# THE INITIAL STAGE OF TEACHING PROOF IN MATHEMATICS COURSE

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***Abstract.*** *This article discusses techniques for organization of propaedeutic stage of teaching proof in mathematics course. It identifies types of tasks that allow students of 5–6 classes to form the ability to carry out simple proofs. This article describes each type of tasks features, it gives some examples.*

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The problem of teaching pupils the proof has always taken an important place in mathematics methodology. Analyzing works on this problem, we can conclude that its decision is made in two ways:

- investigation of logical aspects in proof building process (V. Baidak, G. Bevz, M. Burda, G. Bresler, A. Kolmogorov, Z. Slyepkan, N. Tarasenkova, I. Gastropoda et al.);
- investigation of heuristic aspects (J. Hrudonov, V. Dalinher, J. Kulyutkyn, V. Milushev, D. Poyya, G. Sarantsev, O. Skafa et al.).

As stated in the Program of Mathematics for secondary schools in Ukraine [1], intellectual development of pupils, which includes forming of their abilities to analyze, classify, generalize, make analog deductions, obtain consequences of given prerequisites by consistent reasoning, etc., is one of the main goals of teaching mathematics. To form these skills it is possible and advisable to solve mathematical problems, especially problems of proof. V. Krutetskyy considers such problems as those that develop active, independent, creative thinking. He believes that with their help pupils gain ability of logical reasoning and argumentation [3, p. 247].

It should be noted that according to official requirement to the level of teaching of pupils in Ukraine, such problems are offered for the first time in geometry course at the 7th grade (class). The research that we carry out, reveals that pupils at the 7th grade are not ready for finding proofs. For most pupils theorem proving and solving problems that contain a requirement to "prove" in the formulation, is completely a new form of work and causes some difficulties. They learn theorem proving mechanically by heart, and not even try to solve problems of evidence at all, considering them too complicated. Therefore a question of propaedeutic stage of teaching

evidence and possibility along with reasonability to include problems of proving in mathematics course for 5–6 classes has been raised.

It should be noted that mathematics textbooks for 5–6 classes in Ukraine contained the problem of evidence, but there are some difficulties. Firstly, this type of problems is found only in some topics, and secondly, textbooks do not provide an explanation of how to conduct proving in such tasks.

According to V. Dalinher's opinion [2, p. 25], which we support, teacher must begin to prepare pupils for theorem proving at 5–6th classes level. Propaedeutical work should include the following steps:

- forming the ability to notice patterns based on the observation, calculation, conversion, mapping in pupils;
- training pupils to understand the necessity of evidence;
- teaching pupils to select conditions and requirements in statements;
- introducing pupils with some simple and complex types of statements and their truth values;
- fostering in pupils the intuitive understanding of concepts with denying statements and statements that contain contradiction;
- teaching pupils to use the counterexamples;
- teaching pupils to use geometric drawings and to read them.

**The aim of the article** is the selection of certain types of problems for propaedeutic stage of teaching of evidence. We can add these tasks to mathematics course for 5–6 classes for teaching pupils to perform simple logical steps and make simple deductive conclusions, to form in the necessity of logical reasoning, to teach some heuristic techniques during solving problems.

In our work we use the following tasks:

- task-questions which require reasonable answers;
- tasks with incomplete data;
- tasks for choosing correct statement;
- tasks for compiling counterexamples to proposed incorrect or incomplete statement;
- tasks for finding errors in the formulation of rules or during problem solving;
- tasks for proving simple statement.

We do not advise to offer difficult task to pupils at 5–6th grade. The evidences must contain two or three steps and they should be performed using examples or counterexamples.

Here are examples of such tasks.

**Task-questions which require reasonable answers.**

The tasks of this type include oral exercises. In our opinion, answering such question requires that pupils not only answered "Yes" or "No", also substantiate their answer. In our opinion, if the teacher sets for 5–6 grade pupils tasks like "Explain, why it is so", "Justify your statement, "Prove that it is correct", then pupils will learn theorems and solve problem of evidence in algebra and geometry courses more easily. For example, problem 1.

