LINEAR CONNECTIONS ON NORMAL ALMOST CONTACT MANIFOLDS WITH NORDEN METRIC [1]

Marta Teofilova

Abstract. *Families of linear connections are constructed on almost contact manifolds with Norden metric. An analogous connection to the symmetric Yano connection is obtained on a normal almost contact manifold with Norden metric and closed structural 1-form. The curvature properties of this connection are studied on two basic classes of normal almost contact manifolds with Norden metric.*

Keywords: almost contact manifold, Norden metric, B-metric, linear connection

2010 Mathematics Subject Classification: 53C15, 53C50, 53B30

Introduction

The geometry of the almost contact manifolds with Norden metric (B-metric) is a natural extension of the geometry of the almost complex manifolds with Norden metric [2] in the case of an odd dimension. A classification of the almost contact manifolds with Norden metric consisting of eleven basic classes is introduced in [3].

An important problem in the geometry of the manifolds equipped with an additional tensor structure and a compatible metric is the study of linear connections preserving the structures of the manifold. One such connection on an almost contact manifold with Norden metric, namely the canonical connection, is considered in [1, 4].

In the present work we construct families of linear connections on an almost contact manifold with Norden metric, which preserve by covariant differentiation some or all of the structural tensors of the manifold. We obtain a symmetric connection on a normal almost contact manifold with Norden metric, which can be considered as an analogue to the well-known Yano connection [6, 7].

1. Preliminaries

Let $(M, \varphi, \xi, \eta, g)$ be a $(2n+1)$-dimensional *almost contact manifold with Norden metric*, i.e. (φ, ξ, η) is an *almost contact structure*: φ is an endomorphism of the tangent bundle of M, ξ is a vector field, η is its dual 1-form, and g is a pseudo-Riemannian metric, called a *Norden metric* (or a *B-metric*),

[1]This work is partially supported by The Fund for Scientific Research of the University of Plovdiv, Bulgaria, Project RS09-FMI-003.

such that

$$(1.1)\qquad \begin{aligned} &\varphi^2 x = -x + \eta(x)\xi, \qquad \eta(\xi) = 1, \\ &g(\varphi x, \varphi y) = -g(x, y) + \eta(x)\eta(y) \end{aligned}$$

for all $x,\ y \in \mathfrak{X}(M)$.

From (1.1) it follows $\varphi\xi = 0$, $\eta\circ\varphi = 0$, $\eta(x) = g(x, \xi)$, $g(\varphi x, y) = g(x, \varphi y)$.

The associated metric $\tilde{g}$ of g is defined by $\tilde{g}(x, y) = g(x, \varphi y) + \eta(x)\eta(y)$ and is a Norden metric, too. Both metrics are necessarily of signature $(n + 1, n)$.

Further, $x,\ y,\ z,\ u$ will stand for arbitrary vector fields in $\mathfrak{X}(M)$.

Let ∇ be the Levi-Civita connection of g. The fundamental tensor F is defined by

$$(1.2)\qquad F(x, y, z) = g\left((\nabla_x \varphi)y, z\right)$$

and has the following properties

$$(1.3)\qquad \begin{aligned} &F(x, y, z) = F(x, z, y), \\ &F(x, \varphi y, \varphi z) = F(x, y, z) - F(x, \xi, z)\eta(y) - F(x, y, \xi)\eta(z). \end{aligned}$$

Equalities (1.3) and $\varphi\xi = 0$ imply $F(x, \xi, \xi) = 0$.

Let $\{e_i, \xi\}$, $(i = 1, 2, \dots, 2n)$ be a basis of the tangent space T_pM at an arbitrary point p of M, and g^{ij} be the components of the inverse matrix of (g_{ij}) with respect to $\{e_i, \xi\}$.

The following 1-forms are associated with F:

$$\theta(x) = g^{ij}F(e_i, e_j, x), \qquad \theta^*(x) = g^{ij}F(e_i, \varphi e_j, x), \qquad \omega(x) = F(\xi, \xi, x).$$

A classification of the almost contact manifolds with Norden metric is introduced in [3]. Eleven basic classes $\mathcal{F}_i$ $(i = 1, 2, \dots, 11)$ are characterized there according to the properties of F.

