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ON LIMITING CASE OF THE STEIN-WEISS TYPE INEQUALITY FOR THE B-RIESZ POTENTIALS *)

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Abstract

In this paper we study the Riesz potentials (B-Riesz potentials) generated by the Laplace-Bessel differential operator $\Delta_B = \sum\limits_{k=1}^{n} \frac{\partial^2}{\partial x_k^2} + \frac{\gamma}{x_n}\frac{\partial}{\partial x_n}$, $\gamma > 0$, in the weighted Lebesgue spaces $L_{p,|x|^\beta,\gamma}$. We establish an inequality of Stein-Weiss type for the B-Riesz potentials in the limiting case, and obtain the boundedness of the B-Riesz potential operator from the space $L_{p,|x|^\beta,\gamma}$ to $BMO_{|x|^{-\lambda},\gamma}$.

Mathematics Subject Classification: Primary 42B20, 42B25, 42B35

Key Words and Phrases: Laplace-Bessel differential operator, generalized shift operator, B-Riesz potential, weighted B-BMO spaces, Stein-Weiss type inequality, weighted Lebesgue space

Introduction

The classical Riesz potential is an important technical tool in harmonic analysis, theory of functions and partial differential equations. The potential and related topics associated with the Laplace-Bessel differential operator

$$\Delta_B = \sum_{k=1}^{n} \frac{\partial^2}{\partial x_k^2} + \frac{\gamma}{x_n}\frac{\partial}{\partial x_n}, \quad \gamma > 0$$

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have been the research areas of many mathematicians such as K. Stempak [11], I. Kipriyanov [8], A.D. Gadjiev and I.A. Aliev [1], A.D. Gadjiev and V.S. Guliyev [2], E.V. Guliyev [3], V.S. Guliyev [4]-[6] and others.

In this paper we study Riesz potentials (B-Riesz potentials) generated by the Laplace-Bessel differential operator Δ_B in weighted Lebesgue spaces. We establish the inequality of Stein-Weiss type (see [10]) for B-Riesz potentials in the limiting case. We obtain the boundedness of the B-Riesz potential operator from the spaces $L_{p,|x|^\beta,\gamma}$ to $BMO_{|x|^{-\lambda},\gamma}$ in the limiting case.

1. Definitions, notation and preliminaries

Let $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n\ ;\ x = (x_1, ..., x_n), x_n > 0\}$ and $B(x,r) = \{y \in \mathbb{R}^n_+\ :\ |x-y| < r,\ r > 0\}$, $B_r \equiv B(0,r)$, and let ${}^{\complement}B(x,r) = \mathbb{R}^n_+ \setminus B(x,r)$.

For a measurable set $A \subset \mathbb{R}^n_+$, let $|A|_\gamma = \int_A x_n^\gamma dx$, then $|B_r|_\gamma = \omega(n,\gamma) r^{n+\gamma}$, where

$$\omega(n,\gamma) = \int_{B_1} x_n^\gamma dx = \frac{\pi^{(n-1)/2}\Gamma\left((\gamma+1)/2\right)}{2\Gamma\left((n+\gamma-2)/2\right)}.$$

Denote by T^y the generalized shift operator (B-shift operator) acting according to the law

$$T^y f(x) = C_\gamma \int_0^\pi f\left(x'-y', (x_n,y_n)_\beta\right) \sin^{\gamma-1}\beta d\beta,$$

where

$$(x_n,y_n)_\beta = \sqrt{x_n^2+y_n^2-2x_ny_n\cos\beta} \quad \text{and} \quad C_\gamma = \frac{\Gamma\left((\gamma+1)/2\right)}{\sqrt{\pi}\,\Gamma\left(\gamma/2\right)} = \frac{2}{\pi}\,\omega(2,\gamma).$$

We remark that the generalized shift operator T^y is closely connected with the Laplace-Bessel differential operator Δ_B (for example, $n = 1$ – see [9], and $n > 1$ – [8] for details).

Let $L_{p,\gamma}(\mathbb{R}^n_+)$ be the space of measurable functions on $\mathbb{R}^n_+$ with finite norm

$$\|f\|_{L_{p,\gamma}} = \|f\|_{L_{p,\gamma}(\mathbb{R}^n_+)} = \left(\int_{\mathbb{R}^n_+} |f(x)|^p x_n^\gamma dx\right)^{1/p}, \quad 1 \le p < \infty.$$

For $p = \infty$ the space $L_{\infty,\gamma}(\mathbb{R}^n_+)$ is defined by means of the usual modification

$$\|f\|_{L_{\infty,\gamma}} = \|f\|_{L_\infty} = \operatorname*{ess\,sup}_{x\in\mathbb{R}^n_+} |f(x)|.$$

Lemma 1. ([2]) *Let* $0 < \alpha < n+\gamma$. *Then*

$$\left|T^y|x|^{\alpha-n-\gamma} - |y|^{\alpha-n-\gamma}\right| \le 2^{n+\gamma+1-\alpha}|y|^{\alpha-n-\gamma-1}|x| \tag{1}$$

for $2|x| \le |y|$.

