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Нови еквидистантни константно-тегловни кодове над азбуки с три, четири и пет елемента

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New equidistant constant weight codes over alphabet of three, four and five elements

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New Equidistant Constant Weight Codes over the alphabet of three, four and five elements

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Abstract

In this paper we consider the problem of finding bounds on the size of ternary and quaternary equidistant constant weight codes with $2 \leq w < n \leq 10$. Optimal ternary and quaternary equidistant constant weight codes have been constructed by combinatorial and computer methods. Tables of the best known bounds for ternary and quaternary equidistant constant weight codes are presented.

Keywords: equidistant codes, constant weight codes, bounds of codes

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1 Introduction

Consider a finite set of q elements and containing a distinguished element "zero". The choice of a set does not matter in our context and we will use the set Z_q of integers modulo q. Let Z_q^n be the set of n-tuples (or vectors) over Z_q and $Z_q^{n,w}$ be the set of n-tuples over Z_q of Hamming weight w.

A code is called *constant weight* if all the codewords have the same weight w. A code is called *equidistant* if all the distances between distinct codewords are d. Let $B_q(n,d)$ denote the maximum number of codewords in an equidistant code over Z_q of length n and distance d (called an $(n,M,d;q)$ equidistant code or EC) and $B_q(n,d,w)$ denote the maximum number of codewords in an equidistant constant weight code over Z_q of length n, distance d, and weight w (called an $(n,M,d,w;q)$ equidistant constant weight code or ECWC).

Equidistant codes have been investigated by a large number of authors, mainly as examples of designs and other combinatorial objects. Some works published on this topic are [8], [9], [11], [16]. Constant weight codes have been studied by many authors. For some references for the binary case, see Brouwer *et al.* [4], Agrell [1] and for the ternary case, see Bogdanova [3] and Svanström [15]. A few papers study codes which are both equidistant and of constant weight, for example [10], [14] and [7]. The same problem is considered in this paper.

2 Preliminaries

Some bounds for ECWC and EC are given by the following theorems:

Theorem 1 *[7]* $$B_q(n,d) = 1 + B_q(n,d,d).$$

Theorem 2 (Plotkin) *[13, 11]*

$$B_q(n,d) \le \frac{dq}{dq - n(q-1)},$$

if the denominator is positive.

Theorem 3 (Delsarte) *[6]* $$B_q(n,d) \le (q-1)n + 1.$$

Theorem 4 $$B_q(n,n,w) \le q,$$

$$B_q(n+1,d,w) \ge B_q(n,d,w), \qquad B_q(n+1,d,w+1) \ge B_q(n,d,w).$$

The proof of Theorem 4 is easy and we omit it here.

Theorem 5 *(Trivial values)*
$B_3(n,d,w) = 1$ *if* $d > 2w$, $\qquad B_q(n,d,n) = B_{q-1}(n,d)$

Theorem 6 (the Johnson bounds for ECWC)
The maximum number of codewords in a q-ary ECWC satisfy the inequalities:

$$B_q(n,d,w) \le \frac{n}{n-w} B_q(n-1,d,w),$$

$$B_q(n,d,w) \le \frac{n(q-1)}{w} B_q(n-1,d,w-1).$$

The proof of the Theorem 6 is the same as the proof of Johnson bound for constant-weight codes [15].

Theorem 7 *[7] For* $k = 1,2,...,n$, *if* $P_k^2(w) > P_k(d)\,P_k(0)$, *then*

$$B_q(n,d,w) \le \frac{P_k^2(0) - P_k(d)\,P_k(0)}{P_k^2(w) - P_k(d)\,P_k(0)}.$$

Here $P_k(x)$ is the Krawtchouk polynomial defined by

$$P_k(x) = \sum_{i=0}^{k} \binom{x}{i} \binom{n-x}{k-i} (-1)^i (q-1)^{k-i}$$

and

$$P_k(0) = \binom{n}{k} (q-1)^k.$$

According to [10] there are no ECWC of order q, length $q+1$, distance q and weight $q-1$ which have more than $\frac{(q^2+q)}{2}$ codewords, regardless whether q is even or odd.

