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A SMOOTH FOUR-DIMENSIONAL G-HILBERT SCHEME

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Abstract. When the cyclic group G of order 15 acts with some specific weights on affine four-dimensional space, the G-Hilbert scheme is a crepant resolution of the quotient $\mathbb{A}^4/G$. We give an explicit description of this resolution using G-graphs.

1. Introduction. Let G be a finite group acting faithfully on a quasi-projective smooth scheme X. Consider the quotient X/G, the space of orbits, which is in general a singular scheme. We use a variant of the quotient, to resolve the singularities: we consider not only the set of orbits, but also a whole collection of zero-dimensional G-invariant subsets of X with the associated ring of global sections of the structure sheaf, isomorphic to the regular representation ring of the group G. This is formalized in the notion of a G-Hilbert scheme, introduced in [9]. Apart from the symplectic case treated in [1], the G-Hilbert scheme of a quasi-projective variety X is, in general, a "very" singular variety, especially in higher dimensions. The known cases of smooth G-Hilbert schemes are the minimal resolutions of Klein singularities and the crepant resolutions

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of the quotient $\mathbb{C}^3/G$, with G being a finite subgroup of $SL_3(\mathbb{C})$. The three-dimensional case is due to Craw, Ito, Markouchevitch and Roan by a case-by-case analysis, and to Bridgeland-King-Reid as a consequence of a more general result (see [2]). Unfortunately, none of the current methods gives an answer to the following question: which finite subgroups of $GL_n(\mathbb{C})$, $n \geq 4$, admit G-Hilb$\mathbb{A}^n$ as a crepant resolution of the quotient $\mathbb{A}^n/G$? The only attempt is given in [6], and it is reduced to the two-dimensional case.

In order to give some positive answer to the above question, one needs a better description of the G-Hilbert scheme. Suppose – from now on – that G is a finite abelian subgroup of $GL_n(\mathbb{C})$, consisting of diagonal matrices. Then there is a toric approach to the question given by Nakamura (see [10]).

We now introduce some notation needed in the present paper. We recall that the affine space $\mathbb{A}^n$ is a toric variety associated to the lattice $L := \mathbb{Z}^n$ and the cone $\gamma := \langle e_1, \dots, e_n \rangle$, where $e_1, \dots, e_n$ is the standard basis of $\mathbb{R}^n$ (cf. [8], §2). Suppose that G acts faithfully on $\mathbb{A}^n$. Let r denote the order of G. We write each (diagonal) matrix g of G in the form $g = \mathrm{diag}(\varepsilon^{a_1}, \dots, \varepsilon^{a_n})$, with $\varepsilon := e^{\frac{2\pi i}{r}}$ — a fixed r^{th} primitive root of unity. We associate to such a matrix g a vector $v_g := \frac{1}{r}(a_1, \dots, a_n) \in \mathbb{Q}^n$, and define the lattice:

$$N := L + \sum_{g \in G} \mathbb{Z} v_g.$$

We denote by $N^\vee = \mathrm{Hom}_\mathbb{Z}(N, \mathbb{Z})$ its dual. Then, the quotient $\mathbb{A}^n/G$ is a toric variety with lattice N and fan reduced to the cone γ. We denote by $\{f_1, \dots, f_n\}$ the dual basis of $\{e_1, \dots, e_n\}$ and by M_0 the additive semi-group generated by 0 and the f_i's. We define a semi-group homomorphism from M_0 to the semi-group M of all monomials in n variables (endowed with multiplication), by sending f_i to X_i. In this way, we identify a vector with n non-negative integer coordinates with a monomial. For a monomial p of $\mathbb{C}[X_1, \dots, X_n]$ (or "Laurent monomial" p of $\mathbb{C}[X_1, \dots, X_n][X_1^{-1}, \dots, X_n^{-1}]$), we denote by $v(p)$ the associated vector. Let $G^\vee$ be the set of all irreducible characters of the group G. We introduce the following definition:

Definition 1.1. *Given $\chi \in G^\vee$ and $p = X_1^{i_1} \dots X_n^{i_n}$ a monomial of $\mathbb{C}[X_1, \dots, X_n]$ (or a Laurent monomial), we say that p and χ are* ***associated*** *if we have:*

$$g \cdot p = \chi(g) p, \quad \forall g \in G\,.$$

Definition 1.2. *A subset* Γ *of* M *is called a* $G-$***graph*** (*cf.* [10], *Def.* 1.4) *if the following conditions hold:*

(1) *it contains the constant monomial* 1*;*

(2) *if* p *is in* Γ *and a monomial* q *divides* p*, then* q *is also in* Γ*;*

(3) *the map* $\mathrm{wt}: \Gamma \to G^{\vee}$ *sending a monomial to its associated character (as in Definition 1.1), is a bijection.*

Denote by $\mathrm{Graph}(G)$ the set of all G-graphs.