DEFINITION 1. Let $1 \le p < \infty$. We denote by $WL_{p,\gamma}(\mathbb{R}^n_+)$ the weak $L_{p,\gamma}$ space defined as the set of locally integrable functions f with the finite norms

$$\|f\|_{WL_{p,\gamma}} = \sup_{r>0} r f^{1/p}_{*,\gamma}(r),$$

where $f_{*,\gamma}(r) = \left|\left\{x \in \mathbb{R}^n_+ \ : \ |f(x)| > r\right\}\right|_\gamma$.

Let v be a non-negative and measurable function on $\mathbb{R}^n_+$, and $L_{p,v,\gamma}(\mathbb{R}^n_+)$ be the weighted $L_{p,\gamma}$-space of all measurable functions f on $\mathbb{R}^n_+$ for which

$$\|f\|_{L_{p,v,\gamma}} \equiv \|f\|_{L_{p,v,\gamma}(\mathbb{R}^n_+)} = \|vf\|_{L_{p,\gamma}(\mathbb{R}^n_+)} < \infty.$$

We denote by $WL_{p,v,\gamma}(\mathbb{R}^n_+)$ $(1 \le p < \infty)$ the weighted weak Lebesgue space which is the class of all measurable functions $f : \mathbb{R}^n_+ \to \mathbb{R}$, for which

$$\|f\|_{WL_{p,v,\gamma}} \equiv \|f\|_{WL_{p,v,\gamma}(\mathbb{R}^n_+)} = \|vf\|_{WL_{p,\gamma}(\mathbb{R}^n_+)} < \infty.$$

The $B-BMO$ space (see [5]) $BMO_\gamma(\mathbb{R}^n_+)$, and weighted $B-BMO$ space, $BMO_{w,\gamma}(\mathbb{R}^n_+)$, are defined as the set of locally integrable functions f with finite norms

$$\|f\|_{*,\gamma} = \sup_{r>0,\ x\in\mathbb{R}^n_+} |B_r|^{-1}_\gamma \int_{B_r} |T^y f(x) - f_{B_r}(x)| y_n^\gamma dy < \infty,$$

and

$$\|f\|_{*,w,\gamma} = \sup_{r>0,\ x\in\mathbb{R}^n_+} w(B_r)^{-1} \int_{B_r} |T^y f(x) - f_{B_r}(x)| y_n^\gamma dy < \infty,$$

respectively, where

$$f_{B_r}(x) = |B_r|^{-1}_\gamma \int_{B_r} T^y f(x) y_n^\gamma dy \ \text{ and } \ w(B_r) = \int_{B_r} w(x) x_n^\gamma dx.$$

Consider the B-Riesz potential

$$I_{\alpha,\gamma} f(x) = \int_{\mathbb{R}^n_+} T^y |x|^{\alpha-n-\gamma} f(y) y_n^\gamma dy, \quad 0 < \alpha < n+\gamma$$

and the modified B-Riesz potential

$$\widetilde{I}_{\alpha,\gamma} f(x) = \int_{\mathbb{R}^n_+} \left(T^y |x|^{\alpha-n-\gamma} - |y|^{\alpha-n-\gamma} \chi_{\complement_{B_1}}(y)\right) f(y) y_n^\gamma dy.$$

For the B-Riesz potential the following Stein-Weiss type theorem was proved by A.D. Gadjiev and V.S. Guliyev in [2].

Theorem A. *Let* $0 < \alpha < n+\gamma$, $1 \le p \le q < \infty$, $\beta < \frac{n+\gamma}{p'}$ *(*$\beta \le 0$, *if* $p = 1$*),* $\lambda < \frac{n+\gamma}{q}$ *(*$\lambda \le 0$, *if* $q = \infty$*),* $\alpha \ge \beta + \lambda \ge 0$ *(*$\beta + \lambda > 0$, *if* $p = q$*).*

1) If $1 < p < \frac{n+\gamma}{\alpha-\beta-\lambda}$, *then the condition* $\frac{1}{p} - \frac{1}{q} = \frac{\alpha-\beta-\lambda}{n+\gamma}$ *is necessary and sufficient for the boundedness of* $I_{\alpha,\gamma}$ *from* $L_{p,|x|^\beta,\gamma}(\mathbb{R}^n_+)$ *to* $L_{q,|x|^{-\lambda},\gamma}(\mathbb{R}^n_+)$.