Definition 1 *A balanced incomplete block design with parameters* (v, b, k, r, λ) *(BIB design* (v, b, k, r, λ)*) is defined as an array of* v *different symbols or elements in* b *subsets or blocks such that every block contains* $k < v$ *different elements, each element occurs in* r *blocks, and each pair of elements occurs in* λ *block. [16]*

Definition 2 *A BIB design is called resolvable (an RBIB design), if its* b *blocks can be separated into* r *groups or repetitions of* q *blocks in such a way that each of the* v *elements occurs exactly once in each repetition.[16]*

Theorem 8 *[16] The optimal equidistant* $(n, qt, d)_q$ *codes and RBIB designs* $(v = qk, b, k, r, \lambda)$ *are equivalent to one another and their parameters are connected by the conditions* $v = M, \quad b = nq, \quad k = t, \quad r = n, \quad \lambda = n - d.$

Theorem 9 *If there exists an* $(n, M, d, w)_q$ *code, then there exists a* $(\lambda n, M, \lambda d, \lambda w)_q$ *code for all integers* $\lambda \geq 1$.

3 Some combinatorial bounds and constructions of ECWC

Proposition 10 *There exists a family of optimal ternary ECWC with parameters* $(n, 3, 3, 2; 3)$ *for every integer* $n \geq 3$.

Proof: Let u be a fixed codeword with length n and weight 2. Consider how many codewords are at distance exactly 3 from u we obtain that $B_3(n, 3, 2) = 3$. ∎

Proposition 11 *For* $w = 2, .., n, \quad P_n(w) = (-1)^w (q-1)^{n-w} \neq 0.$

Proof:

$$P_n(w) = \sum_{i=0}^{n} \binom{w}{i} \binom{n-w}{n-i} (-1)^i (q-1)^{n-i}.$$

For the validity of binomial coefficients the conditions:

$$\left| \begin{array}{l} i \leq w \\ n - i \leq n - w \end{array} \right.$$

must be satisfied. Therefore $i = w$ and $P_n(w) = (-1)^w (q-1)^{n-w} \neq 0$. ∎

Proposition 12 *For* $w = 3, .., n$ *and* $k = n$,

$$B_q(n, 3, w) < (q-1)^3 + 1.$$

Proof: By Theorem 7 with $k = n$ and Proposition 11 we have

$$B_q(n, d, w) = \frac{P_k^2(0) - P_k(0)\, P_k(d)}{P_k^2(w) - P_k(0)\, P_k(d)}.$$

Therefore

$$P_n(3) = \sum_{i=0}^{n} \binom{3}{i} \binom{n-3}{n-i} (-1)^i (q-1)^{n-i} = -(q-1)^{n-3} \Rightarrow$$

$$P_n(0) = \binom{n}{n} (q-1)^n = (q-1)^n$$

and consequently

$$B_q(n, d, w) \le \frac{(q-1)^{2n} + (q-1)^{2n-3}}{P_k^2(w) + (q-1)^{2n-3}} < (q-1)^3 + 1.$$

■

Corollary 13 *There exists a family of optimal ternary ECWC with parameters* $(4+\lambda, 8, 3, 3+t; 3)$ *for every integer* $\lambda \ge 0$ *and* $0 \le t \le n-3$.

Proof: From the Simplex code, which has parameters $(4, 9, 3; 3)$ we construct ECWC $C = \{0111, 0222, 1012, 2021, 1201, 2102, 1120, 2210\}$. From the code C we construct a family of $(4+\lambda, 8, 3, 3+t; 3)$ ECWC C' in the following way:

$$C' = \left\{ \left(\underbrace{00 \ldots 0}_{\lambda - t} \underbrace{11 \ldots 1}_{t}, c \right) | c \in C \right\},$$

where $\lambda \ge 0$ and $0 \le t \le n-3$. Therefore $B_3(n, 3, w) \ge 8$. For these parameters Proposition 12 gives $B_3(n, 3, w) < 9$. So

$$B_3(n, 3, w) = 8.$$

■

4 New results for ternary and quaternary ECWC

For codes of small size we apply combinatorial reasoning. For the rest of the values of M we use our own, specifically developed, computer algorithms.

