# TESSELLATION WITH SOME IRREGULAR POLIGONS

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**Abstract.** *An idea for the use of problems of tessellation with irregular polygons in the optional activities in Information Technologies teaching are presented.*

**Keywords:** tessellation, regular polygon, irregular polygon, rotation
**2010 Mathematics Subject Classifications:** 51F20

Modern students live in the world of information and Internet technologies. They are accustomed to using the computer as means of entertainment – games, communication, music. This attitude can be taken advantage of in the optional activities in Information Technologies (IT) by introducing new knowledge in the way of games/ amusing and original tasks, interesting problems which have been preserved since the antiquity and demonstrate the application of IT in the field of decorative art /. Such tasks develop intelligence, skills to generate ideas, inventiveness.

Some of the tasks, that provoke creativity and make work pleasant, are for plane tessellation. They enlarge students' knowledge and develop their skills in Mathematics and IT, enabling them to acquire knowledge through active, purposeful actions. One of the programs suitable for this purpose is THE GEOMETER'S SKETCHPAD [1, 2, 3]. It allows the students to explore different possibilities and to discover new ideas [1, 2, 3]. As it is a classical program for geometric construction its studying can be in aid of teaching Mathematics [3].

The use of specialized software and dynamic structures supports educational research, encourages experimentation, helps to overcome the fear of errors by allowing to move one step back - "undo". The accomplishment of an individual project by each student enhances the pursuit of self-knowledge and expression, gaining understanding of their own experience.

The regular tessellations are familiar to students, because they are taught in IT in $6^{th}$ grade. Tessellation /tiling/ is covering the plane with figures without gaps and overlaps. Even the Pythagoreans knew that around one vertex six equilateral triangles, or four squares, or three regular hexagons can be arranged. It is easy to demonstrate that the sum of the internal angles of regular polygons around the same vertex must be 360 ° [4, pp. 23].

It is evident that 360 ° must be exactly divisible without a remainder of the internal angle of the polygon. It can be shown that it is impossible to cover a plane with another kind of identical regular polygons. Tilling a plane with identical

regular polygons is called regular tessellation. The regular polygons are of the same kind and their arrangement in each vertex of such mosaic patterns is the same /fig.1 /.