By condition (3) in Definition 1.2, there is a unique monomial of Γ associated to any character of $G^{\vee}$. Thus, we define a map $\mathrm{wt}_{\Gamma}: M \to \Gamma$, by sending a monomial to the unique element of Γ with the same associated character.

Definition 1.3. *Given a monomial* p *of* M*, we call the fraction* $p/\mathrm{wt}_{\Gamma}(p) \in \mathbb{C}(X_1, \dots, X_n)$ *the* ***ratio*** *of* p *with respect to* Γ.

Finally, we associate to a G-graph Γ, a cone, a semi-group and an ideal as follows. We denote by $\langle , \rangle$ the scalar product in $\mathbb{R}^n$. We define a cone in $\mathbb{R}^n$ by $\sigma(\Gamma) := \{u \in \mathbb{R}^n / \langle u, v(p/\mathrm{wt}_{\Gamma}(p))\rangle \geq 0, \forall p \in M\}$. The dual of a cone $\sigma(\Gamma)$ is given by $\sigma^{\vee}(\Gamma) := \{v \in \mathbb{R}^n / \langle u, v\rangle \geq 0, \forall u \in \sigma(\Gamma)\}$. We denote by $S(\Gamma)$ the sub-semi-group of $N^{\vee}$ generated by vectors $v(p/\mathrm{wt}_{\Gamma}(p))$, where p runs over the set M. We call $I(\Gamma)$ the ideal of $\mathbb{C}[X_1, \dots, X_n]$ generated by all the monomials of M that are not in Γ.

μ_{15}-Hilb$\mathbb{C}^4$ scheme and its properties. This section deals with an explicit description of the μ_{15}-Hilb$\mathbb{C}^4$ scheme. In the sequel, we denote by $G = \mu_{15}$ the cyclic group of order 15, with generator $\varepsilon := e^{\frac{2\pi i}{15}}$. Let this group act by weights $1, 2, 4$ and 8 on the affine space $\mathbb{A}^4$. We identify G with the finite abelian subgroup of $SL_4(\mathbb{C})$ with generator $g = \mathrm{diag}(\varepsilon, \varepsilon^2, \varepsilon^4, \varepsilon^8)$ and the action of G on $\mathbb{A}^4$ with the natural action of multiplication of an element of $\mathbb{A}^4$ by a matrix.

The quotient $\mathbb{A}^4/G$ is a Gorenstein canonical singularity (cf. [12]) and has only one isolated singularity at the origin. With the notations of Section 1, it is a toric variety with lattice $N = \mathbb{Z}^4 + \frac{1}{15}(1, 2, 4, 8)\mathbb{Z}$ and fan the cone $\gamma = \langle e_1, e_2, e_3, e_4\rangle$. In order to resolve the singularities, we will provide a simplicial decomposition of γ into sub-cones such that the resulting variety is G-Hilb$\mathbb{C}^4$. We prove that G-Hilb$\mathbb{C}^4$ is smooth and it has the crepancy property. Here, crepancy means that the canonical sheaf $\omega_{G\text{-Hilb}\mathbb{C}^4}$ and the structure sheaf $\mathcal{O}_{G\text{-Hilb}\mathbb{C}^4}$ are isomorphic. We remark that the methods of [2] can't be applied in this case

— the condition on the fiber product of Theorem 1.1 of the cited paper is not satisfied. Using Definition 5.2 of [7] and Watanabe's classification Theorem (see Theorem 5.3 of the same paper), we see that the group G above doesn't give rise to a complete intersection singularity. In particular, the techniques of [6] are not applicable. We are not in the symplectic case, so [1] does not apply either.

