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GLOBAL RESULTS FOR SOLUTION TO THE MASS-CRITICAL SCHRÖDINGER EQUATION WITH CONVOLUTION NONLINEARITY IN $\mathbb{R}^{N*}$

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We are concerned with a class of L^2-critical nonlinear Schrödinger equations in $\mathbb{R}^{1+n}$ with convolution nonlinearity of Hartree type. We aim to establish local and global existence and well-posedness of solutions in a small neighborhood of the origin in $L^2(\mathbb{R}^n)$. As a natural consequence of the global results, we prove the existence of scattering operator for small initial data.

1. Introduction. In this paper we consider the Cauchy problem for the defocussing mass-critical Schrödinger equation with convolution nonlinearity of the form

$$i\partial_t\psi + \triangle\psi = \left(\frac{1}{|x|^{n-\gamma}} * |\psi|^\alpha\right)\psi, \quad \psi(0,x) = \psi_0(x), \tag{1.1}$$

$(t,x) \in \mathbb{R}_+ \times \mathbb{R}^n$ for $n \geq 3$, where $\alpha > 0$ and $0 < \gamma < n$. Here $\psi = \psi(t,x)$ is a complex valued function, the initial data ψ_0 takes value in $L^2(\mathbb{R}^n)$ and $*$ denotes the convolution in space. Equation (1.1) can be written in terms of the wave function ψ and the potential V as the following system

$$i\partial_t\psi + \triangle\psi = V\psi, \quad (-\triangle)^{\frac{\gamma}{2}}V = C_n|\psi|^\alpha, \tag{1.2}$$

where the constant $C_n = C_n(\gamma)$ in the second equation can be calculated explicitly (see Chapter V in Stein [5]). Due to the appearance of convolution operator, equation (1.1) is known as the Schrödinger equation with nonlocal nonlinearity (or Hartree equation). As a special case of (1.1), the Schrödinger equation of Hartree type in $\mathbb{R}^3$, say $\alpha = 2$, with the Coulomb convolution kernel $|x|^{-1}$ is derived from the Maxwell-Schrödinger system with zero magnetic field.

In this paper we study the local and global existence, well-posedness and scattering of solutions to (1.1) with small initial data. The scaling argument, i.e. the scaling symmetry $\psi_\lambda(t,x) = \lambda^{\frac{n}{2}}\psi(\lambda^2 t, \lambda x)$, for $\lambda > 0$ gives the value of the mass-critical power $\alpha_c = \dfrac{2(2+\gamma)}{n}$. It is obvious that the scalling trasformation preserves the L^2-norm and leaves equation (1.1) unperturbed. For dimension $n = 3$ and $\gamma = 2$ the system

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(1.2) becomes the mass-critical Schrödinger-Poisson system, which has been extensively studied in [8, 9].

It is essential in our study that, in general, equation (1.1) does not possess Hamiltonian structure, i.e. it is not a Hamiltonian completely integrable dynamical system. We prove that except for the value $\gamma = n-2$, equation (1.1) has only one conserved quantity – the mass, while its solution does not satisfy the energy conservation law. This fact prevents one from studying (1.1) for general powers α of the nonlinear term.

In general, the scaling symmetry relates (1.1) to a wide class of equations, referred to as the mass-critical (L^2-critical or pseudoconformal) nonlinear Schrödinger equations. The name comes from the fact that the above transform leaves both the equation and the mass (the L^2-norm) invariant. Mass is one of the basic structures used in physics and is defined by $M(\psi(t)) = \int_{\mathbb{R}^n} |\psi(t,x)|^2 dx$. For (1.1), we prove (see Theorem 1.1 bellow) that the mass is a conserved quantity, i.e. $M(\psi(t)) = M(\psi_0)$. As in the papers [2, 8, 9], our results make use of mixed spaces of the type $L^q([0,T], L^r(\mathbb{R}^n))$ for admissible q and r. Thus, we use the following