If two indexes are given in a cell, then the upper one is the method used to find an upper bound, and the lower one is method used to find a lower bound for $B_3(n, d, w)$ and $B_4(n, d, w)$ respectively.

In Table 1 and Table 2 are displayed the exact values of $B_3(n, d, w)$ and $B_4(n, d, w)$ for ternary and quaternary ECWC.

All the numbers in column $d = 3$ for $q = 3$ are obtained by Corollary 13.

Some of the values in Table 1 and Table 2 are represented by the next proposition.

Proposition 14 *There exist optimal ECWC with parameters:*

a) $(5+\lambda,5,4,3;3)$ *for* $\lambda=0,1$

$(5,5,4,3;3)$: {00111, 01022, 10202, 12010, 21100}.

b) $(6+\lambda,6,4,4+\lambda;3)$ *for* $\lambda=0,1,2$

$(6,6,4,4;3)$: {001111, 010122, 012201, 100221, 102102, 220101}.

c) $(7+\lambda,7,4,3+t;3)$ *for* $\lambda=0,1,2,3$ *and* $t=0,1$

$(7,7,4,3;3)$: {0000111, 0011001, 0101010, 0110100, 1001100, 1010010, 1100001 }.

d) $(8+\lambda,8,4,5+t;3)$ *for* $\lambda=0,1,2$

$t=0,$ *for* $\lambda=0$

$t=0,1,$ *for* $\lambda=1,2$

$(8,8,4,5;3)$: {00011111, 00012222, 00101122, 00102211, 01001212, 01002121, 10001221, 10002112}.

e) $(8+\lambda,8,4,7+t;3)$ *for* $\lambda=0,1,2$ *and* $0\le t\le\lambda$

$(8,8,4,7;3)$: {01111111, 01112222, 01221122, 01222211, 02121212, 02122121, 02211221, 02212112}.

f) $(7+\lambda,7,5,4+t;3)$ *for* $\lambda=0,1,2,3$ *and*

$t=0,2,$ *for* $\lambda=0$

$t=0,1,2,$ *for* $\lambda=1,2,3$

$(7,7,5,4;3)$: {0001111, 0110012, 0121200, 1010201, 1022010, 1100120, 1201002}.

g) $(8+\lambda,8,5,5+t;3)$ *for* $\lambda=0,1,2$ *and* $2\le t\le\lambda+2$

$(8,8,5,7;3)$: {01111111, 10111222, 11122012, 11212120, 11221201, 12012211, 12120121, 12201112}.

h) $(5+\lambda,9,3,4+t;4)$ *for* $\lambda=0,1,2,3,4,5$ *and* $0\le t\le\lambda$

$(5,9,3,4;4)$: { 01111, 01222, 01333, 02123, 02231, 02312, 03132, 03213, 03321}.

i) $(5+\lambda,15,4,4+t;4)$ *for* $\lambda=0,1,2,3,4,5$ *and* $0\le t\le\lambda$

$(5,15,4,4;4)$: { 01111, 02222, 03333, 10123, 11032, 12301, 13210, 20231, 21320, 22013, 23102, 30312, 31203, 32130, 33021}.

j) $(8+\lambda,11,5,5+t;4)$ *for* $\lambda=0,1,2$ *and*

$t=0,2,$ *for* $\lambda=0$

$t=0,1,2,$ *for* $\lambda=1$

$t=0,1,2,3,$ *for* $\lambda=2$

$(8,11,5,5;4)$: {00011111, 00022222, 00101233, 00210323, 00323013, 01002313, 02013203, 03020133, 10003123, 20012033, 30021303 }.

k) $(8+\lambda, 12, 6, 6+t; 4)$ *for* $\lambda = 0, 1$ *and* $0 \le t \le \lambda + 1$

$(8, 12, 6, 6; 4)$: {00111111, 01012222, 01103333, 02220123, 02332310, 10031323, 11310013, 12013130, 12102021, 21301120, 33002113, 33110320 }.

l) $(9+\lambda, 11, 7, 6+t; 4)$ *for* $\lambda = 0, 1$ *and* $0 \le t \le \lambda + 2$

$(9, 11, 7, 6; 4)$: {000111111, 001022222, 010033333, 022200123, 023313200, 101301303, 102133020, 110202210, 212021100, 221003011, 320122003 }.

