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ON SOLUTIONS OF THE RATIONAL TYPE TO MULTICOMPONENT NONLINEAR EQUATIONS

Tihomir Valchev

In this report we shall propose an algorithm to construct rational type solutions to multicomponent nonlinear evolution equations solvable through inverse scattering transform. The algorithm to be demonstrated is based on Zakharov-Shabat's dressing technique. As an illustration of our approach we shall consider in more detail the derivation of rational solutions to a generalized Heisenberg ferromagnet equation.

1. Introduction

In the late 70's Airault, McKean and Moser [2] demonstrated that the well-known Korteweg-de Vries equation

$$u_t + u_{xxx} + 6uu_x = 0, \qquad u : \mathbb{R}^2 \to \mathbb{C}. \tag{1}$$

has a rational solution of the form:

$$u(x,t) = -\frac{2}{x^2}. \tag{2}$$

The latter represents a "long wave" limit of the 1-soliton solution to the Korteweg-de Vries equation. Later, Ablowitz and Satsuma [1] found an algorithm to derive rational solutions to wide classes of nonlinear evolution equations (NLEEs) based on Hirota's direct method.

Though they may be obtained from solitons, rational solutions are rather different class of solutions, e.g. typically those are **not** traveling waves. Thus their

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study contributes to classification of solutions to NLEEs. Moreover, rational solutions have become a topic of increasing interest after it was observed [3, 4, 13] that rogue waves in the open ocean could be modeled by rational solutions to systems of nonlinear Schrödinger equations (NLS). There exists certain evidence [5, 12] that similar phenomena in optical waveguides and plasmas could be described by rational solutions to other NLEEs like derivative nonlinear Schrödinger equation and 3-wave equation.

The purpose of this report is to demonstrate a method to derive rational type solutions to multicomponent NLEEs integrable by means of inverse scattering transform (IST). The method to be discussed is based on Zakharov-Shabat's dressing technique and seems to be better suited for treating multicomponent NLEEs compared to Ablowitz-Satsuma's approach.

In order to construct rational solutions it suffices to impose certain degeneracy in the spectrum of the corresponding scattering operator. This can be achieved by requiring that the poles of dressing factor belong to the continuous spectrum of the scattering operator.

The report is organized as follows. Next section is preliminary in nature. It introduces some notations and discusses basic facts form the theory of linear pencils of Lax operators. Our main results are presented in Section 3. We will describe there a general algorithm to construct rational type solutions and will consider as an illustration multicomponent NLS and a two-component generalization of Heisenberg's ferromagnet model.

2. Preliminaries

In this section we shall briefly introduce some notations and basic conventions for linear pencils to be used further in text. For more detailed explanations we refer the reader to monograph [8] and paper [9].

Let us consider the Lax operators

$$L(\lambda) = \mathrm{i}\partial_x + U_0(x,t) + \lambda U_1(x,t), \tag{3}$$

$$M(\lambda) = \mathrm{i}\partial_t + \sum_{k=0}^{N} V_k(x,t)\lambda^k, \tag{4}$$

where subscripts mean differentiation in independent variables x and t; $\lambda \in \mathbb{C}$ is spectral parameter and all coefficients $U_{0,1}(x,t)$ and $V_k(x,t)$ belong to a matrix simple Lie algebra $\mathfrak{g}$ over $\mathbb{C}$. The zero curvature condition of (3) and (4)

$$U_t - V_x + \mathrm{i}[U,V] = 0. \tag{5}$$

is equivalent to a nonlinear differential constraint, i.e. a NLEE for dynamical fields.

Making use of the root space decomposition of $\mathfrak{g}$ we can represent each matrix coefficient in the form:

$$U_{0,1} = U_{0,1}^{\mathrm{n}} + U_{0,1}^{\mathrm{d}}, \qquad U_{0,1}^{\mathrm{d}} \in \mathfrak{h}, \quad U_{0,1}^{\mathrm{n}} \in \sum_{\alpha\in\Delta} \mathfrak{g}_\alpha,$$

where $\mathfrak{h} \subset \mathfrak{g}$ is the Cartan subalgebra of $\mathfrak{g}$, Δ is the root space of $\mathfrak{g}$ and $\mathfrak{g}_\alpha \subset \mathfrak{g}$ is the root space corresponding to root α, see [10] for more explanations.

