THE B-TERMINAL BUSY PROBABILITY PREDICTION

Stoyan Poryazov

Abstract: *In the teletraffic engineering of all the telecommunication networks, parameters characterizing the terminal traffic are used. One of the most important of them is the probability of finding the called (B-terminal) busy. This parameter is studied in some of the first and last papers in Teletraffic Theory. We propose a solution in this topic in the case of (virtual) channel systems, such as PSTN and GSM. We propose a detailed conceptual traffic model and, based on it, an analytical macro-state model of the system in stationary state, with: Bernoulli–Poisson–Pascal input flow; repeated calls; limited number of homogeneous terminals; losses due to abandoned and interrupted dialling, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned conversation. Proposed in this paper approach may help in determination of many network traffic characteristics at session level, in performance evaluation of the next generation mobile networks.*

Keywords: *terminal teletraffic, call blocking, human behavior, nonlinear system of equations.*

AMS Subject Classification: *68N01, 65H10, 94C99, 60K30*

1. Introduction

In the teletraffic engineering of all the telecommunication networks, parameters characterizing the terminal traffic are used. One of the most important of them is the probability of finding the called (B-terminal) busy. This parameter is studied in some of the first [Johannesen 1908] and last [Zeng et al 2002] papers in Teletraffic Theory. We propose a solution of this topic in the case of (virtual) channel systems, such as PSTN and GSM.

We propose a detailed conceptual traffic model of a (virtual) circuit switching telecommunication network and, based on it, an analytical macro-state model of the system in stationary state, with: BPP (Bernoulli–Poisson–Pascal) input flow; repeated calls; limited number of homogeneous terminals; losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned conversation.

Proposed in this paper approach may help in determination of many network traffic characteristics at session level, in performance evaluation of the next generation mobile networks ("...dealing with traffic modelling in NG All-IP networks we have to consider three dependent components: mobility behavior of the user, session level teletraffic demands and packet level teletraffic demands."[Koucheryavy et al 2004]).

2. The Conceptual Model

In this paper two types of virtual devices are used: base and comprising base devices.

2.1. Base Virtual Devices and Their Parameters

We will use base virtual device types with names and graphic notation shown on Fig.1. For every device we propose the next notation for its parameters: Letter F stands for intensity of the flow [calls/sec.], P = probability for direction of the external flow to the device considered, T = mean service time in the device of one served call [sec.], Y = intensity of the device traffic [Erl], N = number of service places (lines, servers) in the virtual device (capacity of the device). In the normalized models [Poryazov 2001], used in this paper, every base virtual device, except the switch, has no more of one entrance and/or one exit. Switches have one entrance and two exits.

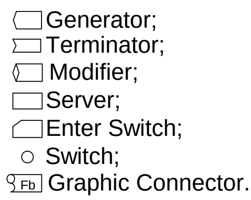

Fig.1. Graphic and text notations of the virtual device types, used in the conceptual model.

Characterizing the intensity of the flow, we are using the next notation: *inc.F* for incoming flow, *dem.F*, *ofd.F* and *rep.F* for demand, offered and repeated flows respectively [ITU E.600]. The same characterization is used for traffic intensity (*Y*).

2.2. The Virtual Base Device Names

In the conceptual model each virtual device has its own name. The names of the devices are constructed on their position in the model basis.

The model is divided on service stages (**d**ialing, **s**witching, **r**inging and **c**onversation).

Every service stage has branches (enter, abandoned, **b**locked, **i**nterrupted, **n**ot available, **c**arried), correspondingly to the modelled possible cases of ends of the calls' service in the branch considered.

Every branch has two exits (**r**epeated, **t**erminated) which show what happens with the calls after they leave the telecommunication system. Users may make a new bid (repeated call), or to stop attempts (terminated call).

In virtual device name construction the corresponding bold letters from the names of stages, branches end exits are used in the order shown below.