**Problem 1**. Among the fractions choose the proper fractions and explain your choice: $8; \frac{5}{5}; 2{,}4; \frac{2}{7}; \frac{4}{3}$.

*(Expected answer. The proper fraction has numerator that is less than the denominator. Fraction* $\frac{2}{7}$ *has numerator 2, which is less than the denominator 7, so this fraction is proper).*

**Task with incomplete data.**

Such problem doesn't have enough data to be solved . It is important that pupils have to explain why they cannot solve this problem. As an example, consider the problem 2.

**Problem 2**. Two cyclists started moving at the same time to meet each other. They met after 2 hours. The first was driving at 14 km/hr. Find the speed of the second cyclist.

*(Expected answer. The problem is unsolvable, because the condition has insufficient data. You need to know the distance between cities at the beginning of movement to solve the problem).*

**Task for choosing correct statement.** The tasks of this type include tasks containing the request from the list of statements to choose the correct (true) from the list. For example, look the problem 3.

**Problem 3**. Among given statements choose only correct.

1. There is a natural solution of the inequality $x < 3$.

*(Expected answer. This statement is not always correct, because there are two natural numbers 1 and 2, for whom* $1 < 3$ *and* $2 < 3$*).*

2. Some fractions are smaller than $\frac{1}{3}$.

*(Expected answer. This statement is correct, because there are fractions less than* $\frac{1}{3}$*, for example* $\frac{1}{4}$*).*

**Task for making counterexamples to proposed incorrect or incomplete statement.**

This type of task requires from pupils to give a counterexample to wrong or incomplete statements. It should be noted that the teacher may not introduce the term "counterexample", but only offer pupils to give an example that does not satisfy the condition of the statement. Consider the problem 4.

**Problem 4.** Are any of these statements true?

1. Two fractions with equal denominators are equal.

*(Expected answer. This statement is not always correct because one can give you an example that does not satisfy the condition. For example,* $\frac{2}{7} \neq \frac{5}{7}$*).*

2. All solutions of inequality $10 < x < 13$ are multiple of 2.

*(Expected answer. The statement is false because the solution of inequality is 11, which is not a multiple of two).*

**Task for finding errors in the formulation of rules or during solving the problem.**

The problem with the requirement to find a mistake in formulation, rules, or during solving problems refers to such tasks. Pupils are taught the ability to analyze information and consciously apply the theoretical material while solving a task. For example, problem 5.

**Problem 5.** Check did Misha solve an example correctly: $5 - 2\frac{3}{5} = 3\frac{3}{5}$.

*(Expected answer. If you solve the example using the rule of subtracting fractions, you will get a another solution* $2\frac{2}{5}$*, so Misha solved the example incorrectly).*

**Task for proving simple statement.**

Pupils need to prove given statement in this type of tasks. At 5–6 grade evidence should include several steps and carried out in a chain of explanations. Consider the problem 6.

**Problem 6**. Prove that inequality $\frac{5}{8} > \frac{1}{3}$ is correct.

*(Expected answer. Fraction* $\frac{5}{8}$ *is greater than a half, but fraction* $\frac{1}{3}$ *is less than half, so the first fraction is greater than the second. So inequality* $\frac{5}{8} > \frac{1}{3}$ *is correct).*

Our results indicate that pupils can be taught to perform simple logical concepts and simple deductive conclusions; to reason evidently and consistently using such tasks. Note that with this approach they get used to the fact that all their actions in the course of solving the problem must be explained and justified. Therefore, pupils' difficulties during the transition to theorem proving and solving problems for evidence in algebra and geometry courses are rather decreased.

We must admit that this work should be continued in future in following ways: teaching pupils independent analysis of the ready evidence, forming the ability to recognize the idea of proof and to perform proof independently, to use different methods of proving.

## References

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