ON THE MASTERING OF KNOWLEDGE AND SKILLS FOR SOLVING MATHEMATICAL PROBLEMS IN THE CONTEXT OF THE RELATIONSHIP “REFLECTION – SYNERGY”

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Abstract: *On the basis of topical investigations on the reflection in the mathematics education, in this article there are presented some contemporary ideas about refining the methodology of mastering knowledge and skills for solving mathematical problems. The thesis is developed that for the general logical and for some particular mathematical methods to become means of solving mathematical problems, first they need to be a purpose of the education.*

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The mastering of various methods and heuristics, which are applied not only to the realization but also to searching and finding a solution to mathematical problems, has a crucial importance for achieving the aims of the mathematics education. Practice shows that the acquisition of knowledge in a certain subject from the school course in mathematics as well as skills for their usage, on the one hand, and the acquisition of knowledge about the nature of the general logical and the particular mathematical methods and skills for their application in solving problems, on the other hand, can be implemented in conformity with the reflexive-synergetic approach at an appropriate structuring of systems of mathematical problems, adequate to the specific educational and development purposes. Through such systems of mathematical problems there can also be mastered different heuristics for the searching of a solution, which, although not always leading to a positive result, are instrumental too in achieving the aims. The realization of all this, accompanied by the prognostication [12], can contribute to the students’ self-realization to the maximum of both the positive and the negative role of certain methods and heuristics during problem solving.

A fundamental place in the activity of solving mathematical problems is taken by reasoning at atomic, molecular and cellular level (after I. Ganchev [1]). A considerable part of the particular mathematical methods for solving the so-called standard problems (for example ones related to solving linear, quadratic, and biquadratic equations, etc.) are based on respective formulae and specific algorithms are developed for them. Therefore it is relevant to assign the use of such methods to the activities at an atomic level.

The repeated joint application of specific mathematical knowledge and elements of propositional logic (mostly attributes referring to the implications of assertions as well as rules for drawing conclusions) is described for brevity as an activity at a molecular level. We consider it advisable to relate to it the use of general logical methods too, which are based on elements of propositional logic (for example the method of contraposition, etc.)

As an activity at a cellular level is considered not only the eventual implementation of specific parts of the solution by means of activities at an atomic and molecular level, but also every splitting of the solution of a particular mathematical problem into separate solutions of the so-called problems-components and their consecutive realization with a view to obtaining the solution to a particular problem. This activity is usually implemented on the basis of the general logical methods of synthesis, analysis and a combination of these as well as the respective skills for reasoning at an atomic and molecular level, including the use of appropriate particular mathematical methods for solving the differentiated problems-components.

From what we have presented so far there stands out the importance of mastering the general logical and particular mathematical methods, which play the role of fundamental “operative” means for the implementation of the activity of solving mathematical problems. However, in order to become means, it is essential for them to be also a purpose of the education at certain stages.

One of the objectives of this study is to refine the methodology of working with mathematical problems on the basis of the reflexive-synergetic approach, when the activity of solving, and consequently the separate methods of searching, finding and realizing a solution, are a purpose of the education.

The issue of the effective utilization of the reflexive approach potentialities in the mathematics education has been investigated by M. Georgieva in [2]. There, on page 12, is shown a schematic model of the structure of the categories system: “perception”, “memorization”, “understanding”, “reflection”, “application”, and “mastering”, in which there is presented the position of the reflection and the succession of the mental processes preceding the mastering. Some fragments from the results of these investigations can be adapted and used to achieve the above-mentioned purposes. It is clear from the scheme of the model that:

- the understanding and rationalization (as well as the perception and memorization) can be implemented without the use of reflection;
- the reflection is realized by means of the understanding and rationalization;
- the reflection is “at the core of the application and mastering as well as in the formation of reflexive capabilities” [2, p. 12].

From these inferences we reach the conclusion that what is crucial in the mathematics education is “the subject’s internal experience, which is an important condition for the manifestation of the reflection” [same source]. The latter is essential for the purposeful acquisition of “mathematical experience”, in the process of which there are implemented activities, which are varied and requiring high mental efforts. Hence is reached the necessity for an effective utilization of the activity approach possibilities in the mathematics education. In this connection there arises the issue of “which side of the activity approach can serve as a basis for a new strategy of the mathematics education nowadays under the new realities? The main thing when mastering experience in solving different problems is the acquisition of the ***core***, which leads to the formation of theoretical thinking. In this context the priority lies in the relationship between the acquisition and **application** of the experience as a satisfaction of cognitive necessity...” [2, p.13]. This thought supports the connections between the intellectual and the praxicological reflection, which we have demonstrated in the model in Fig. 1 in [9] and have realized in practice in [8].