Virtual Device Name = <BRANCH EXIT><BRANCH><STAGE>

A parameter's name of one virtual device is a concatenation of parameters name letter and virtual device name. For example, "*Yid*" means "traffic intensity in interrupted dialing case"; "*Fid*" means "flow (calls) intensity in interrupted dialing case"; "*Pid*" means "probability for interrupted dialing"; *Tid* = "mean duration of the interrupted dialing"; "*Frid*" = "intensity of repeated flow calls, caused from (after) interrupted dialing".

2.3. The Paths of the Calls

Figure 2 shows the paths of the calls, generated from the A-terminals in the proposed network traffic model and its environment. *Fo* is the intent intensity of calls of one

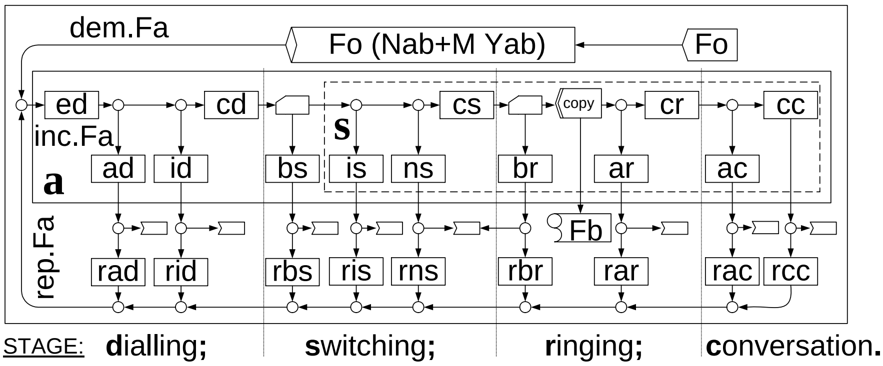

Figure 2. The paths of the calls, generated from the A-terminals in the proposed model.

idle terminal; *M* is a constant, characterizing the BPP flow of demand calls (*dem.Fa*). If *M* = -1, the intensity of demand flow corresponds to Bernoulli (Engset) distribution, if M = 0 - to the Poisson (Erlang), and if *M* = +1 - to the Pascal (Negative Binomial) distribution. In our analytical model every value of *M* in the interval [-1, +1] is allowed. The BPP-traffic model is very applicable [Iversen 2003].

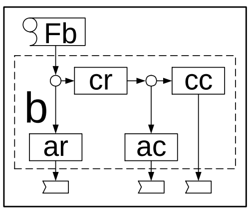

Figure 3. The paths of the calls, occupying B-terminals and corresponding virtual devices.

2.4. The Comprising Virtual Devices and Their Names

The next important virtual devices, comprising several base virtual devices, are considered:

a = a virtual device comprises all the A-terminals (calling) in the system. It is shown with continuous line box in Fig. 2. The devices outside the a-device belong to the network environment. The calls in the environment do not occupy network devices, but they form the incoming to the network flows;

b = a virtual device comprises all the B-terminals (called) in the system. The paths of the calls occupying B-terminals and corresponding virtual devices, included in the box with dashed line, are shown in Fig. 3;

ab = a virtual device comprises all the terminals (calling and called) in the system.

s = a virtual device corresponding to the switching system. It is shown with dashed line box in the Fig. 2.

The flow of calls (B-calls), with intensity Fb, occupying the B-terminals (Fig. 3), is coming from the Copy device (Fig. 2). This corresponds to the fact that at the beginning of the ringing a second (B) terminal in the system becomes busy. The second reason for this conceptual modelling trick is that the paths of the A and B-calls are different in the telecommunication system's environment, after releasing the terminals (compare Figures 2 and 3).

There are two virtual devices from type Enter Switch (Fig. 2) - before Blocked Switching (bs) and Blocked Ringing (br) devices. These devices deflect calls if there is no free line in the switching system and the B-terminal is busy. The correspondent transitions probabilities depend on the macrostate of the system (Yab).