Remark : All $(n+k, M, d, w+t; q)$ codes in the previous proposition, which codewords are not explicitly listed are obtained from $(n, M, d, w; q)$ codes by Theorem 4 and the construction from corollary 13.

We can construct the corresponding EC of an ECWC by adding the all 0's codeword according to Theorem 1. Corresponding EC of some the ECWC that we found are equivalent to RBIB designs according to Theorem 8.

Proposition 15 *There exist optimal EC with parameters:*

a) $(4, 9, 3; 3)$. *This code is equivalent to* $(9, 12, 4, 4, 1)$ *design.*

$(4, 9, 3; 3)$: {0000, 0111, 0222, 1012, 1120, 1201, 2021, 2102, 2210 }.

b) $(5, 6, 4; 3)$. *This code is equivalent to* $(6, 15, 2, 5, 1)$ *design.*

$(5, 6, 4; 3)$: {00000, 01111, 10122, 12201, 21220, 22012}.

c) $(5, 16, 4; 4)$. *This code is equivalent to* $(16, 20, 4, 5, 1)$ *design.*

$(5, 16, 4; 4)$: {00000, 01111, 02222, 03333, 10123, 11032, 12301, 13210, 20231, 21320, 22013, 23102, 30312, 31203, 32130, 33021}.

Table 1. Bounds for optimal ternary ECWC

n	w	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$	$d=8$	$d=9$	$d=10$
3	2	3							
4	2	3	2						
	3	8_z	2						
5	2	3	2						
	3	8	5	2					
	4	8	5_z	2					
6	2	3	3						
	3	8	5	4	2				
	4	8	6	4	3				
	5	8	6	3	2				
7	2	3	3						
	3	8	7_t	4	2				
	4	8	7_t	7_t	3	2			
	5	8	6	6	3	2			
	6	8	6	7_t	2	2			
8	2	3	4						
	3	8	7_t	4	2				
	4	8	7_t	7_t	5	2	2		
	5	8	8_t	7_t	8	3	2		
	6	8	6	7_t	8^p	2	2		
	7	8	8_t	8_t	4	2	2		
9	2	3	4						
	3	8	7_t	4	3				
	4	8	7_t	7_t	9	3	2		
	5	8	8_t	7_t	9	5	3	2	
	6	8	8_t	7_t	11	6^j	3	3	
	7	8	8_t	8_t	12	5	3	2	
	8	8	8_t	8_t	9	3	2	2	
10	2	3	5						
	3	8	7_t	4	3				
	4	8	7_t	7_t	15	5	2		
	5	8	8_t	7_t	12	8	4	2	2
	6	8	8_t	7_t	14	8	5	3	2
	7	8	8_t	8_t	12	9	5	3	2
	8	8	8_t	8_t	15	10	5^p	2	2
	9	8	8_t	8_t	10	5	2	2^p	2

Key to Table 1:
r - ternary constant weight codes; t - Proposition 14; z - Theorem 8;
d - Theorem 3; p - Plotkin bound; j - Theorem 6;
no index -exhaustive search