The Nijenhuis tensor N of the almost contact structure (φ, ξ, η) is defined in [5] by $N(x, y) = [\varphi, \varphi](x, y) + d\eta(x, y)\xi$, i.e. $N(x, y) = \varphi^2[x, y] + [\varphi x, \varphi y] - \varphi[\varphi x, y] - \varphi[x, \varphi y] + (\nabla_x\eta)y.\xi - (\nabla_y\eta)x.\xi$. The almost contact structure is said to be integrable if $N = 0$. In this case the almost contact manifold is called *normal* [5].

In terms of the covariant derivatives of φ and η the tensor N is expressed by $N(x, y) = (\nabla_{\varphi x}\varphi)y - (\nabla_{\varphi y}\varphi)x - \varphi(\nabla_x\varphi)y + \varphi(\nabla_y\varphi)x + (\nabla_x\eta)y.\xi - (\nabla_y\eta)x.\xi$, where $(\nabla_x\eta)y = F(x, \varphi y, \xi)$. Then, according to (1.2), the corresponding tensor of type (0,3), defined by $N(x, y, z) = g(N(x, y), z)$, has the form

$$(1.4)\qquad \begin{aligned} N(x, y, z) = {} & F(\varphi x, y, z) - F(\varphi y, x, z) - F(x, y, \varphi z) \\ & + F(y, x, \varphi z) + F(x, \varphi y, \xi)\eta(z) - F(y, \varphi x, \xi)\eta(z). \end{aligned}$$

The condition $N = 0$ and (1.4) imply

$$(1.5)\qquad F(x, y, \xi) = F(y, x, \xi), \qquad \omega = 0.$$

The 1-form η is said to be closed if $\mathrm{d}\eta = 0$, i.e. if $(\nabla_x \eta)y = (\nabla_y \eta)x$. The class of the almost contact manifolds with Norden metric satisfying the conditions $N = 0$ and $\mathrm{d}\eta = 0$ is $\mathcal{F}_1 \oplus \mathcal{F}_2 \oplus \mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_6$ [3].

Analogously to [2], we define an associated tensor $\widetilde{N}$ of N by

$$\begin{aligned} \widetilde{N}(x, y, z) = & F(\varphi x, y, z) + F(\varphi y, x, z) - F(x, y, \varphi z) \\ & - F(y, x, \varphi z) + F(x, \varphi y, \xi)\eta(z) + F(y, \varphi x, \xi)\eta(z). \end{aligned} \tag{1.6}$$

From $\widetilde{N} = 0$ it follows $F(\varphi x, \varphi y, \xi) = F(y, x, \xi)$, $\omega = 0$. The class with $\widetilde{N} = 0$ is $\mathcal{F}_3 \oplus \mathcal{F}_7$ [3].

The curvature tensor R of ∇ is defined as usually by

$$R(x, y)z = \nabla_x \nabla_y z - \nabla_y \nabla_x z - \nabla_{[x,y]} z,$$

and its corresponding tensor of type (0,4) is given by

$$R(x, y, z, u) = g(R(x, y)z, u).$$

A tensor L of type (0,4) is said to be curvature-like if it has the properties of R, i.e. $L(x, y, z, u) = -L(y, x, z, u) = -L(x, y, u, z)$ and $\underset{x,y,z}{\mathfrak{S}} L(x, y, z, u) = 0$ (first Bianchi identity), where $\mathfrak{S}$ is the cyclic sum by three arguments.

A curvature-like tensor L is called a φ-Kähler-type tensor if

$$L(x, y, \varphi z, \varphi u) = -L(x, y, z, u).$$

2. Connections on almost contact manifolds with Norden metric

Let ∇' be a linear connection with deformation tensor Q, i.e. $\nabla'_x y = \nabla_x y + Q(x, y)$. If we denote $Q(x, y, z) = g(Q(x, y), z)$, then

$$g(\nabla'_x y - \nabla_x y, z) = Q(x, y, z). \tag{2.1}$$

Definition 2.1. *A linear connection ∇' on an almost contact manifold is called an* almost φ-connection *if φ is parallel with respect to ∇', i.e. if $\nabla' \varphi = 0$.*