The macrostate of a (virtual) device is defined as the mean number of simultaneously served calls in this device, in the observed time interval (similar to “mean traffic intensity” in [ITU E.600]).

3. The Analytical Model

3.1. Main Assumptions

For creating a simple analytical model, we make the next system of twelve (A1 - A12) assumptions:

A1. We consider a closed telecommunication system with functional structure shown in Fig. 2 and Fig. 3;

A2. All the terminals are homogeneous, e.g. all relevant characteristics are equal for every terminal;

A3. Every terminal directs the all its calls only to other terminals, not to itself;

A4. All virtual devices in the model (Fig.2 and Fig. 3) have unlimited capacity, with exceptions of the ab-device comprising all the $Nab \in [2, \infty)$ terminals and switching system (s) which has capacity of *Ns* internal switching lines. Every terminal has capacity 1, common for both incoming and outgoing calls;

A5. Every call, from the incoming in the telecommunication system flow (*inc.Fa*), falls only on a free terminal. This terminal becomes busy A-terminal. One call may occupy only one terminal and one terminal may serve only one call;

A6. Every call may occupy one internal switching line, if it find free one, independently from the state of the intent B-terminal (busy or free);

A7. Probabilities of direction of calls to, and duration of occupation of devices ar, cr, ac and cc are the same for A and B-calls (Fig.2 and Fig. 3);

A8. We consider probabilities for direction of calls to, and holding times in the base virtual devices as independent from each other and from intensity $Fa = inc.Fa$ of incoming flow of calls. Values of these parameters are determined from users' behaviour and technical characteristics of the communication system. (This is not applicable to Pbs and Pbr only - see 2.4.);

A9. The system is in stationary state. This means that in every virtual device in the model (including comprising devices like switching system), the intensity of input flow $F(0, t)$, calls holding time $T(0, t)$ and traffic intensity $Y(0, t)$ in the observed interval $(0,t)$ converge to the correspondent finite numbers F, T and Y, when $t \to \infty$. In this case we may apply the theorem of Little [Little 1961] and for every device: $Y = FT$;

A10. The flows directed to A-terminals (*Fa*) and to B-terminals (Fb) are ordinary. For Fa this is usual premise, but for Fb A5 may be acquitted from results like in [Burk 1956] and [Vere-Jones 1968];

A11. The mean probability of a call to find B-terminal busy at the first and at the all next repeated attempts is one and the same. This is the only assumption in this paper, causing systematic error. In [Poryazov 1992] is shown, on the basis of comparison of analytic and simulation [Todorov, Poryazov 1985] results, that this error is stable and don't exceed 5% of the Pbr in the reasonable traffic load interval;

A12. All variables in the analytical model may be random and we are working with their mean values.

3.2. Equations

From definitions of *a* and *b* comprising devices and assumptions A1 and A4, obviously the sum of traffic intensities of A and B-terminals gives the traffic intensity of the all occupied terminals in the system:

$$Ya + Yb = Yab \le Nab\,. \quad (1)$$

Theorem 1. Traffic intensity of B-terminals may be calculated from the equation

$$Yb = FbTb\,, \quad (2)$$

where:

$$Fb = Fa(1 - Pad)(1 - Pid)(1 - Pbs)(1 - Pis)(1 - Pns)(1 - Pbr)\,, \quad (3)$$

$$Tb = ParTar + (1 - Par)[Tcr + PacTac + (1 - Pac)Tcc]\,. \quad (4)$$

Proof: Equation (2) is the formula of Little for device *b* (A9). $Fb = inc.Fb$ is intensity of calls occupying B-terminals and (3) is direct corollary from A1, Fig. 2, A8, and A9.