In what follows, we set $x = X_1$, $y = X_2$, $z = X_3$ and $t = X_4$.

Definition 2.1. *Given a monomial $p = x^\alpha y^\beta z^\gamma t^\delta$ (or a Laurent monomial), the **weight** of p with respect to the group G above is the unique integer $w(p) \in \{0, \ldots, 14\}$, that satisfies $\alpha + 2\beta + 4\gamma + 8\delta \equiv w(p) \pmod{15}$.*

Remark 2.2. We recall that an irreducible character χ_i of G, with $i \in \{0, \ldots, 14\}$, is given by $g \mapsto \varepsilon^i$. So, a monomial $p = x^\alpha y^\beta z^\gamma t^\delta$ is associated to the character χ_i if and only if its weight is i. Thus, a G-graph is a set of 15 monomials of weights 0 to 14, satisfying (1) and (2) of Definition 1.2.

Definition 2.3. *Given S a finite set of monomials in $\mathbb{C}[x, y, z, t]$ and s one of the variables, we set **max.p.**(s) to be $-\infty$ if S doesn't contain any monomial in s and $\max\{l \in \mathbb{N}/s^l \in S\}$ otherwise. We call it the **maximal power** associated to the variable s in the set S.*

Lemma 2.4. *The G-graphs for the group $G = \mu_{15}$ are the following:*

No.	μ_{15}-graph
1	$(1, x, y, y^2, \ldots, y^7, xy, xy^2, \ldots, xy^6)$
2	$(1, x, y, y^2, y^3, t, xy, xy^2, xy^3, xt, yt, y^2t, y^3t, xyt, xy^2t)$
3	$(1, x, y, z, t, xy, xz, xt, yz, yt, zt, xyz, xyt, xzt, yzt)$
4	$(1, x, y, z, z^2, z^3, xy, xz, xz^2, xz^3, yz, yz^2, yz^3, xyz, xyz^2)$
5	$(1, x, x^2, x^3, z, t, xz, x^2z, x^3z, xt, x^2t, x^3t, zt, xzt, x^2zt)$
6	$(1, x, x^2, x^3, z, z^2, z^3, xz, x^2z, x^3z, xz^2, x^2z^2, x^2z^3, x^3z, x^3z^2)$
7	$(1, x, x^2, \ldots, x^7, t, xt, x^2t, \ldots, x^6t)$
8	$(1, x, x^2, \ldots, x^{14})$
9	$(1, y, y^2, \ldots, y^{14})$
10	$(1, y, y^2, y^3, t, t^2, t^3, yt, yt^2, yt^3, y^2t, y^2t^2, y^2t^3, y^3t, y^3t^2)$
11	$(1, y, z, z^2, \ldots, z^7, yz, yz^2, \ldots, yz^6)$
12	$(1, y, z, t, t^2, t^3, yz, y^2z, y^3zyt, yt^2, yt^3, zt, yzt, y^2zt)$
13	$(1, z, t, t^2, \ldots, t^7, zt, zt^2, \ldots, zt^6)$
14	$(1, z, z^2, \ldots, z^{14})$
15	$(1, t, t^2, \ldots, t^{14})$

Table 1. List of G-graphs for $G = \mu_{15}$.

P r o o f. The idea of the proof is to consider, for a given G-graph, the possible choices of max.p.(s), where s is one of the variables x, y, z or t. By (1) in Definition 1.2, the constant monomial 1 is in the G-graph. In particular, for any variable s we have max.p.$(s) \geq 0$. By condition (2) of Definition 1.2 and Remark 2.2, max.p.(s) is also less than 15 for any variable s.

Now, for any G-graph Γ, there are two possible cases: either max.p.$(x) = 0$ – that is x doesn't occur in Γ, or max.p.$(x) > 0$ – meaning that Γ contains at least one monomial in x.