2) If $p = 1$, *then the condition* $1 - \frac{1}{q} = \frac{\alpha-\beta-\lambda}{n+\gamma}$ *is necessary and sufficient for the boundedness of* $I_{\alpha,\gamma}$ *from* $L_{1,|x|^\beta,\gamma}(\mathbb{R}^n_+)$ *to* $WL_{q,|x|^{-\lambda},\gamma}(\mathbb{R}^n_+)$.

Definition 2. The weight function w belongs to the class $A_{p,\gamma}(\mathbb{R}^n_+)$ for $1 < p < \infty$, if

$$\sup_{x\in\mathbb{R}^n_+, r>0} \left(|B(x,r)|_\gamma^{-1} \int\limits_{B(x,r)} w(y) y_n^\gamma dy \right) \times \left(|B(x,r)|_\gamma^{-1} \int\limits_{B(x,r)} w^{-\frac{1}{p-1}}(y) y_n^\gamma dy \right)^{p-1} < \infty,$$

and w belongs to $A_{1,\gamma}(\mathbb{R}^n_+)$, if there exists a positive constant C such that for any $x \in \mathbb{R}^n_+$ and $r > 0$

$$|B(x,r)|_\gamma^{-1} \int_{B(x,r)} w(y) y_n^\gamma dy \le C \operatorname*{ess\,sup}_{y\in B(x,r)} w(y).$$

The properties of the class $A_{p,\gamma}(\mathbb{R}^n_+)$ are analogous to those of the Muckenhoupt classes. In particular, if $w \in A_{p,\gamma}(\mathbb{R}^n_+)$, then $w \in A_{p-\varepsilon,\gamma}(\mathbb{R}^n_+)$ for a certain sufficiently small $\varepsilon > 0$ and $w \in A_{p_1,\gamma}(\mathbb{R}^n_+)$ for any $p_1 > p$.

Note that, $|x|^\alpha \in A_{p,\gamma}(\mathbb{R}^n_+)$, $1 < p < \infty$, if and only if $-\frac{n+\gamma}{p} < \alpha < \frac{n+\gamma}{p'}$ and $|x|^\alpha \in A_{1,\gamma}(\mathbb{R}^n_+)$, if and only if $-n-\gamma < \alpha \le 0$.

For the B-maximal function (see [4, 5])

$$M_\gamma f(x) = \sup_{r>0} |B_r|_\gamma^{-1} \int\limits_{B_r} T^y |f(x)|\, y_n^\gamma dy$$

the following analogue of Muckenhoupt theorem (see [7]) was proved by E.V. Guliyev in [3].

Theorem B. *1. If* $f \in L_{1,w,\gamma}(\mathbb{R}^n_+)$, $w \in A_{1,\gamma}$, *then* $M_\gamma f \in WL_{1,w,\gamma}(\mathbb{R}^n_+)$ *and*

$$\|M_\gamma f\|_{WL_{1,w,\gamma}} \le C_{1,w,\gamma} \|f\|_{L_{1,w,\gamma}}, \tag{2}$$

where $C_{1,w,\gamma}$ *depends only on* w, γ *and* n.

2. If $f \in L_{p,w,\gamma}(\mathbb{R}^n_+)$, $w \in A_{p,\gamma}$, $1 < p < \infty$, *then* $M_\gamma f \in L_{p,w,\gamma}(\mathbb{R}^n_+)$ *and*

$$\|M_\gamma f\|_{L_{p,w,\gamma}} \leq C_{p,w,\gamma} \|f\|_{L_{p,w,\gamma}}, \tag{3}$$

where $C_{p,w,\gamma}$ *depends only on* w, p, γ *and* n.

2. Main result

Our main result is the following Stein-Weiss type theorem for the B-Riesz potential in the limiting case $p = (n+\gamma)/(\alpha - \beta - \lambda)$. We prove that the modified B-Riesz potential operator $\widetilde{I}_\alpha$ is bounded from the space $L_{p,|x|^\beta,\gamma}$ to the weighted B-BMO space $BMO_{|x|^{-\lambda},\gamma}$.