We may receive the expression for B-terminals holding time (4) from the next considerations. From A1, Fig.3, A8 and A9 follow that:

$$Yb = Fb\ Tb = Yar + Ycr + Yac + Ycc; \quad (5)$$

$$Yar = Far\ Tar = Fb\ Par\ Tar; \quad (6)$$

$$Ycr = Fcr\ Tcr = Fb\ (1\text{-}Par)\ Tcr; \quad (7)$$

$$Yac = Fac\ Tac = Fb(1\text{-}Par)Pac\ Tac; \quad (8)$$

$$Ycc = Fcc\ Tcc = Fb(1\text{-}Par)(1\text{-}Pac)\ Tcc. \quad (9)$$

After replacing (6), (7), (8) and (9) in (5), we receive (4).

Theorem 2. A-terminals' traffic intensity (Ya) is determining from the expression:

$$Ya = FaTa = Fa\{(Ted+PadTad+(1\text{-}Pad)(PidTid+(1\text{-}Pid) (Tcd+PbsTbs+(1\text{-}Pbs)(PisTis+(1\text{-}Pis) (PnsTns+(1\text{-}Pns)(Tcs+PbrTbr+(1\text{-}Pbr)Tb))))))\}. \quad (10)$$

The proof of Theorem 2 is very similar to the proof of Theorem 1, but assumption A11 is used in addition to A1, A8 and A9.

Following the same technique one may easy receive equation (11.1) of the system (11):

$$Yab = Fa\ \{Ted + PadTad + (1\text{-}Pad)[PidTid + (1\text{-}Pid)[Tcd + PbsTbs + (1\text{-}Pbs)[PisTis + (1\text{-}Pis)\ [PnsTns + (1\text{-}Pns)[Tcs + PbrTbr + 2(1\text{-}Pbr)Tb]]]]]. \quad (11.1)$$

$$Fa = dem.Fa + rep.Fa. \quad (11.2)$$

$$dem.Fa = Fo\ (Nab + M\ Yab). \quad (11.3)$$

$$rep.Fa = Fa\ \{PadPrad + (1\text{-}Pad)[PidPrid + (1\text{-}Pid)[PbsPrbs + (1\text{-}Pbs) [PisPris + (1\text{-}Pis)[PnsPrns + (1\text{-}Pns)[PbrPrbr + (1\text{-}Pbr) [ParPrar + (1\text{-}Par)[PacPrac + (1\text{-}Pac)Prcc]]]]]]]\}. \quad (11.4)$$

$$Pbr = \frac{Yab\text{-}1}{Nab\text{-}1} \quad if \quad 1 \leq Yab \leq Nab,$$
$$Pbr = 0 \quad if \quad 0 \leq Yab < 1. \tag{11.5}$$

$$ofd.Fs = Fa\ (1\text{-}Pad)(1\text{-}Pid)\ . \tag{11.6}$$

$$Ts = [PisTis + (1\text{-}Pis)[PnsTns + (1\text{-}Pns)[Tcs + PbrTbr + (1\text{-}Pbr)Tb]]]. \tag{11.7}$$

$$ofd.Ys = ofd.Fs\ Ts. \tag{11.8}$$

$$Pbs = Erl\_b\ (Ns,\ ofd.Ys). \tag{11.9}$$

We may remark that the equations:

(11.2)	simply expresses that intensity of the flow of calls occupying A-terminal is sum of primary (demand) and repeated calls [ITU E.600] (see Fig. 2);
(11.3)	shows intensity of demand calls as a function of the intensity of generated calls from one idle terminal (*Fo*) and the macrostate (*Yab*) of the system (BPP - flow, see Fig. 2);
(11.4)	determinates the intensity of repeated calls (*rep.Fa*), as a function of transitions probabilities in the model. It is received in the same way as (11.1) (see Fig. 2);
(11.5)	is discussed in 4;
(11.6)	expresses intensity of the offered to the switching system flow of calls;
(11.7)	*Ts* is the holding time of calls in the switching system, received in the same way as (11.1) (see Fig. 2);
(11.8)	defines offered traffic (*ofd.Ys*) to the switching system [ITU E.600];
(11.9)	expresses usage of the Erlang-B formula for determination of the blocking probability in the switching system, on the basis of the number of internal switching lines (*Ns*) and offered traffic.

