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## FOURIER NEURAL NETWORKS: AN APPROACH WITH SINUSOIDAL ACTIVATION FUNCTIONS<sup>1</sup>

Luis Mingo, Levon Aslanyan, Juan Castellanos,  
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**Abstract:** *This paper presents some ideas about a new neural network architecture that can be compared to a Fourier analysis when dealing periodic signals. Such architecture is based on sinusoidal activation functions with an axo-axonic architecture [1]. A biological axo-axonic connection between two neurons is defined as the weight in a connection in given by the output of another third neuron. This idea can be implemented in the so called Enhanced Neural Networks [2] in which two Multilayer Perceptrons are used; the first one will output the weights that the second MLP uses to computed the desired output. This kind of neural network has universal approximation properties [3] even with lineal activation functions.*

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**Enhanced Neural Networks**

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The only free parameters in the learning algorithm are the weights of one *MLP* since the weights of the other *MLP* are outputs computed by a neural network. This way the backpropagation algorithm must be modified in order to propagate the *Mean Squared Error* through both *MLPs*.

When all activation functions in an axo-axonic architecture are lineal ones ( $f(x)=ax+b$ ) the output of the neural network is a polynomial expression in which the degree  $n$  of the polynomial depends on the number  $m$  of hidden layers [2] ( $n=m+2$ ). This lineal architecture behaves like *Taylor* series approximation but with a global schema instead of the local approximation obtained by Taylor series. All boolean functions  $f(x_1, \dots, x_n)$  can be interpolated with a axo-axonic architecture with lineal activation functions with  $n$  hidden layers, where  $n$  is the number of variables involve in the boolean functions. Any pattern set can be approximated with a polynomial expression, degree  $n+2$ , using an axo-axonic architecture with  $n$  hidden layers. The number of hidden neurons does not affects the polynomial degree but can be increased/decreased in order to obtained a lower *MSE*.

This lineal approach increases *MLP* capabilities but only polynomial approximations can be made. If non lineal activation functions are implemented in an axo-axonic network then different approximation schema can be obtained. That is, a net with sinusoidal functions outputs *Fourier* expressions, a net with *ridge* functions outputs ridge approximation, and so on. The main advantage of using a net is the a global approximation is achieved instead of a local approximation such as in the Fourier analysis.

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## Sinusoidal Activation Functions

In short, considering that only output neurons of the net computing weights have sinusoidal functions, then the output of this net is:

$$o_j = \sum_{i=0}^N w_{ij} o_i, \text{ where } f(x) = \sin(ax + b)$$

Taking into account that weights  $w_{jl}$  of the other net are computing by previous output  $o_j$ , we can say that  $w_{jl}=o_j$ , where  $j=J*N+l$ . Then, desired output follows equation:

$$o_k = \sum_{l=0}^{N_l} w_{kl} o_l, \text{ where } w_{kl}=w_{jl}=o_j=\sin(\cdot)$$

Therefore, considering  $p$  hidden layers in the net, output can be expressed as:

$$o_k = \sum_{n=0}^{n=l+2} A \sin^n(\cdot) + B$$

Considering previous equation, equations involved in *Fourier* analysis are similar to those obtained by axo-axonic networks.

## Results

This section uses three neural network architectures in order to evaluate their forecasting properties. First of all we will present obtained results of a Multilayer Perceptron and a Time-Delay Neural Network, and after that we will show the Time-Delay Enhanced Neural Network results.

The data set has been obtained from the IBEX 35 index. Each pattern represents a day in the stock market and it consists on 5 variables: open value, close value, min value, max value and volume. The pattern set is made up of 8 years, that is 2502 patterns. The desired output is the close value of the IBEX 35 index in time  $t+1$ , that is a forecasting of the future behavior of the signal, without iterative prediction.



**Figure 1.** Desired and MLP net output of IBEX-35 stock market. First figure corresponds to the training set, and the second to the cross validation set. Training MSE=0.007585, Cross Valitation MSE=0.983621.