THEOREM 1. *Let* $0 < \alpha < n+\gamma$, $1 < p = (n+\gamma)/(\alpha - \beta - \lambda)$, $\beta < (n+\gamma)/p'$, $\alpha \geq \beta + \lambda \geq 0$. *Then the operator* $\widetilde{I}_{\alpha,\gamma}$ *is bounded from* $L_{p,|x|^\beta,\gamma}(\mathbb{R}^n_+)$ *to* $BMO_{|x|^{-\lambda},\gamma}(\mathbb{R}^n_+)$.

Moreover, for $f \in L_{p,|x|^\beta,\gamma}(\mathbb{R}^n_+)$ *the integral* $I_{\alpha,\gamma} f$ *exists almost everywhere, then* $I_{\alpha,\gamma} \in BMO_{|x|^{-\lambda},\gamma}(\mathbb{R}^n_+)$ *and the following inequality is valid*

$$\|I_{\alpha,\gamma} f\|_{BMO_{|x|^{-\lambda},\gamma}} \leq C \|f\|_{L_{p,|x|^\beta,\gamma}},$$

where $C > 0$ *is independent of* f.

P r o o f. Let $f \in L_{p,|x|^\beta,\gamma}(\mathbb{R}^n_+)$, $1 < p = (n+\gamma)/(\alpha - \beta - \lambda)$. For given $t > 0$ we denote

$$f_1(x) = f(x)\chi_{B_{2t}}(x), \quad f_2(x) = f(x) - f_1(x), \tag{4}$$

where $\chi_{B_{2t}}$ is the characteristic function of the set B_{2t}. Then

$$\widetilde{I}_{\alpha,\gamma} f(x) = \widetilde{I}_{\alpha,\gamma} f_1(x) + \widetilde{I}_{\alpha,\gamma} f_2(x) = F_1(x) + F_2(x),$$

where

$$F_1(x) = \int_{B_{2t}} \left(T^y |x|^{\alpha-n-\gamma} - |y|^{\alpha-n-\gamma} \chi_{\complement_{B_1}}(y) \right) f(y) y_n^\gamma dy,$$

$$F_2(x) = \int_{\complement_{B_{2t}}} \left(T^y |x|^{\alpha-n-\gamma} - |y|^{\alpha-n-\gamma} \chi_{\complement_{B_1}}(y) \right) f(y) y_n^\gamma dy.$$

Note that the function f_1 has compact support and thus

$$a_1 = -\int_{B_{2t} \setminus B_{\min\{1,2t\}}} |y|^{\alpha-n-\gamma} f(y) y_n^\gamma dy$$

is finite.

Note also that

$$F_1(x) - a_1 = \int_{B_{2t}} T^y|x|^{\alpha-n-\gamma} f(y) y_n^\gamma dy - \int_{B_{2t}\setminus B_{\min\{1,2t\}}} |y|^{\alpha-n-\gamma} f(y) y_n^\gamma dy$$

$$+ \int_{B_{2t}\setminus B_{\min\{1,2t\}}} |y|^{\alpha-n-\gamma} f(y) y_n^\gamma dy = \int_{\mathbb{R}^n_+} T^y|x|^{\alpha-n-\gamma} f_1(y) y_n^\gamma dy = I_{\alpha,\gamma} f_1(x).$$

Therefore

$$|F_1(x) - a_1| \le \int_{\mathbb{R}^n_+} |y|^{\alpha-n-\gamma} \left|T^y f_1(x)\right| y_n^\gamma dy = \int_{B(x,2t)} |y|^{\alpha-n-\gamma} \left|T^y f(x)\right| y_n^\gamma dy.$$

Further, for $x \in B_t$, $y \in B(x, 2t)$ we have

$$|y| \le |x| + |x - y| < 3t.$$

Consequently, we have

$$|F_1(x) - a_1| \le \int_{B_{3t}} |y|^{\alpha-n-\gamma} \left|T^y f(x)\right| y_n^\gamma dy, \tag{5}$$

if $x \in B_t$.

By Theorem B and inequality (5), for $(\alpha - \beta - \lambda)p = n + \gamma$ we have

$$t^{-n-\gamma-\lambda} \int_{B_t} \left|T^z F_1(x) - a_1\right| z_n^\gamma dz$$

$$\le Ct^{-n-\gamma-\lambda} \int_{B_t} T^z \left(\int_{B_{3t}} |y|^{\alpha-n-\gamma} T^y |f(x)| y_n^\gamma dy \right) z_n^\gamma dz$$

$$\le Ct^{\alpha-n-\gamma-\lambda} \cdot t^{(n+\gamma)/p'} \left(\int_{B_t} T^z \left(M_\gamma(f(x))\right)^p z_n^\gamma dz \right)^{1/p}$$