From this point on we shall require that U_1 has eigenvalues that do not depend on x and t. Thus we can put $U_1(x,t)$ into a constant diagonal form by applying gauge transformation

$$L(\lambda) \to \mathcal{L}(\lambda) = \hat{\mathcal{G}}(x,t)L(\lambda)\mathcal{G}(x,t) = \mathrm{i}\partial_x + \mathcal{U}_0(x,t) + \lambda\mathcal{U}_1, \quad \mathcal{U}_1^{\mathrm{d}} = \mathcal{U}_1, \tag{6}$$

$$M(\lambda) \to \mathcal{M}(\lambda) = \hat{\mathcal{G}}(x,t)M(\lambda)\mathcal{G}(x,t) = \mathrm{i}\partial_t + \sum_{k=0}^{N} \mathcal{V}_k(x,t)\lambda^k, \quad \hat{\mathcal{G}} := \mathcal{G}^{-1} \tag{7}$$

on the Lax operators. Gauge transformations preserve zero curvature condition, i.e. $[\mathcal{L},\mathcal{M}] = [L,M] = 0$.

Let us consider now the linear system

$$\mathrm{i}\partial_x\Psi(x,t,\lambda) + U(x,t,\lambda)\Psi(x,t,\lambda) = 0, \tag{8}$$

where it is assumed that U obeys the boundary conditions

$$\lim_{x\to\pm\infty} U(x,t,\lambda) = \lambda\mathcal{G}_\pm\mathcal{U}_1\hat{\mathcal{G}}_\pm, \qquad \mathcal{G}_\pm := \lim_{x\to\pm\infty}\mathcal{G}(x,t). \tag{9}$$

Above $\Psi : \mathbb{R}^2 \times \mathbb{D} \to \mathrm{G}$ is a fundamental set of solutions defined in an open domain $\mathbb{D}$ of the λ-plane and taking values in the Lie group G corresponding to $\mathfrak{g}$. We shall denote by $\mathcal{S}$ the space of all fundamental solutions to (8).

The zero curvature condition (5) implies any fundamental solution to (8) also satisfies the linear system

$$\mathrm{i}\partial_t\Psi + V\Psi = \Psi f \tag{10}$$

for $f(\lambda) = \lim_{|x|\to\infty} \hat{\mathcal{G}}(x,t)V(x,t,\lambda)\mathcal{G}(x,t)$ being dispersion law of NLEE.

Jost solutions Ψ_+ and Ψ_- are usually introduced through the equality:

$$\lim_{x\to\pm\infty} \Psi_\pm(x,t,\lambda)\mathrm{e}^{-\mathrm{i}\lambda\mathcal{U}_1 x}\hat{\mathcal{G}}_\pm = \mathbb{1}. \tag{11}$$

The transition matrix $T(t,\lambda) = \hat{\Psi}_+(x,t,\lambda)\Psi_-(x,t,\lambda)$ represents scattering matrix.

Let us assume a finite group $\mathrm{G_R}$ acts on $\mathcal{S}$ in the following way:

$$\Psi(\lambda) \to \tilde{\Psi}(\lambda) := \mathrm{K_g}[\Psi(\kappa_{\mathrm{g}}(\lambda))], \qquad \mathrm{g} \in \mathrm{G_R}$$

where $\mathrm{K_g} \in \mathrm{Aut}\,(\mathrm{G})$ and $\kappa_{\mathrm{g}} : \mathbb{C} \to \mathbb{C}$ is a conformal mapping. As a result Lax operators acquire certain symmetries reducing the number of independent dynamical fields. This is why $\mathrm{G_R}$ is called reduction group.

