

WEIGHTED ICP ALGORITHM FOR ALIGNMENT OF STARS FROM SCANNED ASTRONOMICAL PHOTOGRAPHIC PLATES*

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ABSTRACT. Given the coarse celestial coordinates of the centre of a plate scan and the field of view, we are looking for a mapping between the stars extracted from the image and the stars from a catalogue, where the stars from both sources are represented by their stellar magnitudes and coordinates, relatively to the image centre. In a previous work we demonstrated the application of Iterative Closest Point (ICP) algorithm for the alignment problem where stars were represented only by their geometrical coordinates. ICP leads to translation and rotation of the initial points—a correction required for one set of stars to fit over the other. This paper extends the previous work by demonstrating significant improvement of ICP by using the stellar magnitudes as point weights. The improvement consists of great decrease of the iteration count until convergence, which helps in the case of highly “misaligned” initial states. The essential aspects of the ICP method like noise tolerance of false or missing stars are still in charge.

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Key words: Iterative Closest Point (ICP) algorithm, stellar alignment, weighted stellar alignment astronomical wide plate registration.

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1. Introduction. The problem of star alignment appears in cases when two or more sets of stars obtained from different sources must be registered in the same database or astronomical catalogue. The contemporary photographic instruments produce high quality images and provide high fidelity measurements characterizing the image as part of the sky. Nevertheless, there are many photographic plates from the observatories' archives that suffer from the quality of the instruments from the past. The Institute of Astronomy at the Bulgarian Academy of Sciences holds a large archive of photographic plates—"Catalogue of Wide-Field Plate Archives" [9] enumerating over 2 million observations obtained since the end of the 19th century from observatories worldwide. This historical heritage is highly important for an astronomer who studies the evolution of the sky map through the years. In the course of the digitization process the photographic plates are scanned and then formatted in FITS format. Along with the image data, the FITS file also holds meta-information (FITS header) characterizing a variety of image attributes. The FITS images must be supplied with specific attributes to be approved for registration in a Virtual Observatory [9] and the minimum consists of the image centre, field of view and, in the case of wide-field plates, nonlinear distortion coefficients. The archived plates contain various kinds of defects, such as obscured surface areas due to physical damage, image noise-artefacts like the artificial *réseau* grid, lost information regarding some of the critical image parameters or completely "lost in space", or "orphan" plates.

This paper contributes to a project¹ aiming to automate the correction of the FITS image header where some of the critical characterizing parameters are not supplied or not accurate. Using the approach of the ICP method we are able to find the translation and rotation necessary to align the stars extracted from an image [2] with the stars queried from a standard star catalogue. That alignment consists of minimizing the *distance* between star pairs for a certain star-to-star mapping. In [7] the algorithm was tested using Euclidian distance between geometrical points. Herein we will demonstrate the significant improvement of the algorithm's performance and accuracy by extending the distance function to use assigned weights to the geometrical points.

For the application of the ICP method the coarse celestial coordinates of the image centre and the field of view must be initially given.

In the case of orphan plates where the corresponding metadata can be completely lost the ICP method is not suitable. Rather, algorithms like [5] must be used prior the presented method, in order to supply ICP with a more com-

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portable initial stage. In addition, the star extractor instrument [2] should also be adapted to provide an estimation of the stellar magnitudes.

The course of wide-field plate registration involves one or more of the following separate steps:

1. Scanning.
2. Filtering the regular noise and artefacts from the input image.
3. Extracting the stars' centres and magnitudes.
4. Searching the stars in a star catalogue to determine the image location on the sky [5].
5. Aligning the extracted stars with the set of stars from the catalogue [7].

Assuming the initial steps 1 to 4 have already been solved, we focus on resolving step 5.

2. Model and problem definition. We are considering the star maps from the image and from the catalogue as two point sets, A and B , in the Euclidian 2-dimensional space E^2 :

$$(1) \quad \begin{aligned} A, B \subset E^2, \quad |A| = n, \quad |B| = m, \quad n \text{ and } m \text{ positive integers} \\ W_A : A \rightarrow R, \quad W_B : B \rightarrow R \end{aligned}$$

Here the functions W_A and W_B represent the assigned weights of the points in A and B respectively; R , the set of real numbers.