We discuss now the case where x occurs in a G-graph Γ. We claim that in this case max.p.(x) cannot be 2, 4, 5, 6, 8, 9, 10, 11, 12, nor 13. To see this, we argue by contradiction. We suppose that there exists a G-graph Γ such that we have max.p.$(x) = 2$. Then, using (2) of Definition 1.2 and Remark 2.2, we see that y, z^4, t^2 cannot occur in any monomial of Γ. In particular, max.p.$(y) = 0$, max.p.$(z) \leq 3$ and max.p.$(t) \leq 1$. On the other hand, there has to be a monomial of weight 3 in Γ and the only possible choice is x^2z^2t. We use (2) of Definition 1.2 to deduce that x^2z^2 and x^2t should both belong to Γ. But these monomials both have the same weight, which contradicts Remark 2.2 and thus the definition of a G-graph. Now, if max.p.(x) were 4, 5 or 6, we see in a similar way that there is no possible choice of monomial of weight 7. When max.p.(x) is one of 8, 9, 10, 11, 12 or 13, there are no monomials divisible by y, z or t. This is because the monomials x^2, x^4 and x^8 are already in Γ and they have the same weights as y, z and t. But then the G-graph Γ contains less then 15 monomials, which is a contradiction.

We discuss now the case max.p.$(x) = 1$. With the notations of Definition 2.1, we have $w(y^8) = w(z^4) = w(t^2) = 1$. We get max.p.$(y) \leq 7$,max.p.$(z) \leq 3$ and max.p.$(t) \leq 1$. A similar discussion as above proves that max.p.(y) equals 1, 3 or 7. Now, if max.p.$(y) = 7$, we get the G-graph Γ_1 of Table 1. If max.p.$(y) = 3$, there are no monomials in z and we obtain Γ_2. For max.p.$(y) = 1$, either max.p.$(z) = 1$ and we get Γ_3, or max.p.$(z) = 3$ and no monomial in t is allowed – we get Γ_4.

For the case max.p.$(x) = 3$, the G-graph contains no monomial in y and has max.p.$(z) \leq 3$ and max.p.$(t) \leq 1$. We obtain the G-graphs Γ_5 and Γ_6 of Table 1.

The case max.p.$(x) = 7$ gives max.p.$(y) = 0$, max.p.$(z) = 0$, and max.p.$(t) \leq 1$. The only possible G-graph is Γ_7, a "planar" G-graph in x and t.

For the last case, i.e. max.p.$(x) = 14$, we get a "linear" G-graph containing only monomials in the variable x – this is the G-graph Γ_8.

Now, if x does not occur in the G-graph, we prove, by a similar argument, that max.p.(y) is one of 1, 3 or 14. We obtain the G-graphs Γ_9, Γ_{10}, Γ_{11}, Γ_{12}. If neither x, nor y occur, then we get again a "planar" G-graph in z and t — this is Γ_{13} and two "linear" graphs – Γ_{14} only in z and Γ_{15} only in t. □

Let Γ be a G-graph as in Table 1. We associate to it a finite set, denoted $E(\Gamma)$, as follows. If the variable x occurs in Γ, let the vector v_g associated to the matrix g be in $E(\Gamma)$, otherwise include in $E(\Gamma)$ the vector e_1 of the canonical base. Similarly, if y is in Γ we take the vector associated to the matrix g^8 in $E(\Gamma)$ and otherwise include the vector e_2. For the variable z, we take either the vector associated to the matrix g^4 or the vector e_3. Finally, if t is in Γ let the vector associated to the matrix g^2 be in $E(\Gamma)$, otherwise include the vector e_4. For example, the set $E(\Gamma_8)$ is $\left\{\frac{1}{15}(1,2,4,8), e_2, e_3, e_4\right\}$.

Lemma 2.5. *With the notations at the end of section* 1*, the cones* $\sigma(\Gamma)$ *are generated by the sets* $E(\Gamma)$*.*

Proof. We want to prove that any vector u of a cone $\sigma(\Gamma)$ is a linear combination with positive coefficients of elements of the associated $E(\Gamma)$. Equivalently, the system:

$$\sum_{v\in E(\Gamma)} x_v \cdot v = u, \tag{2.1}$$

has to admit a solution with x_v positive for any v in $E(\Gamma)$. For this, we note that each x_v is nothing but the scalar product of u and the vector associated to the ratio of a monomial generator of $I(\Gamma)$ (see section 1 for definitions and notations). We use the definition of $\sigma(\Gamma)$ to conclude. □

