[Fernandez, 2006] L.Fernandez, F.Arroyo, J.Castellanos, J.A.Tejedor, I.García, New Algorithms for Application of Evolution Rules based on Applicability Benchmarks, BIOCOMP06 International Conference on Bioinformatics and Computational Biology, Las Vegas (USA), july, 2006 (submitted).

[Paun 1998] Gh. Paun, Computing with membranes, Journal of Computer and System Sciences, 61 (2000), and Turku Center for Computer Science-TUCS Report No 208, 1998.

[Petreska 2003] B.Petreska, C.Teuscher, A hardware membrane system. Preproceedings of the Workshop on Membrane Computing (A.Alhazov, C.Martín-Vide and Gh.Paun, eds) Tarragona, July 17-22 2003, 343-355.

Authors' Information

Victor J. Martinez Hernando – Dpto. Arquitectura y Tecnología de Computadores de la Escuela Universitaria de Informatica de la Universidad Politécnica de Madrid, Ctra. Valencia, km. 7, 28031 Madrid (Spain); e-mail: victormh@eui.upm.es

Luis Fernandez Munoz – Dpto. Lenguajes, Proyectos y Sistemas Informaticos de la Escuela Universitaria de Informatica de la Universidad Politécnica de Madrid; Ctra. Valencia, km. 7, 28031 Madrid (Spain); e-mail: setillo@eui.upm.es

Fernando Arroyo Montoro – Dpto. Lenguajes, Proyectos y Sistemas Informaticos de la Escuela Universitaria de Informatica de la Universidad Politécnica de Madrid, Ctra. Valencia, km. 7, 28031 Madrid (Spain); e-mail: farroyo@eui.upm.es

Abraham Gutierrez – Dpto. Informatica Aplicada de la Escuela Universitaria de Informatica de la Universidad Politécnica de Madrid, Ctra. Valencia, km. 7, 28031 Madrid (Spain); e-mail: abraham@eui.upm.es

CONTRADICTION VERSUS SELFCONTRADICTION IN FUZZY LOGIC*

Carmen Torres, Susana Cubillo, Elena Castineira

Abstract: Trillas et al. introduced in [7] and [8] the concepts of both self-contradictory fuzzy set and contradiction between two fuzzy sets. Later, in [1] and [2] the necessity of determine not only the contradiction, but also the degree in that this property occurs, was considered. This paper takes up again these subjects, and firstly we study if there exists some connection between the two first notions. After that, taking into account that self-contradiction of a fuzzy set could be understood as the contradiction with itself, and starting from the degrees of contradiction between two fuzzy sets proposed in [5], we obtain degrees of self-contradiction. Finally, preservation of some intuitive properties both in the use of connectives and in the obtaining of new knowledge throughout compositional rule of inference, are tested.

Keywords: fuzzy sets, t-norm, t-conorm, strong fuzzy negations, contradiction, measures of contradiction, fuzzy relation, compositional rule of inference.

ACM Classification Keywords: F.4.1 Mathematical Logic and Formal Languages: Mathematical Logic (Model theory, Set theory); I.2.3 Artificial Intelligence: Deduction and Theorem Proving (Uncertainty, “fuzzy” and probabilistic reasoning); I.2.4 Artificial Intelligence: Knowledge Representation Formalisms and Methods (Predicate logic, Representation languages).

* This work is partially supported by CICYT (Spain) under project **TIN 2005-08943-C02-001** and by UPM-CAM (Spain) under project **R05/11240**.

Introduction and Preliminary Definitions

This paper begins, as a previous step, with a study on the relation between self-contradiction and contradiction between two fuzzy sets. Then, taking into account that the self-contradiction of a fuzzy set could be understood as the contradiction with itself, remembering some contradiction degrees defined in [5], the corresponding self-contradiction degrees for a fuzzy set, will be proposed, firstly, depending on a given strong negation, and later, without depending on any fixed negation. In the following section, the problem of consistency with connectives, will be managed. In fact, it is necessary to obtain non-contradictory knowledge, when the premises of non-contradictory information are relaxed. And, in a similar way, the information obtained adding contradictory premises, must also be contradictory. Finally, last section will be devoted to study how contradictoriness is transmitted in the reasoning throughout the Compositional Rule of Inference.

Previously, we will remember some definitions and properties necessary throughout this article.

Definition 1.1 ([9]) A fuzzy set (FS) *P*, in the universe $X \neq \emptyset$, is a set given as $P=\{(x,\mu(x)): x \in X\}$ such that, for all $x \in X$, $\mu(x) \in [0,1]$, and where the function $\mu \in [0,1]^X$ is called membership function. We denote $\mathcal{F}(X)$ the set of all fuzzy sets on X.

