamma_3=\frac{11+57\nu+74\nu^2+39\nu^3+32\nu^4+12\nu^5-9\nu^6)}{(1-\nu)^5(1+\nu)}\frac{1}{c^4}+$$

$$+\frac{(3+24\nu+33\nu^2+45\nu^3+84\nu^4+6\nu^5)\sqrt{1-\nu^2}}{(1-\nu)^5(1+\nu)}\frac{1}{c^3},$$

$$\Gamma_4=\frac{4+4\nu-\nu^2+\nu^4}{(1-\nu)^4}\frac{1}{c^3}+\frac{(2\nu+3\nu^2+2\nu^3+\nu^4)\sqrt{1-\nu^2}}{(1-\nu)^4}\frac{1}{c^2}.$$

Assumptions (B):

$(B1)$ the function $x(t)$ and $E_x(t)$ are bounded.

$(B2)$ the function $w_0(t)$ satisfies the inequalities

$$\frac{\frac{|e|}{m_0}|E_x(t)|+\omega^2|x(t)|+\frac{e^2}{m_0}\frac{V_1}{\tau_0^2}}{q-\frac{e^2}{m_0}\frac{V_2}{\tau_0}}\leq w_0(t)\leq\frac{q-\frac{e^2}{m_0}\frac{\Gamma_4}{\tau_0}}{\frac{e^2}{m_0}[\Gamma_1 a+(a+1)^2\Gamma_2+(a+1)\Gamma_3]},$$

where

$$\left[\frac{|e|}{m_0}|E_x(t)|+\omega^2|x(t)|+\frac{e^2}{m_0}\frac{V_1}{\tau_0^2}\right]\frac{e^2}{m_0}\left[a\Gamma_1+(a+1)^2\Gamma_2+(a+1)\Gamma_3\right]<\left[q-\frac{e^2}{m_0}\frac{V_2}{\tau_0}\right]^2$$

and $\frac{e^2}{m_0}\frac{V_2}{\tau_0}\leq q$, where $q<1$. (It follows $\frac{e^2}{m_0}\frac{\Gamma_4}{\tau_0}\leq q$, because $\Gamma_4<V_2$).

(B_3) the initial function $w_0^{(0)}(t)$ satisfies $|w_0^{(0)}(t)|\leq w_0(t)$.

Theorem 5 *Under assumptions (W) and (B) initial value problem (9) has at least one solution belonging to $L^\infty_{loc}(\mathbf{R})$.*

Proof: Consider the linear space $X=L^\infty_{loc}(\mathbf{R})$ with a saturated family of seminorms $\mathcal{A}=\{\|\;\|_a : a\in A\}$ where the index set A consists of all compact intervals of $\mathbf{R}$ and $\|w\|_a=esssup\{|w(t)|:t\in a\}$.

Define the operator $T:X\to X$ by the right-hand side of (9) for $t>0$ and by $w_0^{(0)}(t)$ for $t\leq 0$. It is easy to check that T maps X into itself. The problem of existence of composition of measurable functions can be solved as in [23].

Introduce the set $M\subset X$: $M=\{w\in X:|w(t)|\leq w_0(t)\}$. It is easy to verify that M is closed convex and bounded.

The inequalities (cf. $(B2)$)

$|(Tw)(t)|\leq\frac{|e|}{m_0}|E_x(t)|+\omega^2|x(t)|+\frac{e^2}{m_0}\frac{V_1}{\tau_0^2}+\frac{e^2}{m_0}\frac{V_2}{\tau_0}w_0(t)\leq w_0(t)$ for $t>0$

and $|w_0^{(0)}(t)|\leq w_0(t)$ for $t\leq 0$ imply $T(M)\subset M$.

Present the operator T as a sum of $\bar{T}$ and $\widetilde{T}$ where:

$$(\bar{T}w)(t)=\begin{cases}[(\Delta^{ret})^2A_{ret}-(\Delta^{adv})^2A_{adv}]\frac{e^2\Delta^2}{2m_0c^2}-\omega^2x(t), & t>0,\\ 0,\quad t\leq 0\end{cases}$$

$$(\widetilde{T}w)(t)=\begin{cases}\frac{e}{m_0}E_x(t)+\frac{e^2\Delta^2}{2m_0c^2}\{D^{ret}[a^{ret}+E_{ret}]w(t-\tau^{ret})-\\ \qquad -D^{adv}[a^{adv}+E_{adv}]w(t+\tau^{adv})\},\quad t>0,\\ w_0^{(0)}(t),\quad t\le 0\end{cases}$$

Using the inequalities $\frac{d\tau^{ret}(t)}{dt}\le\frac{\sqrt{3}(c+\bar{c}_I)}{(c-\bar{c}_I)}$, $\frac{d\tau^{adv}(t)}{dt}\le\frac{\sqrt{3}(c+\bar{c}_I)}{(c-\bar{c}_I)}$, $(t\in I\in A)$ one can show that T is completely continuous (as in[21]).