We want to match the stars from the input image with the stars from the catalogue in order to estimate their mutual correspondence and align their coordinates more precisely, including the image centre. This alignment can be achieved by moving the stars from the input image uniformly to optimally adjust them to the corresponding stars from the catalogue. Thus we consider two functions μ and λ as defined below:

$$(2) \quad \mu : A \rightarrow B, \quad \mu(a) = b,$$

$$(3) \quad \lambda = \lambda(R, t) : A \rightarrow E^2, \quad \lambda(a|R, t) = Ra - t,$$

where $a \in A$, $b \in B$ are arbitrary elements of A and B , $t \in E^2$ is an arbitrary translation vector, and R is a $[2 \times 2]$ rotation matrix, i.e., $\det(R) = 1$, $RR^T = I$, I

the $[2 \times 2]$ unit matrix. We measure the adjustment between the two point sets using the sum S of squared distances between corresponding points:

$$(4) \quad S = \sum_a \|\lambda(a) - \mu(a)\|^2,$$

which we are interested in minimizing. Thus, our problem is to find such λ and μ which satisfy the criterion:

$$(5) \quad \min_{\lambda, \mu} S = \min_{\lambda, \mu} \sum_a \|\lambda(a) - \mu(a)\|^2$$

3. Weighted iterative closest point algorithm. Since the introduction of the basic concepts of the ICP algorithm by Chen et al. [4] and Besl et al. [3], multiple variants of the algorithm have been developed, a classification of which can be found in Rusinkiewicz et al. [8]. An analytical solution finding an optimal rotation matrix R and a translation vector t has been developed by Arun et al. [1]. The implementation of the algorithm is based on the work of the authors mentioned above with the optimization improvements by Kapoutsis et al. [6].

Basically the ICP algorithm is expressed by the following sequence:

1. Initialization: $\lambda : (R, t) = (I, 0)$
2. Matching: $\mu := \text{Arg min}_{\mu} \sum_a \|\lambda(a) - \mu(a)\|^2$
3. Transforming: $\lambda := \text{Arg min}_{\lambda} \sum_a \|\lambda(a) - \mu(a)\|^2$
4. Loop to step 2 until convergence or exceeding the limit of iterations.

Involving the point weights we change the way the points map to each other. When 2 points have similar assigned weights they will map with greater priority, but without losing the influence of the Euclidian distance. We want the distance between the points to increase by an amount reflecting the difference between their weights and exactly equal to the Euclidian distance when their weights are the same. This argument leads us to the following new criterion:

$$(6) \quad S_{new} = \sum_a [\max(W_A(a), W_B(\mu(a))) / \min(W_A(a), W_B(\mu(a)))]^2 \|\lambda(a) - \mu(a)\|^2$$

We evaluate the convergence at step 4 by the minimum of S_{new} .

The correction coefficient of the norm in (6) is simply the quotient of the largest weight and the smallest one that results a value equal or greater than 1. Only the matching μ is affected by this change, and the previous algorithmic steps mentioned above remain the same [1, 4, 6, 7, 8].

A crucial drawback of the ICP algorithm is the high computational complexity of the matching operator μ . Kapoutsis et al. [6] propose a morphological ICP algorithm solving the problem for the 3D space by using an array representing the 3D volume by a Voronoi tessellation method on the model points within that volume. Then the ICP algorithm is employed, where the distance calculations are substituted by simple array references. The complete algorithm is thoroughly described in [6]. In contrast to the μ operator, λ can be computed directly, as follows:

Let \bar{a} and \bar{b} be the centroids of the respective point sets A and B from (1):

$$(7) \quad \bar{a} = \frac{1}{n} \sum_{a \in A} a, \quad \bar{b} = \frac{1}{m} \sum_{b \in B} b$$

and a', b' are the respective central coordinates of $a \in A$ and $b \in B$:

$$(8) \quad a' = a - \bar{a}, \quad b' = b - \bar{b}$$

Hence,

$$(9) \quad \sum_a \|\lambda(a) - \mu(a)\|^2 = \sum_{a, b=\mu(a)} \|Ra - t - b\|^2 = \\ = \sum_{a, b=\mu(a)} \|(Ra' - b') + (R\bar{a} - \bar{b} - t)\|^2.$$