Example 1. Generalized Heisenberg ferromagnet equation
Let us consider the following system of coupled NLEEs:

$$\begin{aligned} \mathrm{i}u_t + u_{xx} + (\epsilon_1 u u_x^* + \epsilon_2 v v_x^*)u_x + (\epsilon_1 u u_x^* + \epsilon_2 v v_x^*)_x u &= 0, \\ \mathrm{i}v_t + v_{xx} + (\epsilon_1 u u_x^* + \epsilon_2 v v_x^*)v_x + (\epsilon_1 u u_x^* + \epsilon_2 v v_x^*)_x v &= 0, \qquad \epsilon_{1,2}^2 = 1 \end{aligned} \tag{12}$$

where $*$ means complex conjugation and at least one of $\epsilon_{1,2}$ equals 1. The infinitely smooth functions $u : \mathbb{R}^2 \to \mathbb{C}$ and $v : \mathbb{R}^2 \to \mathbb{C}$ are not independent but obey the constraint $\epsilon_1|u|^2 + \epsilon_2|v|^2 = 1$. The system above turns into the one studied in [9] when setting $\epsilon_1 = \epsilon_2 = 1$. It possesses a Lax pair in the form:

$$L(\lambda) = \mathrm{i}\partial_x + \lambda U_1, \qquad U_1 = \begin{pmatrix} 0 & u & v \\ \epsilon_1 u^* & 0 & 0 \\ \epsilon_2 v^* & 0 & 0 \end{pmatrix}, \tag{13}$$

$$M(\lambda) = \mathrm{i}\partial_t + \lambda V_1 + \lambda^2 V_2, \qquad V_1 = \begin{pmatrix} 0 & a & b \\ \epsilon_1 a^* & 0 & 0 \\ \epsilon_2 b^* & 0 & 0 \end{pmatrix}, \tag{14}$$

$$V_2 = \begin{pmatrix} -1/3 & 0 & 0 \\ 0 & 2/3 - \epsilon_1|u|^2 & -\epsilon_1 u^* v \\ 0 & -\epsilon_2 v^* u & 2/3 - \epsilon_2|v|^2 \end{pmatrix}, \tag{15}$$

$$a = \mathrm{i}u_x + \mathrm{i}\,(\epsilon_1 u u_x^* + \epsilon_2 v v_x^*)\,u, \qquad b = \mathrm{i}v_x + \mathrm{i}\,(\epsilon_1 u u_x^* + \epsilon_2 v v_x^*)\,v. \tag{16}$$

That Lax pair is subject to the reductions:

$$CL(-\lambda)C = L(\lambda), \qquad CM(-\lambda)C = M(\lambda), \qquad C = \mathrm{diag}\,(1, -1, -1).$$

Moreover, it is easily seen that the following relations are fulfilled:

$$\mathcal{E}U_1^\dagger\mathcal{E} = U_1, \qquad \mathcal{E}V_{1,2}^\dagger\mathcal{E} = V_{1,2}, \qquad \mathcal{E} = \mathrm{diag}\,(1, \epsilon_1, \epsilon_2)$$

for $\dagger$ standing for Hermitian conjugation.

In what follows we shall require that $\mathcal{U}_1$ is real. In that case the Jost solutions are defined on the real line of the λ-plane which coincides with the continuous spectrum of $L(\lambda)$. Using Ψ_+ and Ψ_-, however, one can construct another pair of fundamental solutions X^+ and X^- analytic in the upper and lower half-plane respectively. X^+ and X^- are interrelated through

$$X^-(x,t,\lambda) = X^+(x,t,\lambda)G(\lambda), \qquad \operatorname{Im}\lambda = 0. \tag{17}$$

Hence they can be viewed as solutions to a local Riemann-Hilbert factorization problem.

3. Main Results

In this section we shall describe an algorithm to construct rational type solutions based on Zakharov-Shabat's dressing method. Rational solutions of NLEEs integrable by means of IST are usually obtained through a limiting procedure. For instance, one can find soliton type solutions and then evaluate the long-wave limit in the corresponding expressions. Another option consists in performing the limiting procedure into the method of integration from the very beginning. It is our intention here to demonstrate how that concept works when dressing linear pencils of Lax operators.