In general, the system of equations (11) has:

- 9 equations;
- 9 output parameters with unknown values: *Yab, Fa, dem.Fa, rep.Fa, Pbs, Pbr, ofd.Fs, Ts, ofd.Ys*;
- 32 input parameters with known (given) values: *Nab, Ns, M, Fo, Ted, Pad, Tad, Prad, Tcd, Pid, Tid, Prid, Tbs, Prbs, Pis, Tis, Pris, Pns, Tns, Prns, Tcs, Tbr, Prbr, Par, Tar, Prar, Tcr, Pac, Tac, Prac, Tcc, Prcc*.

4. The B-Terminal Busy Probability Prediction

Theorem 3. The probability of finding the B-terminal busy (Pbr) if $1 \leq Yab \leq Nab$, is:

$$Pbr = \frac{Yab - 1}{Nab - 1} \tag{12}$$

Proof: According assumptions A1 all the calls are directed to the terminals inside the investigated system. Follow A2, all the terminals have equal probability to be called, but A3 excludes the calling A-terminal. Similar natural assumption is made from [Ionin 1978]. Consequently every call is directed with equal probability to $Nab\text{-}1$ terminals, with $Yab\text{-}1$ of them busy. From A10 it follows that two calls can't come simultaneously and hence their probabilities to find B-terminal busy are independent from each other (they depend from the system state only).

Let call number i finds $Yab(i)$ busy terminals. Than its probability to find B-terminal busy ($Pbr(i)$), under the assumptions made, is:

$$Pbr(i) = \frac{Yab(i) - 1}{Nab - 1} \tag{13}$$

If we consider n calls reaching B-terminals, then their mean probability Pbr to find B-terminals busy is:

$$Pbr = \frac{1}{n}\sum_{i=1}^{n} Pbr(i) = \frac{1}{n}\sum_{i=1}^{n}\frac{Yab(i)-1}{Nab-1} = \frac{1}{(Nab-1)}\frac{\sum_{i=1}^{n}(Yab(i)-1)}{n}. \quad (14)$$

From the A9 follows that the system is in a stationary state. Therefore, the mean value of the intensity of the terminal traffic exists and equals to the Yab. In the other words:

$$Yab = \lim_{n\to\infty}\frac{\sum_{i=1}^{n} Yab(i)}{n}. \quad (15)$$

From (15) when $n \to \infty$ and (14), we receive (12).

Theorem 4. A threshold ($thr.Fa > 0$) of the intensity of the input flow Fa exists, so that in the interval $Fa \in [0, thr.Fa]$ busy terminals exist, but there are not losses due the finding B-terminal busy. In this case:

$$\text{thr}.Fa = \frac{1}{S_1}, \quad (16)$$

where:

$S_1 = Ted + Pad\ Tad + (1 - Pad)(Pid\ Tid + (1 - Pid)(Tcd + Pbs\ Tbs + (1-Pbs)\ (Pis\ Tis + (1-Pis)(Pns\ Tns + (1 - Pns)\ (Tcs + 2\ Tb))))).$

Proof: We may present (11.1) in the form:

$$Yab = Fa(S_1 - S_2 Pbr), \quad (17)$$

where:

$S_2 = (1 - Pad)(1 - Pid)(1 - Pbs)\ (1 - Pis)(1 - Pns)(2\ Tb - Tbr).$

If we change Pbr with 0 in (17), we receive:

$$Yab = FaS_1. \quad (18)$$

Equation (18) is received without assumptions of any dependence between Fa and Pbr and therefore is true in any cases, when $Pbr = 0$. From (18) it is obviously that the value of $thr.Fa$ from (16) ($S_1 > 0$ in the all working systems) is the only when $Yab = 1$ and $Pbr = 0$.