Now we can establish that $\widetilde{T}$ is contractive operator.

Choose an arbitrary compact interval $a\in A$ and introduce the notations
$I_a=sup\{t:t\in a\}, I_{ret}=[0,\tau_a^{ret}], I_{adv}=[0,\tau_a^{adv}]$,
$\tau_a^{ret}=sup\{t-\tau^{ret}(t):t\in a\}, \tau_a^{adv}=sup\{t+\tau^{adv}(t):t\in a\}$,
$j_{ret}=\{t-\tau^{ret}(t):t\in a\}, j_{adv}=\{t+\tau^{adv}(t):t\in a\}$,
$\bar{a}=[0,I_a]\cup I_{ret}\cup I_{adv}\cup j_{ret}\cup j_{adv}$.

Define the mapping $j:A\to A$ in the following way:

$$j(a)=\begin{cases}\bar{a}, & if\, a\subset \mathrm{R}^+\ or\ a\cap \mathrm{R}^+\neq\emptyset\\ a, & if\, a\subset \mathrm{R}^-.\end{cases}$$

For $t\in a$ we have

$$|(\widetilde{T}w)(t)-(\widetilde{T}\bar{w})(t)|\le\frac{e^2}{2m_0}\{\frac{w_0(t)\Gamma_1}{\tau^{ret}(t)}|u(t)-\bar{u}(t)|+\frac{w_0(t)\Gamma_2}{[\tau^{ret}(t)]^2}|x(t-\tau_{ret})-\bar{x}(t-\tau_{ret})|+$$
$$+\frac{w_0(t)\Gamma_3}{\tau^{ret}(t)}|u(t-\tau^{ret})-\bar{u}(t-\tau^{ret})|+\frac{\Gamma_4}{\tau^{ret}(t)}|w(t-\tau^{ret})-\bar{w}(t-\tau^{ret})|\}+$$
$$+\frac{e^2}{2m_0}\{\frac{w_0(t)\Gamma_1}{\tau^{adv}(t)}|u(t)-\bar{u}(t)|+\frac{w_0(t)\Gamma_2}{[\tau^{adv}(t)]^2}|x(t+\tau_{adv})-\bar{x}(t+\tau_{adv})|+$$
$$+\frac{w_0(t)\Gamma_3}{\tau^{adv}(t)}|u(t+\tau^{adv})-\bar{u}(t+\tau^{adv})|+\frac{\Gamma_4}{\tau^{adv}(t)}|w(t+\tau^{adv})-\bar{w}(t+\tau^{adv})|\}\le$$
$$\le\{\frac{e^2}{m_0}[a\Gamma_1+(a+1)^2\Gamma_2+(a+1)\Gamma_3]w_0(t)+\frac{e^2}{m_0}\frac{\Gamma_4}{\tau_0}\}\|w-\bar{w}\|_{j(a)}$$

which implies $\|(\widetilde{T}w)(t)-(\widetilde{T}\bar{w})(t)\|_a\le q\|w-\bar{w}\|_{j(a)}$.
Then T is densifying operator (in wiew of Theorem 2) with respect to a measure of noncompactness in the space prescribed in [21].

Finally we observe that the function $w_0(t)$ should be globally essentially bounded which is implied by $(B2)$. Then denoting by γ_a the measure of noncompactness above mentioned we obtain $\gamma_{j^n(a)}(M)\le Q<\infty(n=0,1,2,...)$ where $Q=2\bar{w}_0, \bar{w}_0=esssup\{w_0(t):t\in\mathbf{R}\}$. Since all conditions of Theorem 3 are satisfied initial value problem (9) has at least one solution belonging to M.

Theorem 5 is thus proved.

Now we are able to calculate the spectral density function $\rho(\omega)=u'(\omega)$. Let $w(t)$ be any solution of (9), i.e. it satisfies the equation

$$w(t)+\omega^2x(t)-R_*^{rad}(t)=\frac{e}{m_0}E_x(t)\,(t>0)$$

hence $E_x^2(t)=\frac{m_0^2}{e^2}[w(t)+\omega^2x(t)-R_*^{rad}(t)]^2$.
The energy density function can be obtained by the formula

$u=\frac{3}{4\pi}\bar{E}_x^2=\frac{3}{4\pi}\frac{1}{T}\int_0^T E_x^2\,dt$ where $T=\frac{2\pi}{\omega_0}$, i.e.