Because of (1.2), equality (2.1) and $\nabla' \varphi = 0$ imply the condition for Q

$$F(x, y, z) = Q(x, y, \varphi z) - Q(x, \varphi y, z). \tag{2.2}$$

Theorem 2.1. On an almost contact manifold with Norden metric there exists a 10-parametric family of almost φ-connections ∇' of the form (2.1) with deformation tensor Q given by

$$
\begin{aligned}
Q(x,y,z) &= \tfrac{1}{2}\{F(x,\varphi y,z)+F(x,\varphi y,\xi)\eta(z)\}-F(x,\varphi z,\xi)\eta(y)\\
&+t_1\{F(y,x,z)+F(\varphi y,\varphi x,z)-F(y,x,\xi)\eta(z)-F(\varphi y,\varphi x,\xi)\eta(z)\\
&-F(y,z,\xi)\eta(x)-F(\xi,x,z)\eta(y)+\eta(x)\eta(y)\omega(z)\}\\
&+t_2\{F(z,x,y)+F(\varphi z,\varphi x,y)-F(z,x,\xi)\eta(y)-F(\varphi z,\varphi x,\xi)\eta(y)\\
&-F(z,y,\xi)\eta(x)-F(\xi,x,y)\eta(z)+\eta(x)\eta(z)\omega(y)\}\\
&+t_3\{F(y,\varphi x,z)-F(\varphi y,x,z)-F(y,\varphi x,\xi)\eta(z)+F(\varphi y,x,\xi)\eta(z)\\
&-F(y,\varphi z,\xi)\eta(x)-F(\xi,\varphi x,z)\eta(y)+\eta(x)\eta(y)\omega(\varphi z)\}\\
&+t_4\{F(z,\varphi x,y)-F(\varphi z,x,y)-F(z,\varphi x,\xi)\eta(y)+F(\varphi z,x,\xi)\eta(y)\\
&-F(z,\varphi y,\xi)\eta(x)-F(\xi,\varphi x,y)\eta(z)+\eta(x)\eta(z)\omega(\varphi y)\}\\
&+t_5\{F(\varphi y,z,\xi)+F(y,\varphi z,\xi)-\eta(y)\omega(\varphi z)\}\eta(x)\\
&+t_6\{F(\varphi z,y,\xi)+F(z,\varphi y,\xi)-\omega(\varphi y)\eta(z)\}\eta(x)\\
&+t_7\{F(\varphi y,\varphi z,\xi)-F(y,z,\xi)+\eta(y)\omega(z)\}\eta(x)\\
&+t_8\{F(\varphi z,\varphi y,\xi)-F(z,y,\xi)+\omega(y)\eta(z)\}\eta(x)\\
&+t_9\omega(x)\eta(y)\eta(z)+t_{10}\omega(\varphi x)\eta(y)\eta(z), \qquad t_i\in\mathbb{R},\quad i=1,\,2,\,\dots,\,10.
\end{aligned}
\tag{2.3}
$$

Proof. The proof of the statement follows from (2.2), (2.3) and (1.3) by direct verification that $\nabla'\varphi = 0$ for all t_i.

□

Let $N = \mathrm{d}\eta = 0$. By (1.4), (1.5) from Theorem 2.1 we obtain

Corollary 2.1. Let (M,φ,ξ,η,g) be a $\mathcal{F}_1\oplus\mathcal{F}_2\oplus\mathcal{F}_4\oplus\mathcal{F}_5\oplus\mathcal{F}_6$-manifold. Then, the deformation tensor Q of the almost φ-connections ∇' defined by (2.1) and (2.3) has the form

$$
\begin{aligned}
Q(x,y,z) &= \tfrac{1}{2}\{F(x,\varphi y,z)+F(x,\varphi y,\xi)\eta(z)\}-F(x,\varphi z,\xi)\eta(y)\\
&+s_1\{F(y,x,z)+F(\varphi y,\varphi x,z)\}\\
&+s_2\{F(y,\varphi x,z)-F(\varphi y,x,z)\}\\
&+s_3F(y,\varphi z,\xi)\eta(x)+s_4F(y,z,\xi)\eta(x),
\end{aligned}
\tag{2.4}
$$

where $s_1=t_1+t_2$, $s_2=t_3+t_4$, $s_3=2(t_5+t_6)-t_3-t_4$, $s_4=-t_1-t_2-2(t_7+t_8)$.

Definition 2.2. *A linear connection ∇' is said to be* almost contact *if the almost contact structure (φ,ξ,η) is parallel with respect to it, i.e. if $\nabla'\varphi = \nabla'\xi = \nabla'\eta = 0$.*

Then, in addition to the condition (2.2), for the deformation tensor Q of an almost contact connection given by (2.1) we also have

$$
F(x,\varphi y,\xi) = Q(x,y,\xi) = -Q(x,\xi,y). \tag{2.5}
$$

Definition 2.3. *A linear connection on an almost contact manifold with Norden metric* (M,φ,ξ,η,g) *is said to be* natural *if* $\nabla'\varphi=\nabla'\eta=\nabla'g=0$.