Definition 1.2 $P \in \mathcal{F}(X)$ with membership function $\mu \in [0,1]^X$ is said to be a normal fuzzy set if $\text{Sup}\{\mu(x) : x \in X\}=1$.

Definition 1.3 A fuzzy negation (FN) is a non-increasing function $N: [0,1] \to [0,1]$ with $N(0)=1$ and $N(1)=0$. Moreover, N is a strong fuzzy negation if the equality $N(N(y))=y$ holds for all $y \in [0,1]$.

N is a strong negation if and only if, there is an order automorphism g in the unit interval (that is, $g:[0,1] \to [0,1]$ is an increasing continuous function with $g(0)=0$ and $g(1)=1$) such that $N(y)=g^{-1}(1-g(y))$ for all $y \in [0,1]$ (see [6]); from now on, let us denote $N_g=g^{-1}(1-g)$. Furthermore, the only fixed point of N_g is $n_g=g^{-1}(1/2)$.

Definition 1.4 ([4]) A function $T: [0,1] \times [0,1] \to [0,1]$ is said to be a *t-norm* if it is a commutative, associative and non-decreasing in both variables function verifying $T(y,1)=y$ for all $y \in [0,1]$.

Definition 1.5 ([4]) A function $S: [0,1] \times [0,1] \to [0,1]$ is said to be a *t-conorm* if it is a commutative, associative and non-decreasing in both variables function verifying $S(y,0)=y$ for all $y \in [0,1]$.

Definition 1.6 ([7]) Given $\mu, \sigma \in [0,1]^X$ and a strong negation N_g, then μ and σ are N_g-contradictory if and only if $\mu(x) \leq N_g(\sigma(x))$, for all $x \in X$. This inequality is equivalent to $g(\mu(x))+g(\sigma(x)) \leq 1$, for all $x \in X$.

Definition 1.7 ([7]) Given $\mu \in [0,1]^X$ and a strong negation N_g, μ is said to be N_g-self-contradictory if and only if $\mu(x) \leq N_g(\mu(x))$, for all $x \in X$. This inequality is equivalent to $g(\mu(x)) \leq 1/2$, for all $x \in X$.

Therefore, the definition of N_g-self-contradictory fuzzy set is a particular case of the N_g-contradictory fuzzy sets definition, where the two sets are the same.

Definition 1.8 $\mu, \sigma \in [0,1]^X$ are contradictory if they are N_g-contradictory regarding some strong FN N_g. And μ is self-contradictory if it is N_g-self-contradictory for some strong FN N_g. This condition is equivalent to the fact that μ is not a normal fuzzy set ($\text{Sup}\{\mu(x) : x \in X\}<1$). Again, the definition of self-contradiction is a particular case of the contradiction definition.

Self-contradiction and Contradiction between Two FS

The goal of this section is to study if there exists some direct relation between the self-contradiction of two fuzzy sets and the contradiction between them. In fact, we have the following properties.

Proposition 2.1 Given $\mu, \sigma \in [0,1]^X$, if μ and σ are N_g–self-contradictory, for some strong fuzzy negation N_g, then μ, σ are N_g–contradictory.

The following example shows that reciprocal is not true.

Example 2.2 Let us consider the set $X=\{x,y\}$ and $\mu, \sigma \in [0,1]^X$ such that $\mu(x)=3/4$, $\mu(y)=0$ and $\sigma(x)=0$, $\sigma(y)=3/4$; and the standard negation $N_s=1\text{-id}$. Then $\mu(x)+\sigma(x)=3/4$ and $\mu(y)+\sigma(y)=3/4$ and so μ, σ are N_s-contradictory between them. Nevertheless μ and σ are not N_s-self-contradictory ($\mu(x)>1/2$ and $\sigma(y)>1/2$).

Proposition 2.3 Given $\mu, \sigma \in [0,1]^X$, if μ and σ are self-contradictory, then μ, σ are contradictory.

Proof: As μ, $\sigma \in [0,1]^X$ are self-contradictory there exist order automorphisms g and g' on [0,1], such that $g(\mu(x)) \le 1/2$ and $g'(\sigma(x)) \le 1/2$ for all $x \in X$. Let us take the following function on [0,1], g''=Min{g,g'}. This function is continuous because g and g' are continuous; g''(0)=0, g''(1)=1. Let us see that g'' is increasing: let $y_1, y_2 \in [0,1]$ be such that $y_1 < y_2$, then $g''(y_1)=\text{Min}\{g(y_1),g'(y_1)\} \le g(y_1) < g(y_2)$ and $g''(y_1)=\text{Min}\{g(y_1),g'(y_1)\} \le g'(y_1) < g'(y_2)$. Therefore $g''(y_1) < \text{Min}\{g(y_2),g'(y_2)\} = g''(y_2)$. So g'' is an order automorphism in the unit interval, and moreover $g''(\mu(x))+g''(\sigma(x)) \le g(\mu(x))+g'(\sigma(x)) \le ½+½ = 1$ for all $x \in X$. Therefore, μ, σ are $N_{g''}$-contradictory and so μ, σ are contradictory.