$$u=\frac{3m_0^2}{4\pi e^2}\frac{1}{T}\int_0^T[w(t)+\omega^2x(t)-R_*^{rad}(t)]^2\,dt=\frac{3m_0^2}{8\pi^2e^2}\omega_0\int_0^{\frac{2\pi}{\omega_0}}[w(t)+\omega^2x(t)-R_*^{rad}(t)]^2\,dt.$$

Then for $\rho(\omega)$ we have

$$\rho(\omega)=\frac{du}{d\omega}=\frac{3m_0^2}{8\pi^2e^2}\omega_0\int_0^{\frac{2\pi}{\omega_0}}4\omega[w(t)+\omega^2x(t)-R_*^{rad}(t)]^2x(t)\,dt=$$

$$=\frac{3m_0^2}{2\pi^2e^2}\omega_0\omega\int_0^{\frac{2\pi}{\omega_0}}[w(t)+\omega^2x(t)-R_*^{rad}(t)]^2x(t)\,dt.$$

Consequently

$$|\rho(\omega)|\le\frac{3m_0^2}{2\pi^2e^2}\omega_0\omega\int_0^{\frac{2\pi}{\omega_0}}[2w_0(t)+\omega^2|x(t)|]|x(t)|\,dt\le$$

$$\le\frac{3m_0^2}{2\pi^2e^2}[2\omega_0\omega\int_0^{\frac{2\pi}{\omega_0}}w_0(t)|x(t)|\,dt+\omega_0\omega^3\int_0^{\frac{2\pi}{\omega_0}}|x(t)|^2\,dt].$$

Without loss of generality we can assume that $x_0=u_0=0$, i.e. $x(t)=\int_0^t\int_0^s w(\theta)\,d\theta\,ds$ and therefore

$$|x(t)|\le\bar{w}_0(t)\int_0^t\int_0^s d\theta\,ds=\bar{w}_0\int_0^t s\,ds=\bar{w}_0\frac{t^2}{2},\ w_0(t)\le\bar{w}_0.$$

For the spectral density function we obtain the following estimate:

$$|\rho(\omega)|\le\frac{3m_0^2}{2\pi^2e^2}\left[2\omega_0\omega\bar{w}_0\int_0^{\frac{2\pi}{\omega_0}}|x(t)|\,dt+\omega_0\omega^3\int_0^{\frac{2\pi}{\omega_0}}|x(t)|^2\,dt\right]\le$$

$$\le\frac{3m_0^2}{2\pi^2e^2}\left[2\omega_0\omega\bar{w}_0\int_0^{\frac{2\pi}{\omega_0}}\frac{t^2}{2}\,dt+\omega_0\omega^3\int_0^{\frac{2\pi}{\omega_0}}\frac{t^4}{4}\,dt\right]\le\frac{3m_0^2}{2\pi^2e^2}\left[\frac{(2\pi)^3\bar{w}_0}{3}\frac{\omega}{\omega_0^2}+\frac{(2\pi)^5}{20}\frac{\omega^3}{\omega_0^4}\right]\le$$

$$\le\frac{3m_0^2}{2\pi^2e^2}max\left\{\frac{\bar{w}_0(2\pi)^3}{3},\frac{(2\pi)^5}{20}\right\}\left[\frac{\omega}{\omega_0^2}+\frac{\omega^3}{\omega_0^4}\right]\equiv l(\bar{w}_0)\left[\frac{\omega}{\omega_0^2}+\frac{\omega^3}{\omega_0^4}\right].\qquad(10)$$

Finally we show that Ehrenfest paradox does not exist. Indeed, let us recall that Rayleigh-Jeans law is obtained by the assumption that there is some relation between ω and ω_0, i.e. $\omega_0\approx\frac{1}{n}\omega$. We can choose $\omega_0=\omega_0(\omega)$ with an asymptotic behaviour $\omega_0(\omega)=O(\omega^{1+\alpha})$ for $\omega\to\infty$ with $\alpha>0$. For $\alpha>0$ sufficiently small $\omega_0\approx\omega^{1+\alpha}$, this means $\omega_0\approx\omega$. Then it is easy to verify that $|\rho(\omega)|\le const.\left[\frac{1}{\omega^{1+2\alpha}}+\frac{1}{\omega^{1+4\alpha}}\right]$ and hence $\int_0^\infty\rho(\omega)\,d\omega<\infty$ - ultra-violet catastrophe does not exist. And what is more, by a particular choice of the function $\omega_0(\omega)$ we can obtain formulas (4) and (5). Let us put

$$\omega_0(\omega)=[l(\bar{w}_0)]^{\frac{1}{2}}[\frac{kT}{n^2c^3}\omega]^{-\frac{1}{2}}.$$

Then we replace ω_0 in (10) and obtain

$$|\rho(\omega)|\le\frac{kT\omega^2}{n^2c^3}+\frac{\omega^5}{l(\bar{w}_0)}(\frac{kT}{n^2c^3})^2\approx\frac{kT\omega^2}{n^2c^3}\quad(cf.[4]).$$