Using the triangular inequality we can prove that the minimum of (9) is reached at $(R\bar{a} - \bar{b} - t) = 0$, i.e., for t we have:

$$(10) \quad t = R\bar{a} - \bar{b}.$$

Thus, the minimization of (4) is reduced to a minimization of $\sum_a \|Ra' - b'\|^2$

instead of $\sum_a \|\lambda(a) - \mu(a)\|^2$. Hence,

$$\begin{aligned}
 (11) \quad & \sum_a \|Ra' - b'\|^2 = \sum_a (Ra' - b')^T (Ra' - b') = \\
 & = \sum_a (a'^T R^T Ra' + b'^T b' - a'^T R^T b' - b'^T Ra') = \\
 & = \sum_a \|a'\|^2 + \sum_a \|b'\|^2 - 2 \text{Trace}(RH) ,
 \end{aligned}$$

where

$$(12) \quad a'^T R^T b' = b'^T Ra' = \text{Trace}(R(a'b'^T))$$

and

$$(13) \quad H = \sum_a a'b'^T.$$

Hence the problem of minimizing (4) is reduced to finding:

$$(14) \quad \max_R \text{Trace}(RH).$$

Arun et al. [1] prove that (12) is maximized by

$$(15) \quad R = VU^T$$

where

$$(16) \quad H = U\Sigma V^T$$

is the singular value decomposition of H .

4. Experimental results. For the experiment we are using scanned a wide-field photographic plate with 1975 extracted stars up to stellar magnitude 14. All stars are rotated by 15° and translated by $t = (0.2, 0.2)$ in a normalized space $[1 \times 1]$ resulting in the set A (see Fig. 1a). In this way translational and rotational noise is simulated. Note that translational and rotational invariance is supported by the proposed method, but not scaling.

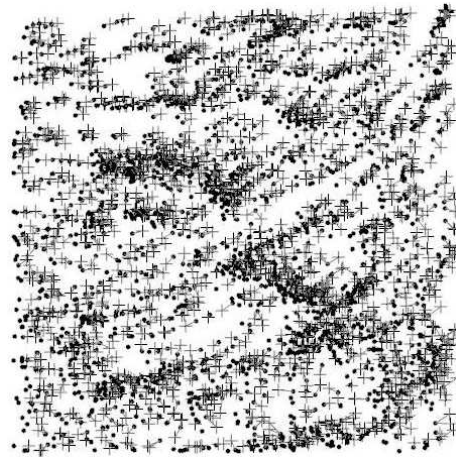
If B is the initial star map we want to align the set A over set B .

First we are running the simulation without taking into consideration the stars' magnitudes. The result of the execution is shown in Fig. 1, where convergence to the desired match is achieved in 51 iterations.

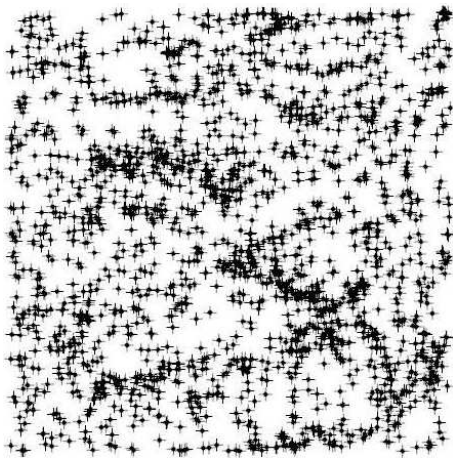
Set A is marked with black crosses and set B with black dots that help to better visibility of the initial and final stages of processing illustrated in Figs 1 and 2. Figures 1a, 1b, and 1c, represent the initial, intermediate and final ICP stage, while Fig. 1d shows the monotonically decreasing distance between the sets at each stage. The distance at the y -axis is calculated as a sum of all distances between pairs of points determined by μ_i (the minimal μ at iteration i).