Newly, reciprocal is not true as the following example shows.

Example 2.4 Let us consider the set $X = \{x_n\}_{n \in IN} \cup \{y_n\}_{n \in IN}$ and μ, $\sigma \in [0,1]^X$ such that $\mu(x_n)=n/(n+1)$, $\mu(y_n)=1/(n+2)$ and $\sigma(x_n)=1/(n+2)$, $\sigma(y_n)=n/(n+1)$. Then $\mu(x_n)+\sigma(x_n)=(n^2+3n+1)/(n^2+3n+2)<1$ and $\mu(y_n)+\sigma(y_n)=(n^2+3n+1)/(n^2+3n+2)<1$ and it follows that μ, σ are N_s-contradictory between them. Nevertheless μ and σ are not self-contradictory ($\text{Sup}\mu(x)=1$ and $\text{Sup}\sigma(x)=1$).

N_g-self-contradiction and Self-contradiction Degrees

Clearly, self-contradiction of a fuzzy set could be viewed as contradiction of the set with itself. Taking this into account, the degrees of contradiction defined in above papers, provide us the respective degrees of self-contradiction, as in this section is shown. In [5], some functions were defined as a model to determine different degrees of N_g-contradiction between two fuzzy sets.

Definition 3.1 Given $\mu, \sigma \in [0,1]^X$ and N_g a strong FN , we define the following contradiction measure functions:

i) $C_1^{N_g}(\mu,\sigma) = \text{Max}\left(0, \underset{x \in X}{\text{Inf}}\left(N_g(\sigma(x)) - \mu(x)\right)\right)$

ii) $C_2^{N_g}(\mu,\sigma) = \text{Max}\left(0, \underset{x \in X}{\text{Inf}}\left(N_g(\mu(x)) - \sigma(x)\right)\right)$

iii) $C_3^{N_g}(\mu,\sigma) = \text{Max}\left(0, 1 - \underset{x \in X}{\text{Sup}}\left(g(\mu(x)) + g(\sigma(x))\right)\right)$

iv) $C_4^{N_g}(\mu,\sigma) = \dfrac{d\left(X_{\mu\sigma}, R_{N_g}\right)}{d\left((0,0), R_{N_g}\right)}$, where d is the Euclidean distance, $X_{\mu\sigma} = \{(\mu(x),\sigma(x)) : x \in X\}$ and

$R_{N_g} = \{(y_1, y_2) \in [0,1]^2 : N_g(y_1) < y_2\}$ is the region free of contradiction. Therefore,

$d\left(X_{\mu\sigma}, R_{N_g}\right) = \text{Inf}\left\{d\left((\mu(x),\sigma(x)), (y_1,y_2)\right) : x \in X, (y_1,y_2) \in R_{N_g}\right\}$ and

$d\left((0,0), R_{N_g}\right) = \text{Inf}\left\{d\left((0,0),(y_1,y_2)\right) : (y_1,y_2) \in R_{N_g}\right\}$.

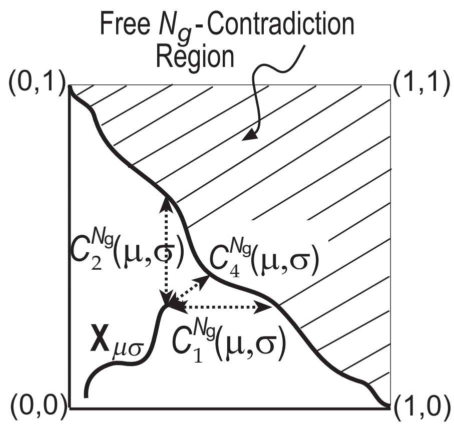

Figure 1: Region free of contradiction and different contradiction degrees

Another new function could serve as definition of contradiction degree:

v) $C_5^{N_g}(\mu,\sigma) = N_g\left(1 - \frac{d\left(\mathrm{X}_{\mu\sigma}, R_{N_g}\right)}{d\left((0,0), R_{N_g}\right)}\right) = N_g\left(1 - C_4^{N_g}(\mu,\sigma)\right)$

For the standard negation $N_s(y)=1-y$ the equality $C_5^{N_s}(\mu,\sigma) = C_i^{N_s}(\mu,\sigma)$ is verified, for all i=1,2,3,4.