3.1. General remarks on dressing method

Dressing procedure can symbolically be presented as the following sequence of steps:

$$U^{(0)}, V^{(0)} \to L_0, M_0 \to \Psi_0 \xrightarrow{g} \Psi_1 \to U^{(1)}, V^{(1)} \to L_1, M_1.$$

First, one starts from a L_0-M_0 pair in the form (3) and (4) whose potentials $U^{(0)}$ and $V^{(0)}$ are assumed to be known solutions to a given NLEE, see [8, 9]. Then one applies a gauge transform $\Psi_0 \to \Psi_1 = g\Psi_0$ that maps $(U^{(0)}, V^{(0)})$ onto another potentials $(U^{(1)}, V^{(1)})$ considered solutions to the same NLEE. In doing this one uses a (dressing) factor g depending on λ. We shall pick up g in the form:

$$g(x,t,\lambda) = A(x,t) + \sum_j \frac{B_j(x,t)}{\lambda - \mu_j}, \qquad \mu_j \in \mathbb{C}. \tag{18}$$

Thus to find $(U^{(1)}, V^{(1)})$ one needs to know the residues B_j and the λ-free term A. It turns out those can be expressed in terms of a fundamental solution Ψ_0 to the initial (bare) linear system

$$L_0(\lambda)\Psi_0 = \mathrm{i}\partial_x\Psi_0 + U^{(0)}\Psi_0 = 0. \tag{19}$$

There exist two substantially different cases:

1. generic case, when the poles of g lie outside of the continuous spectrum of scattering operator $L(\lambda)$, i.e. the real line (see the comment on the bottom of page 206);
2. degenerate case, i.e. when the poles belong to the continuous spectrum.

The former case leads to soliton type solutions while the latter produces rational type solutions [14]. From now on we shall be interested in the degenerate case.

The algorithm to find the residues of g consists in 2-steps. In the first step one considers the identity $g\hat{g} = \mathbb{1}$ which leads to a set of algebraic relations. Those imply that B_j are degenerate matrices, i.e. we have the decomposition:

$$B_j(x,t) = X_j(x,t)F_j^T(x,t) \tag{20}$$

where $X_j(x,t)$ and $F_j(x,t)$ are rectangular matrices. The algebraic relations also allow one to express X_j through F_j.

In the second step one considers the partial differential equations

$$\mathrm{i}\partial_x g + U^{(1)} g - gU^{(0)} = 0, \tag{21}$$

$$\mathrm{i}\partial_t g + V^{(1)} g - gV^{(0)} = 0 \tag{22}$$

obtained after comparing bare linear systems with dressed ones. Equations (21) and (22) allow one to find F_j in terms of Ψ_0. In the next subsection we shall illustrate that procedure on special examples of linear pencils.

3.2. Generalized Heisenberg ferromagnet equation

Let us consider the generalized Heisenberg system of equations (12) when $\epsilon_1 = -\epsilon_2 = -1$ and impose the following boundary conditions

$$\lim_{|x|\to\infty} u(x,t) = 0, \qquad \lim_{|x|\to\infty} v(x,t) = 1. \tag{23}$$

It can be proved [9] that now the auxiliary linear problem

$$\mathrm{i}\partial_x \Psi + \lambda U_1 \Psi = 0$$

is related to a Riemann-Hilbert problem with normalization at $\lambda = 0$. Thus to derive rational solutions one may use a dressing factor in the form:

$$g(x,t,\lambda) = \mathbb{1} + \frac{\lambda B(x,t)}{\mu(\lambda-\mu)} + \frac{\lambda C B(x,t) C}{\mu(\lambda+\mu)}, \qquad \mu \in \mathbb{R}. \tag{24}$$

while its inverse is given by

$$\hat{g}(x,t,\lambda) = \mathbb{1} + \frac{\lambda \mathcal{E} B^\dagger(x,t)\mathcal{E}}{\mu(\lambda-\mu)} + \frac{\lambda \mathcal{E} C B^\dagger(x,t) C \mathcal{E}}{\mu(\lambda+\mu)}. \tag{25}$$

The dressed potential $U_1^{(1)}$ can be obtained from the bare one $U_1^{(0)}$ through the relation

$$U_1^{(1)} = \left(\mathbb{1} + \frac{B}{\mu} + \frac{CBC}{\mu}\right) U_1^{(0)} \mathcal{E} \left(\mathbb{1} + \frac{B}{\mu} + \frac{CBC}{\mu}\right)^{\dagger} \mathcal{E}. \tag{26}$$