Definition 1.1. *We say that the pair (q,r) of exponents is Schrödinger–admissible if q and r satisfy $\dfrac{2}{q} = n\left(\dfrac{1}{2} - \dfrac{1}{r}\right)$, for $2 \le q \le \infty$.*

Following the strategy developed for the semilinear Schrödinger equation (see for instance [1, 2, 7]), we aim to establish the local well-posedness theory for (1.1) and to construct global solutions for sufficiently small L^2-initial data. More precisely, we use the following

Definition 1.2. *A function $\psi : [0,T^*) \times \mathbb{R}^n \to \mathbb{C}$, is a $L^2(\mathbb{R}^n)$ solution to (1.1) if $\psi \in C^0([0,T], L^2(\mathbb{R}^n)) \cap L^{q_0}([0,T], L^{r_0}(\mathbb{R}^n))$ for $(q_0, r_0) = \left(\dfrac{2(n+\gamma+2)}{n}, \dfrac{2(n+\gamma+2)}{n+\gamma}\right)$ and $0 < T < T^*$. Moreover, we have the following Duhamel's integral representation*

$$\psi(t) = U(t)\psi_0 - i\int_0^t U(t-s)\left(\frac{1}{|x|^{n-\gamma}} * |\psi(s)|^{\alpha_c}\right)\psi(s)ds, \quad t \in [0,T]. \tag{1.3}$$

Here $U(t) = e^{it\Delta}$ is the free Schrödinger evolution group defined via the Fourier transform by $U(t)f = \mathcal{F}^{-1} e^{-it|\xi|^2}\mathcal{F}f$. We say that ψ is a global solution to (1.1) if $T^* = \infty$.

The first main result of the present paper is the following

Theorem 1.1. *Let $0 < \gamma < \sqrt{n^2+1} - 1$ for $n = \{3,4\}$ and $\dfrac{n-4}{2} \le \gamma < \sqrt{n^2+1} - 1$ for $n > 4$. Then, for every initial data $\psi_0 \in L^2(\mathbb{R}^n)$, there exists a unique maximal solution $\psi \in C^0([0,T^*), L^2(\mathbb{R}^n)) \cap L^{q_0}([0,T^*), L^{r_0}(\mathbb{R}^n))$ of (1.1). Furthermore:*

(i) *$\psi \in L^q([0,T], L^r(\mathbb{R}^n))$, for $0 < T < T^*$ and every admissible pair (q,r);*

(ii) *the mass is conserved, i.e. $M(\psi(t)) = M(\psi_0)$ for $t \in [0,T^*)$;*

(iii) *there exists a constant $\varepsilon > 0$ sufficiently small, such that if $\|\psi_0\|_{L^2(\mathbb{R}^n)} < \varepsilon$, then $T^* = \infty$ and $\psi \in L^q(\mathbb{R}_+, L^r(\mathbb{R}^n))$ for every admissible pair (q,r);*

(iv) *if $T^* < \infty$, then $\|\psi\|_{L^q([0,T^*),L^r(\mathbb{R}^n))} = \infty$ for every $r > r_0$;*

(v) *ψ depends continuously on the initial data $\psi_0 \in L^2(\mathbb{R}^n)$ in the space $\psi \in C^0([0,T^*), L^2(\mathbb{R}^n)) \cap L^{q_0}([0,T^*), L^{r_0}(\mathbb{R}^n))$.*

With our second result we develop a scattering theory for (1.1) in $L^2(\mathbb{R}^n)$ with small initial data.