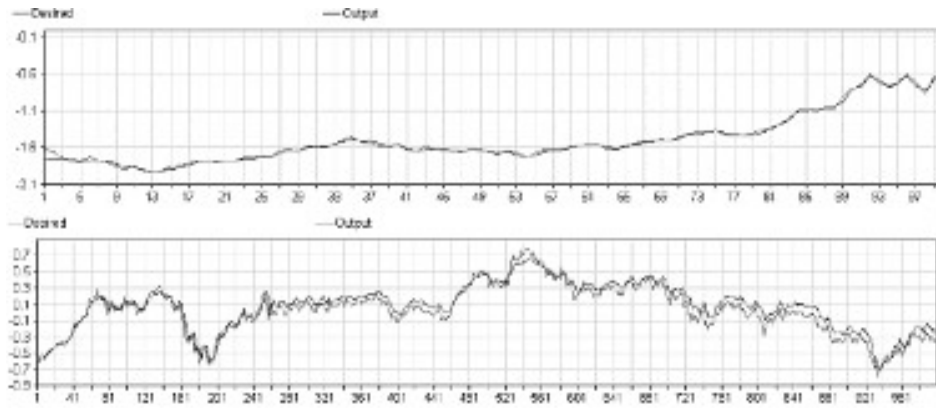


Figure 2. Desired and Time Delay Net output of IBEX-35 stock market. First figure corresponds to the training set, and the second to the cross validation set. Training MSE=0.000585, Cross Validation MSE=0.003621.

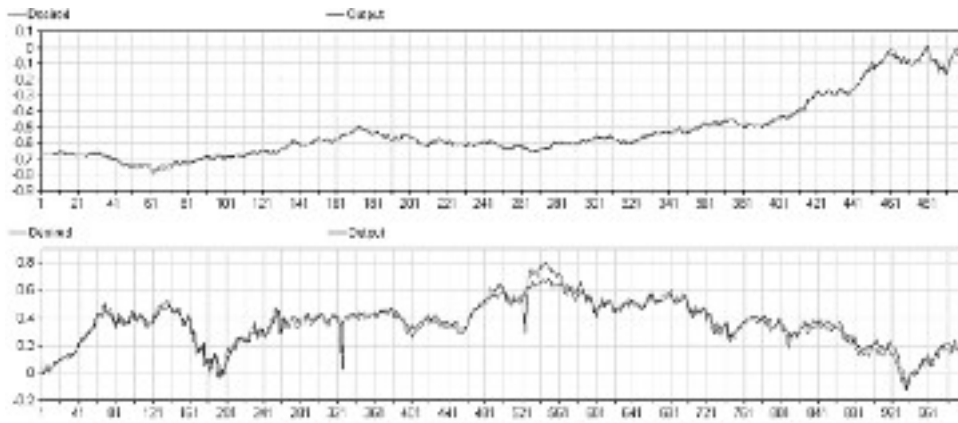


Figure 3. Desired and ENN output of IBEX-35 stock market. First figure corresponds to the training set, and the second to the cross validation set. Training MSE=0.001703, Cross Validation MSE=0.000803.

According to previous figures we can see that ENN with tap delays improves obtained results using TDNN (table 1), this fact is due to the universal approximation capabilities of ENN.

	ENN (fig. 3)	TDNN (fig. 2)
Training set	0.001703	0.000585
CV set	0.000803	0.003621

Table 1. MSE in training and cross validation sets.

### Conclusions

Multilayer perceptrons are a subset of non linear axo-axonic networks, since axo-axonic architectures with a given weights configuration behaves same way as a multilayer net. That is the reason why axo-axonic nets have more approximation capabilities than *MLP*. But the degree of output equation in a *MLP* can not be computed a priori such as in the axo-axonic architectures. Proposed architecture outputs a sinusoidal basis and a non lineal combination of it in order to obtained desired output and it can be compared to a *Fourier* analysis, moreover,

Fourier coefficients can be used to initialize weights of neural networks in order to start the learning process with a low error.

Axo-axonic architectures can be used to forecast signals since they behave like *Fourier* analysis. This kind of connections can also be implemented on *Time-Delay* networks to improve results when dealing periodic signals. Some applications have been developed in order to forecast stock markets, weather and load demand.

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## MUSIC AS THE SOURCE OF INFORMATION INFLUENCE AND SOUL EDUCATION

Larissa Kuzemina

### Extended Abstract

Unlike other works of art (painting, sculpture, etc.) a musical composition should be performed, it should sound to become accessible. Therefore, the role of the musical masterly performance is extremely important. But presently it has increased in importance when music through mass communication media i.e. radio, television, sound recording becomes in the full sense of the word the property of millions.

Art in all its genres as a means of information helps to recreate a picture of one or other epoch as a whole. Moreover, art has a profound impact on education: it can be positive or negative, creative or destructive. Let us dwell on such aspect of music as means of information and the value of musical mastery activity for bringing information to hearers of the alternating generations.

Unlike other works of art (painting, sculpture etc.) a musical composition should be performed, it should sound to become intelligible. Therefore, the role of the musical masterly performance is extremely important. But presently