The condition $\nabla'g=0$ and (2.1) yield

$$Q(x,y,z)=-Q(x,z,y). \tag{2.6}$$

From (1.6) and (2.3) we compute

$$\begin{aligned}
&Q(x,y,z)+Q(x,z,y)=\\
&=(t_1+t_2)\{\widetilde{N}(y,z,\varphi x)-\widetilde{N}(\xi,y,\varphi x)\eta(z)-\widetilde{N}(\xi,z,\varphi x)\eta(y)\}\\
&\quad-(t_3+t_4)\{\widetilde{N}(y,z,x)-\widetilde{N}(\xi,y,x)\eta(z)-\widetilde{N}(\xi,z,x)\eta(y)\}\\
&\quad-(t_5+t_6)\{\widetilde{N}(\varphi^2z,y,\xi)+\widetilde{N}(z,\xi,\xi)\eta(y)\}\eta(x)\\
&\quad+(t_7+t_8)\{\widetilde{N}(\varphi z,y,\xi)-\widetilde{N}(\varphi z,\xi,\xi)\eta(y)\}\eta(x)\\
&\quad+2\{t_9\omega(x)+t_{10}\omega(\varphi x)\}\eta(y)\eta(z).
\end{aligned} \tag{2.7}$$

By (2.1), (2.3), (2.5), (2.6) and (2.7) we prove the following

Proposition 2.1. Let (M,φ,ξ,η,g) be an almost contact manifold with Norden metric, and let ∇' be the 10-parametric family of almost φ-connections defined by (2.1) and (2.3). Then

(i): ∇' are almost contact iff $t_1+t_2-t_9=t_3+t_4-t_{10}=0$;
(ii): ∇' are natural iff $t_1+t_2=t_3+t_4=t_5+t_6=t_7+t_8=t_9=t_{10}=0$.

Taking into account equation (1.4), Theorem 2.1, Proposition 2.1, and by putting $p_1=t_1=-t_2$, $p_2=t_3=-t_4$, $p_3=t_5=-t_6$, $p_4=t_7=-t_8$, we obtain

Theorem 2.2. On an almost contact manifold with Norden metric there exists a 4-parametric family of natural connections ∇'' defined by

$$\begin{aligned}
g(\nabla''_xy-\nabla_xy,z)=&\tfrac{1}{2}\{F(x,\varphi y,z)+F(x,\varphi y,\xi)\eta(z)\}-F(x,\varphi z,\xi)\eta(y)\\
&+p_1\{N(y,z,\varphi x)+N(\xi,y,\varphi x)\eta(z)+N(z,\xi,\varphi x)\eta(y)\}\\
&+p_2\{N(z,y,x)+N(y,\xi,x)\eta(z)+N(\xi,z,x)\eta(y)\}\\
&+p_3\{N(\varphi^2z,y,\xi)+N(z,\xi,\xi)\eta(y)\}\eta(x)\\
&+p_4\{N(y,\varphi z,\xi)+N(\varphi z,\xi,\xi)\eta(y)\}\eta(x).
\end{aligned}$$

Since $N=0$ on a normal almost contact manifold with Norden metric, the family ∇'' consists of a unique natural connection on such manifolds

$$\nabla''_xy=\nabla_xy+\tfrac{1}{2}\{(\nabla_x\varphi)\varphi y+(\nabla_x\eta)y.\xi\}-\nabla_x\xi.\eta(y), \tag{2.8}$$

which is the well-known canonical connection [1].

Because of Proposition 2.1., (2.7) and the condition $\widetilde{N}=0$, the connections ∇' given by (2.1) and (2.3) are natural on a $\mathcal{F}_3\oplus\mathcal{F}_7$-manifold iff $t_1=-t_2$ and $t_3=-t_4$.