Table 2. Bounds for optimal quaternary ECWC

n	w	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$	$d=8$	$d=9$	$d=10$
4	2	3	2						
	3	8	4						
5	2	3	2						
	3	8	10^h	2					
	4	9	15_z	3					
6	2	3	3						
	3	8	10	4	2				
	4	9	15^j	9^j	3				
	5	9	15^j	8	3				
7	2	3	3						
	3	8	10	7^j	2				
	4	9	15_t	9_t	5^j	2			
	5	9	15_t	9_t	7^j	3			
	6	9	15_t	9_t	7^j	3			
8	2	3	4						
	3	8	10	7	2				
	4	9	15_t	9	8	2	2		
	5	9	15_t	11	10	5	2		
	6	9	15_t	9	12	5	4_c		
	7	9	15_t	11	12	4	3		
9	2	3	4						
	3	8	10	7	3				
	4	9	15_t	9	9_l	3	2		
	5	9	15_t	11_t	10_t	9^j	3	2	
	6	9	15_t	11_t	12_t	11	5	3	
	7	9	15_t	11_t	12_t	11	5	3	
	8	9	15_t	11_t	12_t	11	4	3	
10	2	3	5						
	3	8	10	7	3				
	4	9	15_t	9	15	5	2		
	5	9	15_t	11_t	12_r	10_t	6_l^j	2	2
	6	9	15_t	11_t	14_r	11_t	10_c^j	5^j	2
	7	9	15_t	11_t	12_t	11_t	14_l	5^j	3
	8	9	15_t	11_t	15_r	11_t	15_c^j	5^j	3
	9	9	15_t	11_t	12_t	11_t	8	4	3

Key to Table 2:
c - Theorem 9; l - lexicographic codes;
r - ternary equidistant codes; t - Proposition 14; z - Theorem 8; h - see [10]; j - Theorem 6;
no index -exhaustive search

5 Lexicographic codes

Let B be an ordered base $b_1, b_2, ..., b_n$ over Z_q^n and let $x = \lambda_1 b_1 + \lambda_2 b_2 + ... + \lambda_n b_n$ and $y = \mu_1 b_1 + \mu_2 b_2 + ... + \mu_n b_n$ be vectors from Z_q^n.

Definition: We say that x precedes y in lexicographical order if $(\lambda_1, \lambda_2, .., \lambda_n)$ precedes $(\mu_1, \mu_2, .., \mu_n)$ in lexicographical order, i.e. $\lambda_1 \leq \mu_1, \lambda_2 \leq \mu_2, .., \lambda_n \leq \mu_n$.

Lexicographic codes of length n and Hamming distance d are obtained by considering all q-ary vectors with weight w in lexicographic order, and adding them to the code if they are at a distance exactly d from the words that have been added earlier.

In order to find *lexicographic codes* we can start the search in the following ways:

- *Without seeds.* As a result, we construct *lexicographic code* where the first codeword is the first word in the vector space Z_q.
- *With seeds.* In this case, it is important to choose a proper seed from one or more vectors. We apply the following methods: exhaustive search, consecutively choosing the possible seeds from restricted area of Z_q and random search.

6 Computer search results of lexicographic equidistant constant weight codes

We apply greedy search beginning with an empty array and while looping through all possible codewords, we add one if it has weight w and distance d from every member of the current code.

For improving the results we use *lexicographic codes with seed. Lexicographic codes with a seed* are obtained in a similar way as the standard lexicographic codes. The difference is that we use an initial set of vectors (called a seed) instead of the empty set.

We obtain bounds for equidistant constant weight codes using standard lexicographic codes or lexicographic codes with seeds. Improved tables for lexicographic equidistant constant weight codes for $q = 3$, $q = 4$ and $q = 5$ and for $2 \leq w < n \leq 10$ are presented in Table 3, Table 4 and Table 5.

Table 3. Bounds on $B_3(n, d, w)$ for $n \leq 10$.