Examining the cited schematic model [2, p. 12] from the point of view not only of the reflexive but also the reflexive-synergetic approach [11], it becomes possible to indicate the position of the self-organization [3] in the system of cognitive processes preceding the mastering, as well as of the equally important process at a mega level, which is the future applications of acquired knowledge when solving respective problems. In virtue of that we have reached the following improved model, which we are presenting schematically in Fig. 1.

We will point out that the **novelty** of this model is based on the following:

1) The components perception, memorization, rationalization and understanding from the cited model of M. Georgieva [2, p. 12] are inscribed in the self-organization [3], because at the respective stage of the education the subjects can reach self-actualization and self-development;

2) The mastering is not only preceded by the component applications but it also leads to future applications for the solution of problems of various nature.

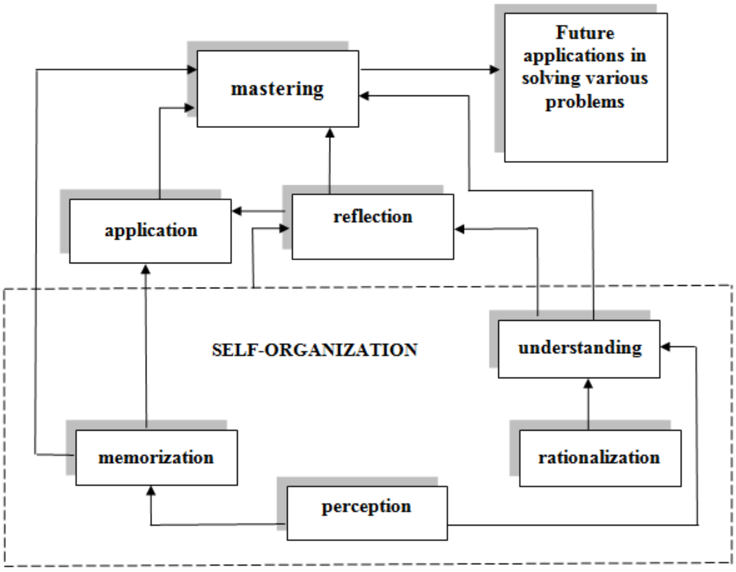

Fig. 1 A schematic model of the position of the reflection and self-organization, and of the sequence of cognitive processes preceding the mastering

It is advisable for the above-mentioned concepts to have an impact on the construction of systems of school mathematical problems, designed for the mastering of particular problem solving methods in such a way that the education accomplished through them should be oriented towards generalized methods of activity ([5], [6], [13], [14]). Furthermore, the interrelations between the separate systems (which differentiate them as components of a more general, “global” system) should be able to ensure **in a stepwise manner** a realization of the following ***phases of education***:

- ***First phase*: self-organization**, expressed by **perception – memorization – rationalization – understanding;**
- ***Second phase*: reflection;**
- ***Third phase*: applications – mastering – future application.**

When doing her research in [2], M. Georgieva leans on three types of relationships: “education – reflection”, “education – development”, and “education – development – reflection”. Working on Talizina’s concept of learning from the point of view of the relationship “education – reflection”, she pays attention to the two varieties of the students’ approach towards studying in the education:

♦ the teacher elaborates on the learning content and presents it to the students, and they do appropriate activities;

♦ the students participate actively in refinding scientific truths and obtaining independent cognitive experience” [2, p.16-17].

Accepting the thesis that **in both varieties the place of reflection can be searched – both above knowledge and above the activity**, in [14] and [10] we have developed and presented models for mastering by means of “refinding” particular mathematical methods.

Based on the close connection between the relationship “education – reflection” and the relationship “education – development”, as well as L. S. Vigotsky’s opinion that “education is good only if it comes before the development”, there can be established the principle and methodological significance of these concepts about the mathematics education, one of which refers to the “interiorization and exteriorization as mechanisms of development in the mathematics education”. The resulting accent placed on the importance of the introduced by I. Ganchev concept “mastering to the degree of random reproduction and applying of the most general knowledge and skills, which are not given directly in the course of education [1, p. 47] holds, in our opinion, especially for the knowledge and skills, connected with the use of the general logical methods. This fact is taken into consideration when developing the methodological system in [10].