Theorem 1.2. *Let $\varepsilon > 0$ be sufficiently small and consider the ball $B_\varepsilon = \{\psi \in L^2(\mathbb{R}^n); \|\psi\|_{L^2} < \varepsilon\}$. Let $\psi \in C^0([0,T^*), L^2(\mathbb{R}^n)) \cap L^{q_0}([0,T^*), L^{r_0}(\mathbb{R}^n))$ be the unique maximal solution of* (1.1)*, given by part* **(iii)** *of Theorem* 1.1*. Then, we have:*

(i) *for any $\psi_\pm \in B_\varepsilon$, there exists unique $\psi_0 \in B_\varepsilon$, such that*

$$\lim_{t\to\pm\infty} \|U(-t)\psi(t) - \psi_\pm\|_{L^2} = \lim_{t\to\pm\infty} \|\psi(t) - U(t)\psi_\pm\|_{L^2} = 0; \tag{1.4}$$

(ii) *for any $\psi_0 \in B_\varepsilon$, there exists unique $\psi_\pm \in B_\varepsilon$, such that* (1.4) *is satisfied;*

(iii) *the wave operators $\Omega_\pm : \psi_\pm \mapsto \phi_0$ and the scattering operator $S = \Omega_+^{-1} \circ \Omega_-$ are homeomorphisms from B_ε onto itself and isometric in the L^2 norm.*

2. Proof of Theorems 1.1 and 1.2. We point out that the mixed space $L^{q_0}L^{r_0}$ for the admissible pair (q_0, r_0) with $q_0 = \dfrac{2(n+\gamma+2)}{n}$ and $r_0 = \dfrac{2(n+\gamma+2)}{n+\gamma}$ plays a fundamental role. This is better understood if we recall the dispersive properties of the Schrödinger operator [4, 10].

Lemma 2.1. *Let (q,r) be an admissible pair. Then, for every $\varphi \in L^2(\mathbb{R}^n)$ the following estimate holds*

$$\|U(t)\varphi\|_{L^q(\mathbb{R}, L^r(\mathbb{R}^n))} \le C_0 \|\varphi\|_{L^2(\mathbb{R}^n)}. \tag{2.1}$$

Moreover, for every admissible pair (θ,ρ) and $f \in L^{\theta'}([0,T], L^{\rho'}(\mathbb{R}^n))$ we have

$$\left\| \int_0^t U(t-s) f(s) ds \right\|_{L^q([0,T], L^r(\mathbb{R}^n))} \le C \|f\|_{L^{\theta'}([0,T], L^{\rho'}(\mathbb{R}^n))}, \quad 0 < T \le \infty. \tag{2.2}$$

Here the constants $C_0, C > 0$ and depend only on the spatial exponents r and ρ.

The arguments of Theorem 1.1 rely primarily on the estimate (2.2), applied to the integral representation (1.3), Hölder inequality and the following Lemma

Lemma 2.2 (Hardy-Littlewood-Sobolev Inequality [6])**.** *For $0 < \gamma < n$ consider the Riesz potential*

$$I_\gamma(f)(x) = \int_{\mathbb{R}^n} \frac{f(y)}{|x-y|^{n-\gamma}} dy. \tag{2.3}$$

Then, for any $1 < r < \theta < \infty$ and $f \in L^r(\mathbb{R}^n)$, we have

$$\|I_\gamma(f)\|_{L^\theta} \le C \|f\|_{L^r}, \quad \frac{1}{\theta} = \frac{1}{r} - \frac{\gamma}{n}. \tag{2.4}$$

Let us denote by $N(\psi)$ the nonlinear term

$$N(\psi) = (|x|^{-(n-\gamma)} * |\psi|^{\alpha_c})\psi \tag{2.5}$$

in the Schrödinger equation (1.1).

Lemma 2.3. *Let $0 < T \le \infty$ and let (q,r) be a Schrödinger-admissible pair. Then, there exists a constant $C > 0$, independent of T, such that for every*

$\psi, \chi \in L^{q_0}([0,T], L^{r_0}(\mathbb{R}^n))$ *we have*

$$\left\| \int_0^t U(t-s)[N(\psi)(s) - N(\chi)(s)]ds \right\|_{L^q([0,T],L^r)} \tag{2.6}$$
$$\leq C \left(\|\psi\|^{\alpha_c}_{L^{q_0}([0,T],L^{r_0})} + \|\chi\|^{\alpha_c}_{L^{q_0}([0,T],L^{r_0})} \right) \|\psi - \chi\|_{L^{q_0}([0,T],L^{r_0})}.$$