Example 2.6. For a more explicit approach, let us actually see what happens for the G-graph Γ_8. The set $E(\Gamma_8)$ is given by the vectors e_2, e_3, e_4 and the vector associated to the matrix g – this is $v_g = \frac{1}{15}(1,2,4,8)$. Let $u = (u_1,u_2,u_3,u_4)$ be a vector of $\sigma(\Gamma_8)$. The solutions of the system (2.1) are:

$$\begin{aligned} x_{v_g} &= \langle u, 15e_1\rangle,\\ x_{e_2} &= 15\langle u, e_2-2e_1\rangle,\\ x_{e_3} &= 15\langle u, e_3-4e_1\rangle,\\ x_{e_4} &= 15\langle u, e_4-8e_1\rangle. \end{aligned} \tag{2.2}$$

The monomial generators of $I(\Gamma_8)$ are x^{15}, y, z, t. The vectors associated to the ratios of those monomial generators are $15e_1, e_2 - 2e_1, e_3 - 4e_1$ and $e_4 - 8e_1$. By definition of $\sigma(\Gamma_8)$, the numbers $\langle u, 15e_1\rangle$, $\langle u, e_2 - 2e_1\rangle$, $\langle u, e_3 - 4e_1\rangle$ and $\langle u, e_4 - 8e_1\rangle$ are positive. Thus the solutions given by (2.2) are also positive numbers, as wanted. □

Corollary 2.7. *The toric variety obtained by gluing together all the affine pieces* $\mathrm{Spec}[\sigma^\vee(\Gamma) \cap N^\vee]$, *where* Γ *runs over* $\mathrm{Graph}(G)$, *is a smooth variety.*

Proof. It is enough to see that every such affine piece is a copy of $\mathbb{C}^4$. For this, we use the "smoothness criterion" of [11], Theorem 1.10, page 10. Now the result follows because each cone $\sigma(\Gamma)$ is generated by the sets $E(\Gamma)$ of the Lemma 2.5 which are a [part of a] basis for the lattice N. □

Lemma 2.8. *With the notations of Corollary* 2.7, *for any G-graph, we have* $S(\Gamma) = \sigma^\vee(\Gamma) \cap N^\vee$.

Proof. By definitions of $\sigma(\Gamma)$ and $S(\Gamma)$, the inclusion $S(\Gamma) \subset \sigma^\vee(\Gamma) \cap N^\vee$ follows immediately. Now, for the reverse inclusion, it is enough to prove that $\sigma^\vee(\Gamma) \cap N^\vee$ is generated by a set of vectors contained in $S(\Gamma)$. For this, we proceed as follows. We call a monomial pure if it involves only one of the variables x, y, z or t. We denote by $v_\Gamma(s)$ the vectors associated to the ratios of the pure monomials s that generate $I(\Gamma)$.

We claim that any vector v in $\sigma^\vee(\Gamma) \cap N^\vee$ is a linear combination with non-negative integer coefficients of the vectors $v_\Gamma(s)$ above. A case-by-case analysis of each $I(\Gamma)$ shows that this is equivalent to resolving a 4×4 system. The proof is very similar to that of Lemma 2.5 and the calculations made in the subsequent Example 2.6, so we omit it. □

Theorem 2.9. *Let the cyclic group of order* 15 *act by weights 1, 2, 4, 8 on the affine four dimensional space. Then, the* μ_{15}*-Hilbert scheme of* $\mathbb{A}^4$ *is a smooth variety and it provides a crepant resolution of the Gorenstein quotient singularity* $\mathbb{A}^4/\mu_{15}$.

Proof. We use (iii) of Theorem 2.11 of [10] to see that μ_{15}-Hilb$\mathbb{A}^4$ is the variety obtained by gluing together all the Spec$\mathbb{C}[S(\Gamma)]$, when Γ runs over $\mathrm{Graph}(\mu_{15})$. By Lemma 2.8, this is the same as the toric variety whose fan is given by the cones $\sigma(\Gamma), \Gamma \in \mathrm{Graph}(\mu_{15})$. We apply Corollary 2.7 to conclude that μ_{15}-Hilb$\mathbb{A}^4$ is smooth. In particular, the μ_{15}-Hilbert scheme of $\mathbb{A}^4$ provides a toric resolution of the quotient $\mathbb{A}^4/\mu_{15}$, by subdivision of the cone γ into the sub-cones $\sigma(\Gamma), \Gamma \in \mathrm{Graph}(\mu_{15})$.