The identity $g(\lambda)\hat{g}(\lambda) = \mathbb{1}$ gives rise to the following algebraic relations:

$$B\mathcal{E}B^{\dagger} = 0, \qquad \Omega\mathcal{E}B^{\dagger} + B\mathcal{E}\Omega^{\dagger} = 0 \tag{27}$$

where

$$\Omega = \mathbb{1} + B/\mu + CBC/2\mu.$$

The former relation in (27) implies that $B(x,t)$ is degenerate so decomposition (20) holds for some vectors X and F. The vector F is subject to the following constraint:

$$F^T \mathcal{E} F^* = 0. \tag{28}$$

From (21) it follows that F depends on Ψ_0 as follows:

$$F^T = F_0^T \hat{\Psi}_0(\mu) \tag{29}$$

where F_0 is an integration constant. On the other hand, from the latter equation in (27) one can find for X the following result

$$X = \left(\alpha_0 - F^T \partial_\lambda|_{\lambda=\mu} \Psi_0(\lambda)\mathcal{E}F_0^* - \frac{F^T C \mathcal{E} F^*}{2\mu} C\right)^{-1} \mathcal{E}F^* \tag{30}$$

where α_0 is another integration constant.

The time dependence in all the expressions above can be recovered after analyzing equation (22). As a result one gets the following correspondence:

$$F_0^T \;\to\; F_0^T \mathrm{e}^{-\mathrm{i}f(\mu)t}, \tag{31}$$

$$\alpha_0 \;\to\; \alpha_0 - \mathrm{i}F_0^T \left.\frac{\mathrm{d}\, f(\lambda)}{\mathrm{d}\,\lambda}\right|_{\lambda=\mu} \mathcal{E}F_0^* t \tag{32}$$

where the dispersion law of (12) is $f(\lambda) = -\lambda^2 \mathrm{diag}\,(1, -2, 1)/3$.

The simplest choice for a bare potential satisfying boundary condition (23) and a bare fundamental solution is

$$U_1^{(0)} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \qquad \Psi_0(x,\lambda) = \frac{\sqrt{2}}{2} \begin{pmatrix} \mathrm{e}^{\mathrm{i}\lambda x} & 0 & -\mathrm{e}^{-\mathrm{i}\lambda x} \\ 0 & \sqrt{2} & 0 \\ \mathrm{e}^{\mathrm{i}\lambda x} & 0 & \mathrm{e}^{-\mathrm{i}\lambda x} \end{pmatrix}.$$

Then the rational solution for (12) can be written down as:

$$u_1(x,t)=\frac{2\sqrt{2}[\gamma(x)+2\mathrm{i}\mu(2\mu t-x\cos\vartheta)]}{[\gamma(x)-2\mathrm{i}\mu(2\mu t-x\cos\vartheta)]^2}\left[\mathrm{e}^{\mathrm{i}\varphi_-(x,t)}\cos\frac{\vartheta}{2}+\mathrm{e}^{\mathrm{i}\varphi_+(x,t)}\sin\frac{\vartheta}{2}\right], \tag{33}$$

$$v_1(x,t)=\frac{\gamma(x)+2\mathrm{i}\mu(2\mu t-x\cos\vartheta)}{[\gamma(x)-2\mathrm{i}\mu(2\mu t-x\cos\vartheta)]^2}[\gamma(x)-4+2\mathrm{i}\mu(2\mu t-x\cos\vartheta)], \tag{34}$$

$$\gamma(x)=1-\sin\vartheta\cos 2\mu(x+x_0),\qquad \varphi_\pm(x,t)=\mp\mu(x\pm\mu t+x_0)$$

where $\vartheta\in[0,\pi]$, μ and x_0 are parameters of the solution. Clearly, this is not a traveling solution. For $\vartheta=0,\pi$ (33) and (34) turns into a purely rational non-singular solution while for $\vartheta=\pi/2$ the denominator of (33) and (34) vanishes for $t=0$ and $x=-x_0+k\pi/\mu$, $k\in\mathbb{Z}$.