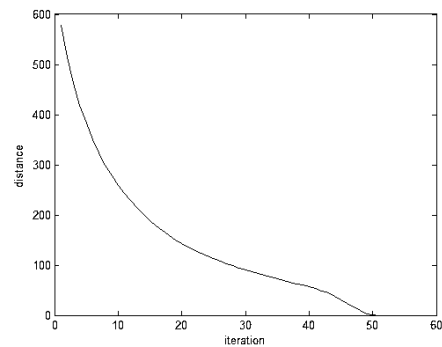
a. Initial stage



b. Intermediate stage



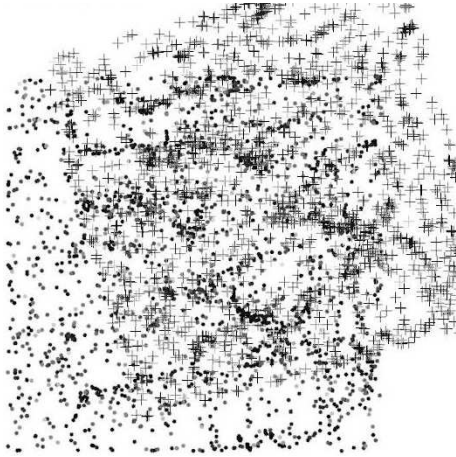
c. Final stage



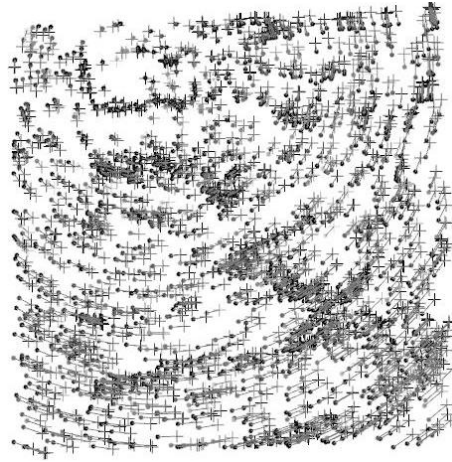
d. Set distance at each iteration

Fig. 1. ICP algorithm process aligning sky maps without using stellar magnitudes

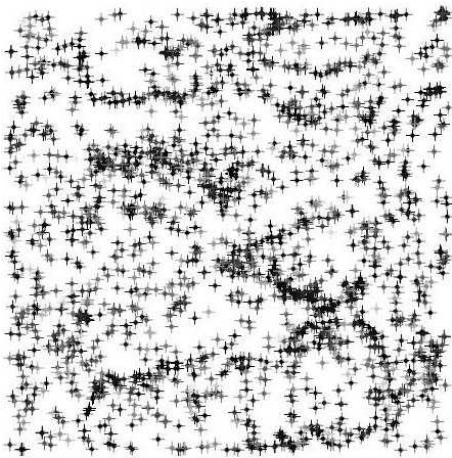
We run the same simulation by involving the star magnitudes as point weights. The result of the process is presented in Fig. 2. The notes from the previous experiment are still valid, except that the marks are in greyscale intensities proportional to the star magnitude, where the brighter stars are marked with darker colours. This time, convergence is achieved in 30 iterations, which is a remarkable improvement.



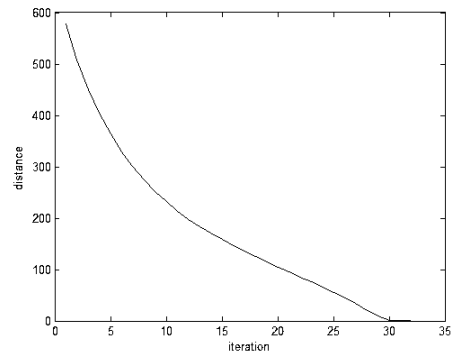
a. Initial stage



b. Intermediate stage



c. Final stage



d. Maps distance at each iteration

Fig. 2. ICP algorithm process aligning sky maps with given stellar magnitudes

5. Conclusion. We illustrated the Iterative Closest Point algorithm applied to the problem of aligning stars in astronomical images by involving the star magnitudes. Two experiments are presented which demonstrate a remarkable improvement in the algorithm performance when the point sets are supplied with weights. In the common case ICP converges to local minima, which is not necessarily the best coverage. But the provision of more information (e.g., point weights) for calculating point distances decreases this risk. Of course, some stars can change their magnitudes in time and the photographic plate can be much older than the catalogue used, but such situations are not very common as a rule. Besides, the star magnitudes are always considered a less informative attribute in comparison to the star coordinates in the catalogue. Thus, using the magnitudes as relative (normalized) weights, we can consider the mentioned phenomenon as an extra (not very high) noise in the proposed calculus.

As future work, the authors envisage research in extensions of ICP considering existing scaling factors between the two point sets.

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