The second summand can be disregarded because it is of order larger tnah two. In the same way the choise

$$\omega_0(\omega) = [l(\bar{w}_0)]^{\frac{1}{2}} \left[\frac{\hbar\omega^2}{n^2 c^3 (e^{\frac{\hbar\omega}{kT}} - 1)} \right]^{-\frac{1}{2}}$$

implies

$$|\rho(\omega)| \leq \frac{\hbar\omega^3}{n^2 c^3 (e^{\frac{\hbar\omega}{kT}} - 1)} + \frac{1}{l(\bar{w}_0)} \frac{\hbar^2\omega^7}{(n^2 c^3)^2 (e^{\frac{\hbar\omega}{kT}} - 1)^2} \approx \frac{\hbar\omega^3}{n^2 c^3 (e^{\frac{\hbar\omega}{kT}} - 1)} \quad (cf.[5]).$$

Summary

The paper is concerned with the oscillator equation assigned to the black body. A new form of the Lorentz radiation term is derived. It corresponds to the original physical assumptions due to Dirac. This leads to a nonlinear functional differential equation of neutral type. By means of a fixed point theorem an existence result has been proved. As a consequence the ultra-violet catastrophe disappears.

References

[1] J. Schwinger: *Quantum Electrodynamics.* New York, Dover, (1958).

[2] J. Schwinger: *Particles, Sources and Fields.*
v. 1, 2, Keading: Addison-Wesley, (1970 and 1973).

[3] R. P. Feynman: *Quantum Elektrodynamics.* New York, Benjamin, (1973).

[4] N. N. Bogoljubov, D. V. Shirkov: *Introduction in the Theory of Quantized Fields.* Moscow, Nauka, (1976). (In Russian)

[5] K. Hepp: *Renormalization Theory.* Berlin, Springer-Verlag, (1969).

[6] R. Jost: *The General Theory of Quantized Fields.* Providence, R. I. AMS, (1965).

[7] I. I. Todorov: *Analytic Properties of Feynman Diagrams in Quantum Field Theory.* Oxford, Pergamon Press, (1971).

[8] C. Itzykson, J. B. Zuber: *Quantum Field Theory.* Mc Graw Hill, Book Company, New York, (1980).

[9] V. G. Pisarenko: *Problems in Many Body Relativistic Dynamics and Nonlinear Fields Theory.* Kiev, (1974). (In Russian)

[10] P. A. M. Dirac: *Classical Theory of Radiating Electronics.* Proc. Roy. Soc. (London), A 167, (1938), 148-169.

[11] M. Born: *Atomic Physics.* Blackie & Sons Limited, London, (1969).

[12] H. A. Lorentz: *Die Elektronentheorie.* Leipzig, (1909).

[13] D. Ivanenko, A. Sokolov: *Classical Field Theory.* Moscow, (1949).

[14] Y. A. Rjabov: *On the Approximation of Solutions of Nonlinear Differential Equations with Retarding Argument.* Trudy seminara po teor. Differential'nikh uravnenii s zapazdiwaychikh arg. Universitet Patris Lumumba, v.3 (1965), 165-185.(in Russian)

[15] A. A. Sokolov, Y. M. Loskutov, I. M. Ternov: *Quantum Mechanics.* Moscow, 1965, (in Russian).

[16] V. G. Angelov: *Fixed point theorems in uniform spaces and applications.* Czechoslovak Math. Journal, v. 37, (1987), 19-33.

[17] A. Weil: *Sur les espaces a structure uniforme at sur la topologie generale.* Hermann & C-ie, Editeurs, Paris, 1937.

[18] J. L. Kelley: *General Topology.* D. van Nostrand Co., Inc. Princeton, New Jersey, (1953).

[19] B. N. Sadovskii: *Limiting Compact and Densifying Operators:* Uspekhi Mat. Nauk, v.27, No.1 (163), (1972), 82-146 (in Russian).

[20] H. H. Schaefer: *Topological Vector Spaces.* McMillan Co., New York, 1966.

[21] V. G. Angelov: *On the Synge Equations in a Three-Dimensional Two-Body Problem of classical electrodynamics.* J. Math. Anal. Appl., v.151, No.2 (1990), 488-511.

[22] J. L. Synge: *On the Electromagnetic Two-Body Problem.* Proc. Roy. Soc. (London), A 177, (1940), 118-139.

[23] V. G. Angelov, D. D. Bajnov: *Absolutely Continuous Global Solution of the Initial Value Problem for Neutral Functional Differential Equations of Mixed Type.* Rend Circolo Mat. Palermo, v.30, ser.II, (1981), 435-452.