3.3. Multi-component NLS

Let us consider now the linear pencil

$$L(\lambda) = \mathrm{i}\partial_x+Q(x,t)-\lambda J,\qquad Q(x,t)=\begin{pmatrix}0 & \mathbf{q}^T(x,t)\\ \mathcal{E}_n\mathbf{q}^*(x,t)\mathcal{E}_m & 0\end{pmatrix} \tag{35}$$

$$M(\lambda) = \mathrm{i}\partial_t+\sum_{k=0}^{N}\lambda^k A_k(x,t),\qquad J=\begin{pmatrix}\mathbb{1}_m & 0\\ 0 & -\mathbb{1}_n\end{pmatrix} \tag{36}$$

$$\mathcal{E}_m = \mathrm{diag}\,(\epsilon_1,\dots,\epsilon_m),\qquad \mathcal{E}_n=\mathrm{diag}\,(\epsilon_{m+1},\dots,\epsilon_{m+n}) \tag{37}$$

related to symmetric space $\mathrm{SU}(m+n)/\mathrm{S}(\mathrm{U}(m)\times\mathrm{U}(n))$, see [7]. Above $\mathbf{q}(x,t)$ is a $n\times m$ matrix; $\mathbb{1}_m$ is the unit matrix of dimension m; $\epsilon_k^2=1$, $k=1,\dots,m+n$. The quadratic flow of Lax pair ($N=2$) leads to the multicomponent NLS equation

$$\mathrm{i}\mathbf{q}_t+\mathbf{q}_{xx}+2\mathbf{q}\mathcal{E}_m\mathbf{q}^\dagger\mathcal{E}_n\mathbf{q}=0. \tag{38}$$

The simplest class of rational solutions to NLS can be constructed by using a dressing factor in the form:

$$g(x,t,\lambda)=\mathbb{1}+\frac{B(x,t)}{\lambda-\mu},\qquad \mu\in\mathbb{R}. \tag{39}$$

A more detailed consideration [14] similar to that one in the previous example shows that $B(x,t)$ is a degenerate matrix again, i.e. the decomposition (20) applies for X and F being complex $m+n$-vectors. When picking up $Q^{(0)}=0$ and $\Psi_0(x,\lambda)=\exp(-\mathrm{i}\lambda Jx)$ one obtains the following singular rational solution to (38):

$$\mathbf{q}_{ij}(x,t)=\frac{\epsilon_i\mathrm{e}^{-2\mathrm{i}\mu(x+2\mu t+\varphi_{ij})}|\mathcal{F}_{0,i}\mathcal{F}_{0,j}|}{x+4\mu t},\qquad \varphi_{ij}\in\mathbb{R}, \tag{40}$$

$$\mathcal{F}_{0,k} = \frac{F_{0,k}}{\sqrt{\sum_{p=1}^{m} \epsilon_p |F_{0,p}|^2}}, \qquad i = 1, \ldots, n, \quad j = 1, \ldots, m.$$

4. Conclusion

We have showed how dressing technique can be applied to construct rational solutions for classes of completely integrable NLEEs connected to linear pencils. In order to achieve this it suffices to use dressing factors with simple poles lying on the continuous spectrum of scattering operator, see (18). To illustrate that idea we have considered the generalized Heisenberg ferromagnet system (12) and the NLS equation (38). Following the algorithm described in Section 3. one can derive explicit formulas for the solutions, see (33), (34) and (40). A common feature of these rational solutions is they are **not** traveling waves. In contrast to rational solutions to multicomponent NLS, however, the rational solutions to (12) can be non-singular for particular values of parameters. The rational solution (40) can naturally be reduced to well-known rational solutions to the scalar NLS.

By utilizing appropriate dressing factors with multiple poles one is able to construct even more complicated families of rational solutions. The main advantage of the procedure sketched in Section 3. is it allows one to do so directly, i.e. without any knowledge of the corresponding generic soliton type solutions.

In the present report we have focused on the simplest boundary conditions (trivial background). One further step to extend our results is by considering nontrivial background. Nontrivial background solutions were obtained in [3, 4, 11] for the case of the scalar NLS and in [6] for 3-wave equation. However, the considerations required in this case are more complicated and are envisaged to be presented elsewhere.

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Institute of Mathematics
and Informatics
Bulgarian Academy of Sciences
Acad. G. Bonchev Str., Bl. 8
1113 Sofia, Bulgaria
e-mail: `tiv@math.bas.bg`