And for N_g with $g(y)=y^2$ is $C_5^{N_g}(\mu,\sigma) = \sqrt{1-\left(1-C_4^{N_g}(\mu,\sigma)\right)^2} = \sqrt{C_3^{N_g}(\mu,\sigma)}$.

Considering N_g-self-contradiction as a particular case of N_g-contradiction between two fuzzy sets with $\mu=\sigma$, the N_g-contradiction degrees given in 3.1 are turned into the following N_g-self-contradiction degrees:

i) $C_{s1}^{N_g}(\mu) = C_1^{N_g}(\mu,\mu) = \text{Max}\left(0, \inf_{x\in X}\left(N_g(\mu(x)) - \mu(x)\right)\right) = C_2^{N_g}(\mu,\mu) = C_{s2}^{N_g}(\mu)$

ii) $C_{s3}^{N_g}(\mu) = C_3^{N_g}(\mu,\mu) = \text{Max}\left(0, 1-2\sup_{x\in X}\left(g(\mu(x))\right)\right)$

This measure of N_g-self-contradiction, $C_{s3}^{N_g}(\mu)$, was also defined in [3].

iii) $C_{s4}^{N_g}(\mu) = C_4^{N_g}(\mu,\mu) = \frac{d\left(\mathrm{X}_{\mu\mu}, R_{N_g}\right)}{d\left((0,0), R_{N_g}\right)} = \frac{d\left(\left(\sup_{x\in X}\mu(x), \sup_{x\in X}\mu(x)\right), R_{N_g}\right)}{d\left((0,0), R_{N_g}\right)}$

iv) $C_{s5}^{N_g}(\mu) = C_5^{N_g}(\mu,\mu) = N_g\left(1 - \frac{d\left(\mathrm{X}_{\mu\mu}, R_{N_g}\right)}{d\left((0,0), R_{N_g}\right)}\right) = N_g\left(1 - \frac{d\left(\left(\sup_{x\in X}\mu(x), \sup_{x\in X}\mu(x)\right), R_{N_g}\right)}{d\left((0,0), R_{N_g}\right)}\right)$

It should be pointed out that this last N_g-self-contradiction measure $C_{s5}^{N_g}(\mu)$, in the case of some determined negations N_g, was already obtained in a previous paper [1], as the following proposition shows.

Proposition 3.2 Let N_g be a Yager strong negation, $N_g(y) = \left(1-y^r\right)^{\frac{1}{r}}$, with $0<r\leq 2$ or $N_\lambda(y) = \frac{1-y}{1+\lambda y}$ with $\lambda>0$ a Sugeno strong negation (see figure 2), then for all $\mu \in [0,1]^X$ it is: $C_{s5}^{N_g}(\mu) = N_g\left(\text{Min}\left(1, \frac{\sup_{x\in X}(\mu(x))}{n_g}\right)\right)$.

Proof: The curves $N_g(y) = \left(1-y^r\right)^{\frac{1}{r}}$, with $0<r\leq 2$ and $N_\lambda(y) = \frac{1-y}{1+\lambda y}$ with $\lambda>0$ are under the limit curve $N_g(y) = \sqrt{1-y^2}$, which distance to (0,0) is just 1 (see the figure), and so for all $\mu \in [0,1]^X$ it is:

$d\left(\mathrm{X}_{\mu\mu}, R_{N_g}\right) = \begin{cases} 0 & \text{if } \sup_{x\in X}\mu(x) \geq n_g \\ d\left(\left(\sup_{x\in X}\mu(x), \sup_{x\in X}\mu(x)\right), \left(n_g, n_g\right)\right) & \text{in other case} \end{cases}$ and $d\left((0,0), R_{N_g}\right) = d\left((0,0), \left(n_g, n_g\right)\right) = \sqrt{2}n_g \leq 1$ and

Consequently, $C_{s5}^{N_g}(\mu) = N_g\left(1 - \text{Max}\left(0, 1 - \frac{\sup_{x\in X}(\mu(x))}{n_g}\right)\right) = N_g\left(\text{Min}\left(1, \frac{\sup_{x\in X}(\mu(x))}{n_g}\right)\right)$

We also note that $C_{s4}^{N_g}(\mu) = \text{Max}\left(0, 1 - \frac{\sup_{x\in X}(\mu(x))}{n_g}\right)$.

However, a similar result is not true for all strong fuzzy negation, as the following example shows.