Let $(M, \varphi, \xi, \eta, g)$ be in the class $\mathcal{F}_1 \oplus \mathcal{F}_2 \oplus \mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_6$. Then, $N = \mathrm{d}\eta = 0$ and hence the torsion tensor T of the 4-parametric family of almost φ-connections ∇' defined by (2.1) and (2.4) has the form

$$\begin{aligned} T(x,y,z) = s_1 \{&F(y,x,z) - F(x,y,z) + F(\varphi y, \varphi x, z) \\ &- F(\varphi x, \varphi y, z)\} + \tfrac{1-4s_2}{2}\{F(x,\varphi y, z) - F(y, \varphi x, z)\} \\ &- s_4\{F(x,z,\xi)\eta(y) - F(y,z,\xi)\eta(x)\} \\ &- (1 - s_2 + s_3)\{F(x,\varphi z,\xi)\eta(y) - F(y,\varphi z,\xi)\eta(x)\}. \end{aligned} \tag{2.9}$$

Then, from (2.9) we derive that $T = 0$ if and only if $s_1 = s_4 = 0$, $s_2 = \frac{1}{4}$, $s_3 = -\frac{3}{4}$. By this way we prove

Theorem 2.3. On a $\mathcal{F}_1 \oplus \mathcal{F}_2 \oplus \mathcal{F}_4 \oplus \mathcal{F}_5 \oplus \mathcal{F}_6$-manifold there exists a symmetric almost φ-connection ∇' defined by

$$\begin{aligned} \nabla'_x y = \nabla_x y + \tfrac{1}{4}\{&2(\nabla_x\varphi)\varphi y + (\nabla_y\varphi)\varphi x - (\nabla_{\varphi y}\varphi)x \\ &+ 2(\nabla_x\eta)y.\xi - 3\eta(x).\nabla_y\xi - 4\eta(y).\nabla_x\xi\}. \end{aligned} \tag{2.10}$$

Let us remark that the connection (2.10) can be considered as an analogous connection to the well-known Yano connection [6, 7] on a normal almost contact manifold with Norden metric and closed 1-form η.

3. Connections on $\mathcal{F}_4 \oplus \mathcal{F}_5$-manifolds

In this section we study the curvature properties of the 4-parametric family of almost φ-connections ∇' given by (2.1) and (2.4) on two of the basic classes normal almost contact manifolds with Norden metric, namely the classes $\mathcal{F}_4$ and $\mathcal{F}_5$. These classes are defined in [3] by the following characteristic conditions for F, respectively:

$$\mathcal{F}_4 : F(x,y,z) = -\tfrac{\theta(\xi)}{2n}\{g(\varphi x, \varphi y)\eta(z) + g(\varphi x, \varphi z)\eta(y)\}, \tag{3.1}$$

$$\mathcal{F}_5 : F(x,y,z) = -\tfrac{\theta^*(\xi)}{2n}\{g(\varphi x, y)\eta(z) + g(\varphi x, z)\eta(y)\}. \tag{3.2}$$

The subclasses of $\mathcal{F}_4$ and $\mathcal{F}_5$ with closed 1-form θ and θ^*, respectively, are denoted by $\mathcal{F}_4^0$ and $\mathcal{F}_5^0$. Then, it is easy to prove that on a $\mathcal{F}_4^0 \oplus \mathcal{F}_5^0$-manifold it is valid:

$$x\theta(\xi) = \xi\theta(\xi)\eta(x), \qquad x\theta^*(\xi) = \xi\theta^*(\xi)\eta(x). \tag{3.3}$$

Taking into consideration (3.1) and (3.2), from (2.1) and (2.4) we obtain

Proposition 3.1. Let $(M, \varphi, \xi, \eta, g)$ be a $\mathcal{F}_4 \oplus \mathcal{F}_5$-manifold. Then, the connections ∇' defined by (2.1) and (2.4) are given by

$$\begin{aligned} \nabla'_x y = \nabla_x y &+ \tfrac{\theta(\xi)}{2n}\{g(x,\varphi y)\xi - \eta(y)\varphi x\} + \tfrac{\theta^*(\xi)}{2n}\{g(x,y)\xi - \eta(y)x\} \\ &+ \tfrac{\lambda\theta(\xi)+\mu\theta^*(\xi)}{2n}\{\eta(x)y - \eta(x)\eta(y)\xi\} + \tfrac{\mu\theta(\xi)-\lambda\theta^*(\xi)}{2n}\eta(x)\varphi y, \end{aligned} \tag{3.4}$$

where $\lambda = s_1 + s_4$, $\mu = s_3 - s_2$.

The Yano-type connection (2.10) is obtained from (3.4) for $\lambda = 0$, $\mu = -1$.