$$\le Ct^\beta \left(\int_{B_t} T^z \left(M_\gamma(f(x))\right)^p z_n^\gamma dz \right)^{1/p} \le C \left(\int_{B_t} |z|^{\beta p}\, T^z \left(M_\gamma(f(x))\right)^p z_n^\gamma dz \right)^{1/p}$$

$$= C \left(\int_{\mathbb{R}^n_+} T^z \left(\chi_{B_t} |x|^{\beta p} \right) \left(M_\gamma(f(x))\right)^p z_n^\gamma dz \right)^{1/p}$$

$$= C \left(\int_{\mathbb{R}^n_+} |z|^{\beta p} \left(M_\gamma(f(x))\right)^p z_n^\gamma dz \right)^{1/p} \le C \|f\|_{L_{p,|x|^\beta,\gamma}}. \tag{6}$$

Denote

$$a_2 = \int_{B_{\max\{1,2t\}}\setminus B_{2t}} |y|^{\alpha-n-\gamma} f(y) y_n^\gamma dy,$$

and estimate $|F_2(x) - a_2|$ for $x \in B_t$:

$$|F_2(x) - a_2| \le \int_{\complement B_{2t}} |f(y)| \left| T^y |x|^{\alpha-n-\gamma} - |y|^{\alpha-n-\gamma} \right| {y_n}^{\gamma} dy.$$

Applying Lemma 1 and Hölder's inequality we get

$$|F_2(x) - a_2| \le 2^{n+\gamma-\alpha+1} |x| \int_{\complement B_{2t}} |f(y)||y|^{\alpha-n-\gamma-1} {y_n}^{\gamma} dy$$
$$\le 2^{n+\gamma-\alpha+1} |x| \left(\int_{\complement B_t} |y|^{\beta p} |f(y)|^p {y_n}^{\gamma} dy \right)^{1/p}$$
$$\times \left(\int_{\complement B_t} |y|^{(-\beta+\alpha-n-\gamma-1)p'} {y_n}^{\gamma} dy \right)^{1/p'}$$
$$\le C|x| t^{\alpha-\beta-1-n-\gamma/p} \|f\|_{L_{p,|x|^\beta,\gamma}} \le C|x| t^{\lambda-1} \|f\|_{L_{p,|x|^\beta,\gamma}} \le C|x|^\lambda \|f\|_{L_{p,|x|^\beta,\gamma}}.$$

Note that if $|x| \le t$ and $|z| \le 2t$, then $T^z|x| \le |x| + |z| \le 3t$. Thus for $(\alpha - \beta - \lambda)p = Q$ we obtain

$$|T^z F_2(x) - a_2| \le T^z \, |F_2(x) - a_2| \le C|x|^\lambda \|f\|_{L_{p,|x|^\beta,\gamma}}. \tag{7}$$

Denote

$$a_f = a_1 + a_2 = \int_{B_{\max\{1,2t\}}} |y|^{\alpha-n-\gamma} f(y) y_n^\gamma dy.$$

Finally, from (6) and (7) we have

$$\sup_{x,t} t^{-n-\gamma-\lambda} \int_{B_t} \left| T^y \widetilde{I}_{\alpha,\gamma} f(x) - a_f \right| y_n^\gamma dy \le C \|f\|_{L_{p,|x|^\beta,\gamma}}.$$

Thus,

$$\left\| \widetilde{I}_{\alpha,\gamma} f \right\|_{BMO_{|x|^{-\lambda},\gamma}}$$
$$\le 2C \sup_{x,t} t^{-n-\gamma-\lambda} \int_{B_t} \left| T^y \widetilde{I}_{\alpha,\gamma} f(x) - a_f \right| y_n^\gamma dy \le C \|f\|_{L_{p,|x|^\beta,\gamma}}.$$

Thus Theorem 1 is proved. ■

Corollary 1. ([4, 5]) *Let $0 < \alpha < n + \gamma$, $1 < p = (n+\gamma)/\alpha$. Then the operator $\widetilde{I}_{\alpha,\gamma}$ is bounded from $L_{p,\gamma}(\mathbb{R}^n_+)$ to $BMO_\gamma(\mathbb{R}^n_+)$.*

Moreover, for $f \in L_{p,\gamma}(\mathbb{R}^n_+)$ the integral $I_{\alpha,\gamma} f$ exists almost everywhere, then $I_{\alpha,\gamma} \in BMO_\gamma(\mathbb{R}^n_+)$ and the following inequality is valid

$$\|I_{\alpha,\gamma} f\|_{BMO_\gamma} \le C \|f\|_{L_{p,\gamma}},$$

where $C > 0$ is independent of f.

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