Example 3.3 Let N_g be with $g(y)=y^3$; in this case $d\big((0,0),(n_g,n_g)\big)=2^{\frac{1}{6}}>1$ ($n_g=\left(\tfrac{1}{2}\right)^{\frac{1}{3}}$) and then $d\big((0,0),R_{N_g}\big)=1$. Given $\mu\in[0,1]^X$ such that $X_{\mu\mu}=\left\{\left(\frac{1}{10},\frac{1}{10}\right)\right\}$ it is $d\big(X_{\mu\mu},R_{N_g}\big)=0.899$ and consequently $C_{s5}^{N_g}(\mu)=N_g\left(1-\frac{d\left(X_{\mu\mu},R_{N_g}\right)}{d\left((0,0),R_{N_g}\right)}\right)=N_g(1-0.899)$, but however $d\big(X_{\mu\mu},(n_g,n_g)\big)=0.981$ and so $C_{s5}^{N_g}(\mu)\neq N_g\left(1-\frac{d\left(X_{\mu\mu},(n_g,n_g)\right)}{d\left((0,0),(n_g,n_g)\right)}\right)=N_g\left(\frac{\operatorname{Sup}_{x\in X}(\mu(x))}{n_g}\right)=N_g\left(\frac{\frac{1}{10}}{n_g}\right)$.

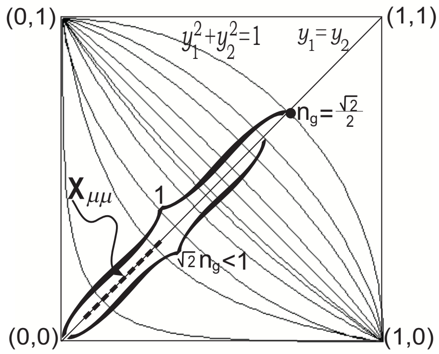

Figure 2: Yager, with 0<r≤2, and Sugeno curves

Until now, we have managed contradiction depending on a fixed strong negation. We continue studying contradiction without depending on any fixed negation. The following degrees of contradiction, between two fuzzy sets, were given in [5].

Definition 3.4 Given $\mu,\sigma\in[0,1]^X$, we have the following contradiction measure functions (see figure 3):

i) $C_1(\mu,\sigma)=\operatorname{Min}\left(\operatorname{Inf}_{x\in X}(1-\mu(x)),\ \operatorname{Inf}_{x\in X}(1-\sigma(x))\right)=\operatorname{Min}\left(d\left(X_{\mu\sigma},L_1\right),d\left(X_{\mu\sigma},L_2\right)\right)$, denoting L_1 the line $y_1=1$ and L_2 the line $y_2=1$.

ii) $C_2(\mu,\sigma)=0$ if there exists $\{x_n\}_{n\in N}\subset X$ such that $\lim_{n\to\infty}\{\mu(x_n)\}=1$ or $\lim_{n\to\infty}\{\sigma(x_n)\}=1$ and, in other case $C_2(\mu,\sigma)=1-\frac{\operatorname{Sup}_{x\in X}(\mu(x)+\sigma(x))}{2}=\frac{d_1\left(X_{\mu\sigma},(1,1)\right)}{d_1((0,0),(1,1))}$, being d_1 the reticular distance.

iii) $C_3(\mu,\sigma)=0$ if there exists $\{x_n\}_{n\in N}\subset X$ such that $\lim_{n\to\infty}\{\mu(x_n)\}=1$ or $\lim_{n\to\infty}\{\sigma(x_n)\}=1$, and, in other case $C_3(\mu,\sigma)=\frac{d\left(X_{\mu\sigma},(1,1)\right)}{d((0,0),(1,1))}$.

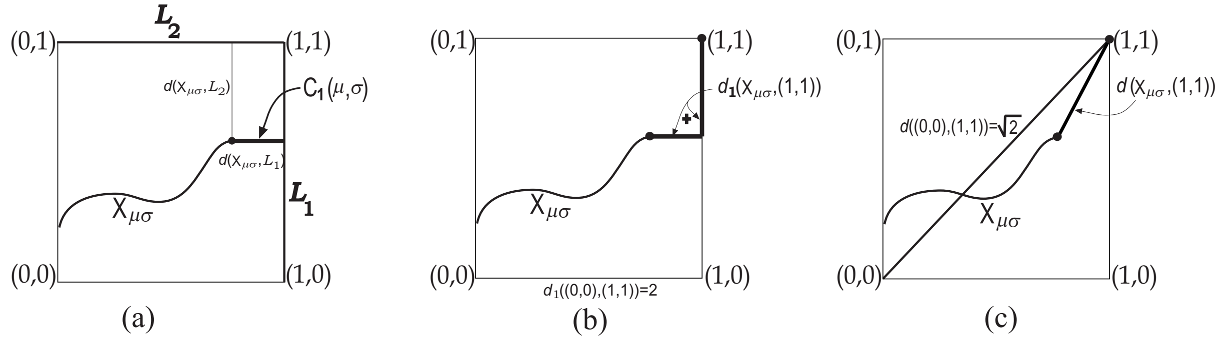

Figure 3: Geometrical interpretation of the measures C_1, C_2 and C_3

Newly, considering self-contradiction as a particular case of contradiction between two fuzzy sets with μ=σ, the contradiction degrees given in 3.4 are turned into the following self-contradiction degrees:

i) $C_{s1}(\mu) = C_1(\mu,\mu) = \operatorname{Inf}_{x\in X}(1-\mu(x)) = 1 - \operatorname{Sup}_{x\in X}(\mu(x)) = C_2(\mu,\mu) = C_{s2}(\mu)$, this measure of self-contradiction was also introduced in [1].