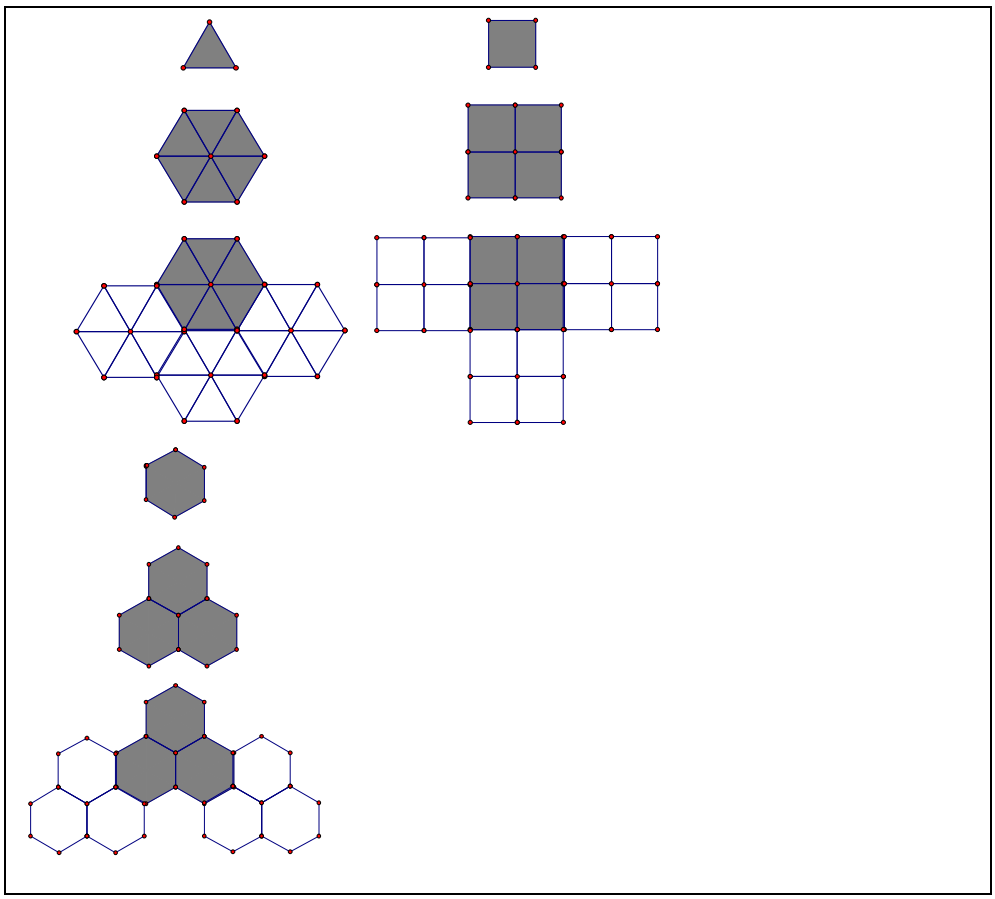


Fig. 1

The question which arises is whether only congruent equilateral triangles, square, hexagons can tessellate a plane. What will happen if we choose to tile with an irregular triangle or quadrilaterals? Is it possible to find a configuration that will cover the plane? The desire to provoke the students' intuition naturally leads to assigning the task to tessellate a plane with same irregular triangles, quadrangles, hexagons [5].

One way to cover the plane with irregular quadrilaterals is to apply on a arbitrary quadrilateral a central symmetry with a centre one of the midpoints of the sides of the quadrilateral /Fig. 2a /. In THE GEOMETER'S SKETCHPAD program the central symmetry can be realized as a rotation by 180°.

The following basic concepts are necessary to fulfill this task:

- Regular and irregular polygons;
- A midpoint of a segment;
- Rotation / center and angle of rotation/.

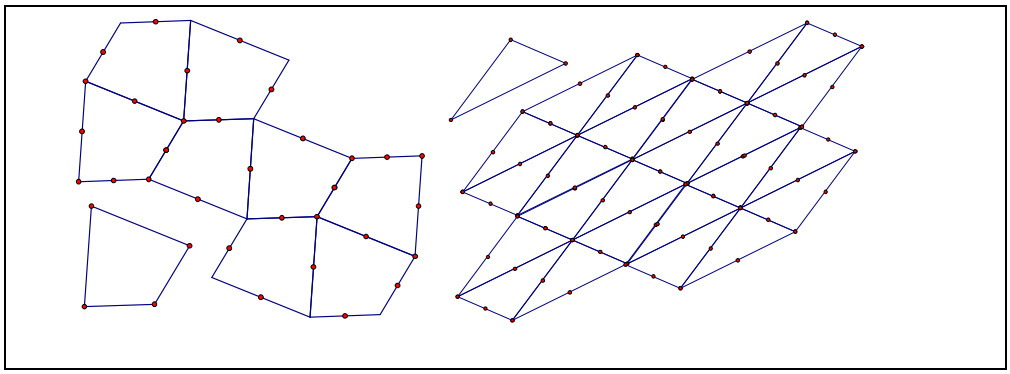
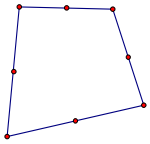


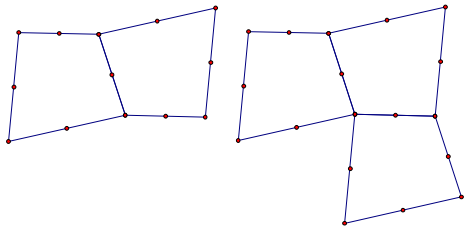
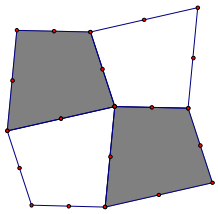
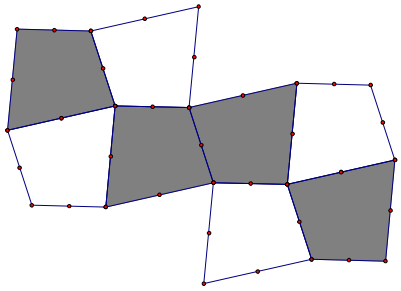
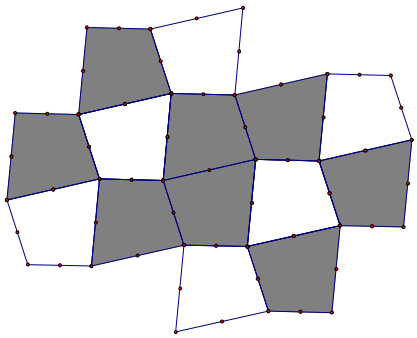
Fig. 2a, 2b

This assignment is suitable for an individual project. Thus, the students are allowed to work at their own pace, the dynamic structures give the possibility for trial and error, to go “back” and “undo” easily. The necessary skills to work with THE GEOMETER’S SKETCHPAD program are:

- Construction of a quadrilateral;
- Construction of a midpoint of a side;
- Rotation;
- Filling the interior of the polygon.