There needs to be paid attention to the key tool for mastering knowledge and skills at a reflexive level called “**method of education by means of generalized reasoning**” [1]. It is based on the presumption that after some specific knowledge or a skill has been mastered, not only that there must be mastered a lot more specific knowledge or skills bearing the characteristics of the first one, but that they must be "broken" – separated from the concreteness of the knowledge and then differentiated and fixed in the subject's consciousness. The performing of such a generalizing activity can be better managed as well as accelerated. In our opinion the method of education by means of generalized reasoning contributes significantly to the development of intellectual reflection in learners. We are making use of the above-mentioned as well as of this fact to match the three phases of a "global" system for acquiring specific general logical methods of solving problems, to the conclusions made by I. Ganchev based on the model he has built about the connection of the education with the mental development, which are also valid for the development of reflexive abilities in students in the mathematics education. In the *first phase*, generally speaking, there is implemented input of knowledge and skills, referring to the application of a certain method (depending on the particular mathematical knowledge and skill for its application) in the student's close development zone (under the teacher's guidance); in the *second phase* a transition takes place from the close development zone (CDZ) to the learner's actual development zone (ADZ) with an intensive use of the reflexive approach; in the *third phase* there takes place a development of the reflexive abilities in the actual development zone, by means of which is achieved "mastering to the level of arbitrariness for the reproduction and application of the most general knowledge and skills" for the general logical methods of solving problems, which, according to the secondary school curricula, are indeed "not given directly in the course of education".

In connection with the formation of intellectual and praxicological reflection, the methodology for developing the students' skills in the mathematics education must comply with certain requirements, part of which are included in I. Ganchev's research work and in his model of the connection between the education and the mental development. One of the requirements is for the learners to be taught to handle Pap's scheme (ascending analysis) for solving problems without announcing its name. According to the author, the skill to reason on this scheme "for the majority of the students is still "bound to" different particular types of problems, i.e. it has not been mastered to the level of arbitrariness" [1, p.145]. Here we will share from our experience that after a systematic preparation by students, aimed at mastering of skills necessary for the implementation of the ascending analysis, and added to sufficient acquired experience, such as students from 12th grade possess (the ones who wish to sit for a school-leaving or candidate-students' entrance examination in mathematics), the announcing of the method name (Pap's scheme) proved fruitful as that was a kind of an "act of breaking" of this method from the concreteness of the different cases, to which it had been applied [4]. We believe that the specified requirement could also be expanded towards the mastering of knowledge and skills for a combined application (in different variants) of the general logical methods for searching and finding of a solution. Bearing in mind that in the secondary school mathematics education the so-called distant propaedeutics is implemented concerning the general logical methods of analysis, synthesis and some combinations of them, then their further mastering can be realized effectively, taking into consideration the conclusions from the study in [1, p.64-65], which are connected to the methodology of the organization of the mathematics education and have a direct relation to the reflection. More specifically, these conclusions are applied when constructing a "global" system of school mathematical problems for the mastering of certain general logical and/or particular mathematical methods for solving problems [10].

The relationship "education – reflection" has a connection with the fourth requirement in the methodology for the development of the students' skills for solving mathematical problems, as was formulated in [1, p.147-154], namely: the construction of didactic systems from symptoms and their use in the process of education under the teacher's guidance (i.e. to work with them as with knowledge and skills from Z_{CDZ}). For this purpose it is of use to have the systems of symptoms systematized and periodically revised and expanded with newly studied symptoms. That gave us an idea when reviewing the ways of solving problems from the same section (for example, a quadratic equation with two unknown quantities) to compose a "list" of the methods, with which the problems from this section are "attacked" and, after familiarizing with each new method, it should be added to that "list". In the process of acquainting the students with a new type of problems, connected with newly studied material, when the problems are solved by means of an already familiar method, we also consider appropriate first to update the knowledge about that particular method as well as the skills for its application by using a few appropriately selected problems from material, which has already been covered, but from a different type, while at the same time accentuating on its wide applicability and comprehensiveness. Afterwards, other types of already covered problems should be described, for the solution of which the same method is applied. As a result of this activity there should be composed a "list" of the types of problems (if one hasn't been made yet), the "new" application of the method under consideration should be examined and the "list" should be supplemented with the type of problems, to which it is applicable.

After adapting some of the conclusions in [2] for this study, we can say that in the teaching of methods and heuristics for solving problems, in most cases the activity has a leading role, because of which the reflection is connected with the ability, the skill, the "activity" mental new formation, and then it is appropriate to use the term reflection above the activities. However, in some cases, the knowledge has a leading role and then "the reflection is viewed as a process, which translates the information into personal knowledge and in these cases we deal with reflection above knowledge" [2, p. 26].

The conceptual model of a technology for reflexive education, developed in the same source (p. 32), is of scientific and practical interest. Its structure includes: motives and objectives of the reflexive activity, methods, didactic means and organizational forms of education, as well as the differentiated four levels – **reproductive, productive, transfer, and creative** ones, for acquiring knowledge and performing activities, through which the respective reflexive skills are formed.

The system of these levels can be used as a model when constructing (selecting, composing and/or transforming problems) of didactic expedient systems of school mathematical problems from different sections of the school mathematics course, the purpose of which is to form a connection with the mastering of specific general logical or particular mathematical methods for solving problems in the context of the reflexive-synergetic approach.

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