ii) $C_{s3}(\mu) = C_3(\mu,\mu) = \dfrac{d\left(X_{\mu\mu},(1,1)\right)}{\sqrt{2}} = \dfrac{d\left(\left(\underset{x\in X}{Sup}\,\mu(x), \underset{x\in X}{Sup}\,\mu(x)\right),(1,1)\right)}{\sqrt{2}} = \dfrac{\sqrt{2}\left(1-\underset{x\in X}{Sup}\,\mu(x)\right)}{\sqrt{2}} = C_{s1}(\mu)$

Then, three measures are coincidental.

Contradiction Degrees and Connectives

In this section, the problem of consistency with connectives, will be managed. In fact, if we have non-contradictory premises, and these ones are relaxed (by an OR connective, that is, by means of a t-conorm), then the new information must also be non-contradictory. And, in a similar way, if we have contradictory premises, and we add new information (by an AND connective, of a t-norm), the information must also be contradictory.

The following results handle this subject.

Proposition 4.1 Given $\mu \in [0,1]^X$, if μ is not N_g–self-contradictory, for a strong fuzzy negation N_g, then S(μ,σ) is not N_g–self-contradictory, for all S t-conorm and for all $\sigma \in [0,1]^X$.

In particular, if μ, $\sigma \in [0,1]^X$ are not N_g-contradictory then μ is not N_g-self-contradictory or σ is not N_g-self-contradictory (by proposition 2.1), and subsequently S(μ,σ) is not N_g-self-contradictory, for all S t-conorm.

Proposition 4.2 Given $\mu \in [0,1]^X$, if μ is not self-contradictory, then S(μ,σ) is not self-contradictory for all S t-conorm and for all $\sigma \in [0,1]^X$.

In particular, if μ, $\sigma \in [0,1]^X$ are not contradictory then μ is not self-contradictory or σ is not self-contradictory (by proposition 2.3), and subsequently S(μ,σ) is not self-contradictory, for all S t-conorm.

Then, it is obtained that the disjunction with non-contradictory information provides non-self-contradictory information. In addition, the definitions of measures of contradiction also must be consistent with the idea that a disjunction with non-contradictory information remains non-contradictory. In general, for all weak measure of self-contradiction (that is, $C:[0,1]^X \to [0,1]$ such that $C(\mu_\emptyset)=1$, $C(\mu)=0$ if μ normal and C anti-monotonic, as defined in [3]) it is verified that: if C(μ)=0 then C(S(μ,σ))=0 for all S t-conorm and $\sigma \in [0,1]^X$. Furthermore, for all weak measure of contradiction (that is, $C:[0,1]^X \times [0,1]^X \to [0,1]$ such that $C(\mu_\emptyset,\mu_\emptyset)=1$, $C(\mu,\mu)=0$ if μ normal and C symmetric and anti-monotonic [3]) it is verified that: if C(μ,σ)=0 then C(S(μ,σ),S(μ,σ))=0 for all S t-conorm. Furtheremore, we have the following result:

Proposition 4.3 Given $C_i^{N_g}$ with i=1,2,3,4,5 (or C_i with i=1,2,3), and $\mu,\sigma\in [0,1]^X$, if $C_i^{N_g}(\mu,\sigma)=0$ (or $C_i(\mu,\sigma)=0$), then for any t-conorm S it holds that $C_{si}^{N_g}(S(\mu,\sigma))=0$ (or $C_{si}(S(\mu,\sigma))=0$).

Proposition 4.4 Given $\mu \in [0,1]^X$, if μ is N_g–self-contradictory, for some strong fuzzy negation N_g, then T(μ,σ) is N_g–self-contradictory, for all t-norm T and for all $\sigma \in [0,1]^X$.

Moreover, if μ, $\sigma \in [0,1]^X$ are N_g-contradictory then T(μ,σ) is N_g-self-contradictory, for all t-norm T.

Proposition 4.5 Given $\mu \in [0,1]^X$, if μ is self-contradictory, then T(μ,σ) is self-contradictory for all t-norm T and for all $\sigma \in [0,1]^X$.