n	w	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$	$d=8$	$d=9$	$d=10$
4	2	3	2						
	3	8	2						
5	2	3	2						
	3	8	5	2					
	4	8	5	2					
6	2	3	3						
	3	8	5	4	2				
	4	8	6	4	3				
	5	8	6	3	2				
7	2	3	3						
	3	8	7	4	2				
	4	8	7	7	3	2			
	5	8	6	6	3	2			
	6	8	6	7	2	2			
8	2	3	4						
	3	8	7	4	2				
	4	8	7	7	5	2	2		
	5	8	8	7	8	3	2		
	6	8	6	7	8	3	2		
	7	8	8	8	4	2	2		
9	2	3	4						
	3	8	7	4	3				
	4	8	7	7	9	3	2		
	5	8	8	7	9	5	3	2	
	6	8	8	7	11	6	3	3	
	7	8	8	8	12	5	3	2	
	8	8	8	8	9	3	2	2	
10	2	3	5						
	3	8	7	4	3				
	4	8	7	7	15	5	2		
	5	8	8	7	11	8	4	2	2
	6	8	8	7	14	8	5	3	2
	7	8	8	8	12	9	5	3	2
	8	8	8	8	12	10	5	2	2
	9	8	8	8	10	5	2	2	2

Table 4. Bounds on $B_4(n, d, w)$ for $n \leq 10$.

n	w	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$	$d=8$	$d=9$	$d=10$
4	2	3	2						
	3	8	4						
	4	9	3						
5	2	3	2						
	3	8	10	2					
	4	9	15	3					
	5	9	6	3					
6	2	3	3						
	3	8	10	4	2				
	4	9	15	9	3				
	5	9	15	8	3				
	6	9	7	4	3				
7	2	3	3						
	3	8	10	7	2				
	4	9	15	9	5	2			
	5	9	15	9	7	3			
	6	9	15	9	7	3			
	7	9	8	7	3	3			
8	2	3	4						
	3	8	10	7	2				
	4	9	15	9	8	2	2		
	5	9	15	11	10	5	2		
	6	9	15	9	12	5	4		
	7	9	15	11	12	4	3		
	8	9	8	8	9	3	3		
9	2	3	4						
	3	8	10	7	3				
	4	9	15	9	9	3	2		
	5	9	15	11	10	9	3	2	
	6	9	15	11	12	11	5	3	
	7	9	15	11	12	11	5	3	
	8	9	15	11	12	11	4	3	
	9	9	8	8	12	6	3	3	
10	2	3	5						
	3	8	10	7	3				
	4	9	15	9	15	5	2		
	5	9	15	11	10	10	6	2	2
	6	9	15	11	14	11	10	5	2
	7	9	15	11	12	11	14	5	3
	8	9	15	11	12	11	15	5	3
	9	9	15	11	12	11	10	4	3
	10	9	8	8	9	10	6	3	3

Table 5. Bounds on $B_5(n, d, w)$ for $n \leq 10$.

n	w	$d=3$	$d=4$	$d=5$	$d=6$	$d=7$	$d=8$	$d=9$	$d=10$
4	2	3	2						
	3	8	4						
	4	9	4						
5	2	4	2						
	3	8	10	2					
	4	9	15	5					
	5	9	16	4					
6	2	3	3						
	3	8	10	4	2				
	4	9	15	9	3				
	5	9	16	24	4				
	6	9	16	9	4				
7	2	4	3						
	3	8	10	7	2				
	4	9	15	13	6	2			
	5	9	16	24	12	3			
	6	9	16	24	14	4			
	7	9	16	10	8	4			
8	2	4	4						
	3	8	10	7	2				
	4	9	15	13	9	2	2		
	5	9	16	24	9	6	2		
	6	9	16	24	12	7	4		
	7	9	16	24	10	4	4		
	8	9	16	10	12	5	4		
9	2	4	4						
	3	8	10	7	3				
	4	9	15	13	11	3	2		
	5	9	16	24	9	8	3	2	
	6	9	16	24	11	10	6	3	
	7	9	16	24	10	4	6	4	
	8	9	16	24	12	4	4	4	
	9	9	16	10	12	4	4	4	
10	2	4	5						
	3	8	10	7	3				
	4	9	15	13	15	5	2		
	5	9	16	24	9	9	6	2	2
	6	9	16	24	11	10	10	3	2
	7	9	16	24	10	4	9	6	3
	8	9	16	24	12	4	4	5	5
	9	9	16	24	12	4	4	4	4
	10	9	16	10	12	4	10	4	4

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