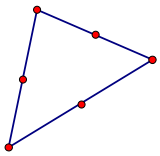
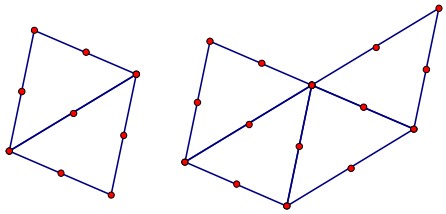
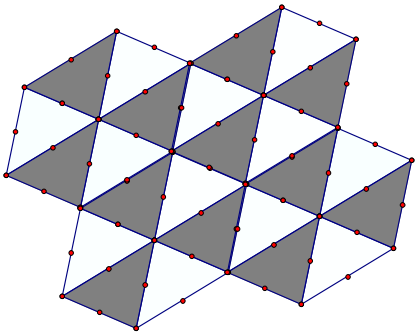
Table 1. Algorithm for tilling the plane with irregular quadrilaterals with THE GEOMETER’S SKETCHPAD program.

| № | Steps of the algorithm | Construction |
|---|---|---|
| 1 | Construct a quadrilateral and midpoints of the sides:<br>• Construct menu/Segments<br>• Construct menu/Midpoints |  |

| | | |
|---|---|---|
| 2 | Rotate the quadrilateral around the midpoint of a side of the quadrilateral by 180°<br>• Transform menu/Mark center,<br>• Transform menu/Rotate by 180° |  |
| 3 | Construct the basic pattern<br>• Construct menu/Quadrilateral interior |  |
| 4 | Rotate the basic pattern around a midpoint on the outside perimeter of the pattern by 180°<br>• Transform menu/Mark center<br>• Edit/Select All<br>• Transform menu/Rotate by 180° |  |
| 5 | Repeat this process until tessellate the plane |  |

Applying a rotation with centre a midpoint of a side of a arbitrary triangle by 180° is tessellating the plane with triangles.

Table 2. Algorithm for tilling the plane with irregular triangles with THE GEOMETER'S SKETCHPAD program.

| № | Steps of the algorithm | Construction |
|---|---|---|
| 1 | Construct a triangle and midpoints of the sides:<br>• Construct menu/Segments<br>• Construct menu/Midpoints |  |
| 2 | Rotate the triangle around a midpoint of the side by 180°<br>• Transform menu/Mark center,<br>• Transform menu/Rotate by 180° |  |
| 3 | Rotate the basic pattern around a midpoint of a side of the triangle by 180° |  |

The tessellating of a plane with irregular hexagons, whose opposite sides are two by two parallel and equal is a consequence of the tiling of the plane with arbitrary triangles /fig. 3/.

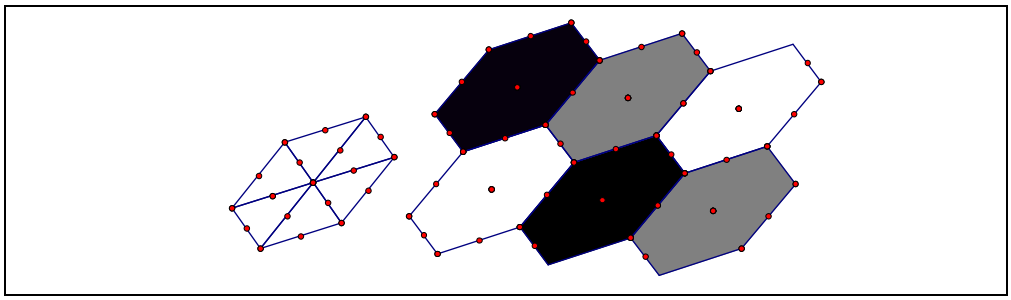


Fig. 3

In classes in Information Technologies as an optional subject, the use of specialized software such as THE GEOMETER'S SKETCHPAD program allows problems to be solved connected with creativity, broadening of students' horizons, as well as contributing to the learning of the basic mathematical concepts and classic software for geometric construction.

## References

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