Moreover, if μ, $\sigma \in [0,1]^X$ are contradictory then they are N_g-contradictory, for some strong fuzzy negation N_g, and consequently T(μ,σ) is N_g-self-contradictory, and therefore T(μ,σ) self-contradictory, for all t-norm T.

Then, it is obtained that the conjunction with contradictory information provides self-contradictory results. Similarly, definitions of measures of contradiction also must be consistent with the idea that a conjunction with contradictory information must remain contradictory. In general, for all weak measure of self-contradiction it is verified that: if C(μ)>0 then C(T(μ,σ))>0 for all t-norm T and $\sigma \in [0,1]^X$. In a similar way, for all weak measure of contradiction it is verified that: if C(μ,σ)>0 then C(T(μ,σ),T(μ,σ))>0 for all t-norm T. Also, we have the following result:

Proposition 4.6 Given $C_i^{N_g}$ with i=1,2,3,4,5 (C_i with i=1,2,3), and μ,σ∈[0,1]^X, if $C_i^{N_g}(\mu,\sigma)>0$ (or $C_i(\mu,\sigma)>0$), then for any t-norm T it holds that $C_{si}^{N_g}(T(\mu,\sigma))>0$ (or $C_{si}(T(\mu,\sigma))>0$).

In particular, if T is a t-norm in the Lukasiewicz's family, that is, $T=g^{-1}\circ W\circ(g\times g)$, with W(x,y)=Max(0,x+y-1), where *g* is an order automorphism in the unit interval, it holds that if $C_i^{N_g}(\mu,\sigma)>0$ then $C_{si}^{N_g}(T(\mu,\sigma))=1$, or equivalently, T(μ,σ)=$\mu_\varnothing$.

Contradiction in Inference

For inference purposes in both classical and fuzzy logic, neither the information itself should be contradictory, nor should any of the items of available information contradict each other. In order to avoid these troubles in fuzzy logic, it is necessary to study self-contradiction and contradiction in the fuzzy inference systems.

The Compositional Rule of Inference ([4]) is based on the Zadeh's Logical Transform:

$$T_J(\mu)(y)=\underset{x\in X}{Sup}\,T(\mu(x),J(x,y))$$

Where $J:X\times X\to[0,1]$ is a given fuzzy relation, T a t-norm and μ ∈ [0,1]^X any fuzzy set. We aim to study the relationship between the contradiction of the input μ and the contradiction of the output $T_J(\mu)$. Also, we want to research the relationship between the degrees of contradiction of the input μ and the degrees of contradiction of the output $T_J(\mu)$.

Proposition 5.1 Given μ ∈ [0,1]^X, if μ is N_g–self-contradictory (or self-contradictory), then $T_J(\mu)$ is N_g–self-contradictory (or self-contradictory), for all t-norm T and all fuzzy relation J.

Reciprocals are not true, as the following example shows.

Example 5.2 Let us consider the set $X=[0,1]$, μ ∈ [0,1]^X such that μ(x)=1-x, J(x,y)=Min(x,y) and T(x,y)=Min(x,y) for all x, y ∈ [0,1]. Therefore, $T_J(\mu)(y)=Min\left(\frac{1}{2},y\right)$ and thus $\underset{y\in[0,1]}{Sup}\,T_J(\mu)(y)=\frac{1}{2}$. Then, $T_J(\mu)$ is N_s-self-contradictory and self-contradictory but μ is neither N_s-self-contradictory nor self-contradictory ($\underset{x\in[0,1]}{Sup}\,\mu(x)=1$).

Moreover, if μ is N_g-self-contradictory (or self-contradictory) then, from proposition, 5.1 and 2.1 (or 2.3), it is obtained that μ and $T_J(\mu)$ are N_g-contradictory (or contradictory) between them, for all t-norm T.

Proposition 5.3 Given μ∈[0,1]^X and a reflexive fuzzy relation J, (J(x,x)=1, ∀x∈X), μ is N_g–self-contradictory (or self-contradictory) if and only if $T_J(\mu)$ is N_g–self-contradictory (or self-contradictory), for all t-norm T.

In addition, if J is a reflexive fuzzy relation, then μ is N_g–self-contradictory (or self-contradictory) if and only if μ and $T_J(\mu)$ are N_g–contradictory (or contradictory) between them, for all t-norm T. Now, let us study if there is some relationship between the contradiction measures of the input μ and those of the inference output $T_J(\mu)$.

Proposition 5.4 Given a reflexive fuzzy relation J and μ∈[0,1]^X such that C(μ)=0 then C($T_J(\mu)$)=0, for all C weak contradiction measure.

If J is not reflexive the last proposition is not true, in general, as the following example shows.