Comment: The fact, that in the interval $Fa \in [0, thr.Fa]$ we have not losses due finding B-terminal busy, must be understand in the asymptotic case, when $t \to \infty$. In the other words, losses may exist, but:

$$Pbr = \lim_{t\to\infty}\frac{Zbr.a(0,t)}{Zbr.a(0,t) + Z.b(0,t)} = 0, \quad (19)$$

where $Zbr.a(0,t)$ notes the number of the all calls finding the B-terminal busy in the interval of observation $(0,t)$ and $Z.b(0,t)$ is the number of calls successfully sizing B-terminals in the same time interval.

Based on Theorem 3 and Theorem 4, we may define (11.5). This definition is used in a very simple teletraffic model [Poryazov 1991] without detailed proof.

5. Conclusions

- Detailed conceptual and analytical models of a telecommunication system are created.
- A mathematical model, which may be used for prediction of the probability of finding B-terminal busy, is proved.

Bibliography

[Burk 1956] Burk, P. J. The output of a queueing system. J. Op. Res. Soc. Amer. 4, 699-704, 1956.

[Ionin 1978] Ионин Г. Л. Исследование необратимых состояний системы массового обслуживания с повторными вызовами и конечным числом источников. В кн.: Информационные сети и их анализ. Москва, «Наука», 1978. с. 13-31.

[ITU E.600] ITU-T Recommendation E.600: Terms and Definitions of Traffic Engineering. (Melbourne, 1988; revised at Helsinki, 1993).

[Iversen 2003] Villy B. Iversen TELETRAFFIC ENGINEERING HANDBOOK. ITU-D SG 2/16 & ITC. Draft, December 2003, pp. 328. http://www.tele.dtu.dk/teletraffic/ (11.12.2004)

[Johannesen 1908] Fr. Johannesen. "Busy" The frequency of reporting "busy" and the cost caused thereby. The Copenhagen Telephone Company, Copenhagen, October 1908, 4 pp.

[Koucheryavy et al 2004] Y. Koucheryavy, D. Moltchanov, J. Harju, G. Giambene. Cross-layer black-box approach to performance evaluation of next generation mobile networks. NEW2AN 2004, 7 pp.

[Little 1961] J. D. C. Little. A Proof of the Queueing Formula L=λW. Operations Research, 9, 1961, 383-387.

[Poryazov 1991] S. Poryazov. Determination of the Probability of Finding B-Subscriber Busy in Telephone Systems with Decentralized Control. Comptes Rendus de l'Academie Bulgare des Sciences – Sofia, 1991, Tome 44, No.3, pp. 37-39.

[Poryazov 1992] S. Poryazov. Determining of the Basic Traffic and Time Characteristics of Telephone Systems with Nonhomogeneous Subscribers. 4th International Seminar on Teletreffic Theory and Computer Modelling – Moscow October 20-25, 1992 – pp. 130-138.

[Poryazov 2001] S. Poryazov. On the Two Basic Structures of the Teletraffic Models. Conference "Telecom'2001" - Varna, Bulgaria, 10-12 October 2001 – pp. 435-450).

[Todorov, Poryazov 1985] Todorov, S. Poryazov. Basic Dependences Characterizing a Model of an Idealised (Standard) Subscriber Automatic Telephone Exchange. Proc. of the 11-th ITC, September 4-11, 1985 Kyoto, Japan, pp 4.3B-3-1- 4.3B-3-8.

[Vere-Jones 1968] Vere-Jones D. Some applications of probability generating functionals to the study of input-output streams. J. Roy. Statist. Soc., 1968, B30, No 2, 321-333.

[Zeng et al 2002] H. Zeng, Y. Fang, I. Chlamtac. Call Blocking Performance Study for PCS Networks under More Realistic Mobility Assunptions. Telecommunication Systems 19:2, 125–146, 2002, Kluwer Academic Publishers.

Author Information

Stoyan A. Poryazov - Institute of Mathematics and Informatics, Bulgarian Academy of Sciences,
Acad. G. Bonchev Str., Block 8, 1113 Sofia, Bulgaria, Tel: (+359 2) 979 28 46,
Fax: (+359 2) 971 36 49, e-mail: stoyan@cc.bas.bg