Let us denote by R' the curvature tensor of ∇', i.e. $R'(x,y)z = \nabla'_x\nabla'_y z - \nabla'_y\nabla'_x z - \nabla'_{[x,y]}z$. The corresponding tensor of type (0,4) with respect to g is defined by $R'(x,y,z,u) = g(R'(x,y)z,u)$. Then, it is valid

Proposition 3.2. Let $(M, \varphi, \xi, \eta, g)$ be a $\mathcal{F}_4^0 \oplus \mathcal{F}_5^0$-manifold, and ∇' be 2-parametric family of almost φ-connections defined by (3.4). Then, the curvature tensor R' of an arbitrary connection in the family (3.4) is of φ-Kähler-type and has the form

$$\begin{aligned} R' = R &+ \tfrac{\xi\theta(\xi)}{2n}\pi_5 + \tfrac{\xi\theta^*(\xi)}{2n}\pi_4 + \tfrac{\theta(\xi)^2}{4n^2}\{\pi_2 - \pi_4\} \\ &+ \tfrac{\theta^*(\xi)^2}{4n^2}\pi_1 - \tfrac{\theta(\xi)\theta^*(\xi)}{4n^2}\{\pi_3 - \pi_5\}, \end{aligned} \tag{3.5}$$

where the curvature-like tensors π_i $(i = 1,2,3,4,5)$ are defined by [4]*:*

$$\begin{aligned} \pi_1(x,y,z,u) &= g(y,z)g(x,u) - g(x,z)g(y,u), \\ \pi_2(x,y,z,u) &= g(y,\varphi z)g(x,\varphi u) - g(x,\varphi z)g(y,\varphi u), \\ \pi_3(x,y,z,u) &= -g(y,z)g(x,\varphi u) + g(x,z)g(y,\varphi u) \\ &\quad - g(x,u)g(y,\varphi z) + g(y,u)g(x,\varphi z), \\ \pi_4(x,y,z,u) &= g(y,z)\eta(x)\eta(u) - g(x,z)\eta(y)\eta(u) \\ &\quad + g(x,u)\eta(y)\eta(z) - g(y,u)\eta(x)\eta(z), \\ \pi_5(x,y,z,u) &= g(y,\varphi z)\eta(x)\eta(u) - g(x,\varphi z)\eta(y)\eta(u) \\ &\quad + g(x,\varphi u)\eta(y)\eta(z) - g(y,\varphi u)\eta(x)\eta(z). \end{aligned} \tag{3.6}$$

Proof. It is known that the curvature tensors of two linear connections related by an equation of the form (2.1) satisfy

$$\begin{aligned} g(R'(x,y)z,u) = R(x,y,z,u) &+ (\nabla_x Q)(y,z,u) - (\nabla_y Q)(x,z,u) \\ &+ Q(x,Q(y,z),u) - Q(y,Q(x,z),u). \end{aligned} \tag{3.7}$$

Then, (3.5) follows from (3.7), (3.4), (3.3) and (3.6) by straightforward computation. □

Let us remark that (3.5) is obtained in [4] for the curvature tensor of the canonical connection (2.8) on a $\mathcal{F}_4^0 \oplus \mathcal{F}_5^0$-manifold, i.e. the connection (3.4) for $\lambda = \mu = 0$.

References

[1] V. Alexiev, G. Ganchev: Canonical connection on a conformal almost contact metric manifold, *Ann. Univ. Sofia-Math.* **81**, (1987), 29–38.

[2] G. Ganchev, A. Borisov: Note on the almost complex manifolds with a Norden metric, *Compt. Rend. Acad. Bulg. Sci.* 39(5), (1986), 31–34.

[3] G. Ganchev, V. Mihova, K. Gribachev: Almost contact manifolds with B-metric, *Math. Balk. N.S.* 7(3-4) (1993), 261–276.

[4] M. Manev: On the curvature properties of almost contact B-metric hypersurfaces of Kaehlerian manifolds with B-metric, *Plovdiv Univ. Sci. Works – Math.* 33(3), (2001), 61-72.

[5] S. Sasaki: *Almost-contact manifolds*, Lecture Notes Math., Inst. Tôhoko Univ. 1-3, 1965, 1967, 1968.

[6] K. Yano: Affine connections in an almost product space, *Kodai Math. Semin. Rep.* 11(1), (1959), 1–24.

[7] K. Yano: Differential geometry on complex and almost complex spaces, *Pure and Applied Math.*, vol. 49, Pergamon Press Book, New York, 1965.

Marta Teofilova
Faculty of Mathematics and Informatics
University of Plovdiv
236 Bulgaria Blvd.
4003 Plovdiv, Bulgaria
e-mail: marta@uni-plovdiv.bg