Example 5.5 Let us consider X, μ, T and J as in the example 5.2; J(x,x)=Min(x,x)=x. Then, J is not reflexive. Moreover, μ (x)=1-x is a normal fuzzy set, so C(μ)=0 for all C weak contradiction measure (in particular for C_{si}), however $C_{si}(T_J(\mu))=1-\underset{y\in[0,1]}{Sup}\,T_J(\mu)(y)=\frac{1}{2}$

Also, if J is a reflexive fuzzy relation it is $\mu\le T_J(\mu)$ and therefore $C(\mu)\ge C(T_J(\mu))$, for all weak contradiction measure C.

Finally, let us see that for the N_g-self-contradiction and self-contradiction degrees considered in this paper, the equality between the contradiction degree of the input μ and the contradiction degree of the output $T_J(\mu)$ is verified; being of interest, for it, to consider a previous proposition.

Proposition 5.6 Given $\mu\in[0,1]^X$, for all J fuzzy relation and all t-norm T, the inequality $\underset{x\in X}{Sup}\, T_J(\mu)(x) \le \underset{x\in X}{Sup}\, \mu(x)$ holds.

Consequently, if J is reflexive, it is $\underset{x\in X}{Sup}\, T_J(\mu)(x) = \underset{x\in X}{Sup}\, \mu(x)$.

Corollary 5.7 Given $\mu\in[0,1]^X$, if J is a reflexive fuzzy relation it is $C_{si}^{N_g}(\mu) = C_{si}^{N_g}(T_J(\mu))$ and $C_{si}(\mu) = C_{si}(T_J(\mu))$ for all i and for all t-norm T, being $C_{si}^{N_g}(\mu)$ and $C_{si}(\mu)$ the N_g-self-contradiction and self-contradiction degrees given in definition 3.1 and 3.4.

Conclusions

This paper deepens on the study of contradictoriness in fuzzy sets. New self-contradiction measures have been obtained by means of contradiction measures between two fuzzy sets when the two sets are the same.

Furthermore, some results about the propagation of contradictoriness throughout connectives (t-norms and t-conorms) have been attained. As it was expected, this results are coherent with the human intuition.

Finally, the compositional rule of inference, commonly used in reasoning processes, is studied from the point of view of the contradiction. Results prove non-contradictoriness of input, assure the same property in the output.

Bibliography

[1] E. Castiñeira, S. Cubillo and S. Bellido. Degrees of Contradiction in Fuzzy Sets Theory. Proceedings IPMU'02, 171-176. Annecy (France), 2002.

[2] E. Castiñeira, S. Cubillo. and S. Bellido. Contradicción entre dos conjuntos. Actas ESTYLF'02, 379-383. León (Spain), 2002, (in Spanish).

[3] S. Cubillo and E. Castiñeira. Measuring contradiction in fuzzy logic. International Journal of General Systems, Vol. 34, Nº1, 39-59, 2005.

[4] H. T. Nguyen and E. A. Walker. A first course in fuzzy logic. CRC Press, 1997.

[5] C. Torres, E. Castiñeira S Cubillo and V. Zarzosa. A geometrical interpretation to define contradiction degrees between two fuzzy sets. International Journal "Information Theories and Applications", Vol. 12, Nº2, 127-134, 2005.

[6] E. Trillas. Sobre funciones de negación en la teoría de conjuntos difusos. Stochastica III/1, 47-60, 1979 (in Spanish). Reprinted (English version) (1998) in Avances of Fuzzy Logic. Eds. S. Barro et altr, 31-43.

[7] E. Trillas, C. Alsina and J. Jacas. On Contradiction in Fuzzy Logic. Soft Computing, 3(4), 197-199, 1999.

[8] E. Trillas and S. Cubillo. On Non-Contradictory Input/Output Couples in Zadeh's CRI. Proceedings NAFIPS, 28-32. New York, 1999.

[9] L. A. Zadeh. Fuzzy Sets. Inf. Control, volume 20, 301-312, 1965.

Authors' Information

Carmen Torres – Dept. Applied Mathematic. Computer Science School of University Politécnica of Madrid. Campus Montegancedo. 28660 Boadilla del Monte (Madrid). Spain; e-mail: ctorres@fi.upm.es

Elena Castineira – Dept. Applied Mathematic. Computer Science School of University Politécnica of Madrid. Campus Montegancedo. 28660 Boadilla del Monte (Madrid). Spain; e-mail: ecastineira@fi.upm.es

Susana Cubillo – Dept. Applied Mathematic. Computer Science School of University Politécnica of Madrid. Campus Montegancedo. 28660 Boadilla del Monte (Madrid). Spain; e-mail: scubillo@fi.upm.es