Now, for crepancy, we use the equivalence stated in [3], page 656. For this, we see that the Euler number of μ_{15}-Hilb$\mathbb{A}^4$ is given by the number of three-dimensional cones of the associated fan. In this case, this is $\#\mathrm{Graph}(\mu_{15}) = \#G = 15$. Together with smoothness, this gives crepancy. □

Remark 2.10. Equivalently, we could use Theorem 4.1 of [7] to prove the crepancy of the desingularization defined by μ_{15}-Hilb$\mathbb{A}^4$. This is because the first skeleton of any cone $\sigma(\Gamma)$, for Γ a μ_{15}-graph, is formed only by elements in the simplex $\Delta_4 := \{(u_1, u_2, u_3, u_4) \in \mathbb{R}^4/u_1 + u_2 + u_3 + u_4 = 1\}$, as proved in Lemma 2.5.

3. Miscellaneous remarks.

Remark 3.1. The H-Hilbert scheme of $\mathbb{A}^4$ for H the cyclic group of order 40 acting by weights 1, 3, 9, 27 on $\mathbb{A}^4$ fails to provide a smooth crepant resolution for the quotient singularity $\mathbb{A}^4/H$. This is mainly because the number of H-graphs in this case is not equal to the cardinal of the group.

Remark 3.2. It is interesting to note that in the situation described in Section 2, the McKay correspondence as stated by Reid holds (see [4] page x for a statement of the conjecture). The Betti numbers b_l of a crepant resolution (in this case μ_{15}-Hilb$\mathbb{A}^4$) are the cardinalities of the conjugacy classes of elements of same "age" l in the group μ_{15}.

Remark 3.3. (cf. with [10] "deformation" and [5] "ratio"). We can recover all the μ_{15}-graphs from a given one, by using ratios of the generating monomials of the associated ideal, as follows. We take for example the μ_{15}-graph Γ_3 in Table 1. The associated ideal is generated by the monomials x^2, y^2, z^2, t^2, $xyzt$. The corresponding ratios are x^2/y, y^2/z, z^2/t, t^2/x, $xyzt/1$. Let us take one of those, say z^2/t.

By repeatedly replacing every occurrence of t in Γ_3 by z^2, we recover the μ_{15}-graph Γ_4. We do the same with the other ratios (note that the ratio $xyzt/1$ provides no μ_{15}-graph), to recover Γ_2, Γ_5 and Γ_{12}. By repeating the procedure, we get all remaining μ_{15}-graphs of Table 1. The result is given in the figure below.

In the Figure 1, the direction of the arrows can be reverted, as follows. We take for example the "bottom" μ_{15}-graph Γ_9. The pure monomial x of $I(\Gamma_9)$ has as associated ratio the fraction x/y^8. Replacing in Γ_9 the monomial y^8 and all its occurrences by x, we obtain the μ_{15}-graph Γ_1.

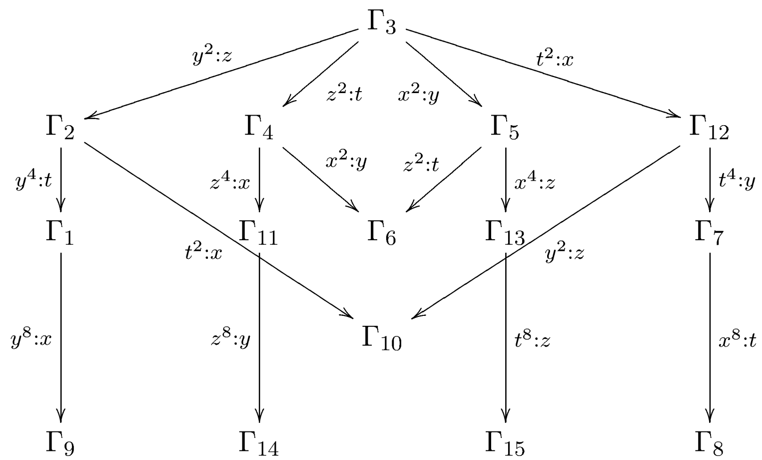

Figure 1. Deforming G-graphs for $G = \mu_{15}$.

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