Proof. To prove the Lemma, it is sufficient to show that

$$\|N(\psi) - N(\chi)\|_{L^{q_0'}([0,T],L^{r_0'})} \tag{2.7}$$
$$\leq C \left(\|\psi\|^{\alpha_c}_{L^{q_0}([0,T],L^{r_0})} + \|\chi\|^{\alpha_c}_{L^{q_0}([0,T],L^{r_0})} \right) \|\psi - \chi\|_{L^{q_0}([0,T],L^{r_0})}.$$

Indeed, we can write

$$\begin{aligned} N(\psi) - N(\chi) &= I_\gamma(|\psi|^{\alpha_c})\psi - I_\gamma(|\chi|^{\alpha_c})\chi \\ &= I_\gamma(|\psi|^{\alpha_c})(\psi - \chi) + I_\gamma(|\psi|^{\alpha_c} - |\chi|^{\alpha_c})\chi, \end{aligned} \tag{2.8}$$

where the operator I_γ is defined by (2.3). Then, using Hölder inequality we estimate

$$\begin{aligned} \|N(\psi) - N(\chi)\|_{L^{r_0'}} &\leq C\|I_\gamma(|\psi|^{\alpha_c})\|_{L^{p_1}}\|\psi - \chi\|_{L^{r_0}} \\ &+\|I_\gamma(|\psi|^{\alpha_c} - |\chi|^{\alpha_c})\|_{L^{p_1}}\|\chi\|_{L^{r_0}}, \end{aligned} \tag{2.9}$$

with $p_1 = \dfrac{n+\gamma+2}{2}$. Now, using Lemma 2.2 we have

$$\|I_\gamma(|\psi|^{\alpha_c} - |\chi|^{\alpha_c})\|_{L^{p_1}} \leq C\||\psi|^{\alpha_c} - |\chi|^{\alpha_c}\|_{L^{p_2}}, \tag{2.10}$$

where $p_2 = \dfrac{n(n+\gamma+2)}{(n+\gamma)(\gamma+2)}$. The condition $1 < p_2 < p_1$ is equivalent to the following algebraic inequality $\gamma^2 + 2\gamma - n^2 < 0$, which gives the upper bound $\gamma < \sqrt{n^2+1} - 1$. On the other hand, the lower bound $\dfrac{n-4}{2} \leq \gamma$ ensures that $\alpha_c \geq 1$ and, thus, we can estimate

$$||\psi|^{\alpha_c} - |\chi|^{\alpha_c}| \leq C \max\left\{|\psi|^{\alpha_c - 1}, |\chi|^{\alpha_c - 1}\right\} |\psi - \chi|. \tag{2.11}$$

Combining the above estimates, we obtain the following inequality

$$\|N(\psi) - N(\chi)\|_{L^{r_0'}} \leq C\left(\|\psi\|^{\alpha_c}_{L^{r_0}} + \|\chi\|^{\alpha_c}_{L^{r_0}}\right)\|\psi - \chi\|_{L^{r_0}}. \tag{2.12}$$

Applying Hölder inequality in time to (2.12), we obtain (2.7). Finally, it is sufficient to use the estimate (2.2) from Lemma 2.1 and the proof is completed. □

Now we prove Theorem 1.1. The proof of **(ii)** is rather technical. We prove the existence of solution to (1.1) by a fix point argument. Let $\psi_0 \in L^2(\mathbb{R}^3)$ be an arbitrary initial data, $R > 0$ and T be a fixed positive time and consider the ball

$$B_R(T) = \{\psi \in C^0([0,T], L^2)) \cap L^{q_0}([0,T], L^{r_0}); \|\psi\|_{L^{q_0}([0,T],L^{r_0})} < R\}, \tag{2.13}$$

endowed with the metric $d(\psi, \chi) = \|\psi - \chi\|_{L^{q_0}([0,T],L^{r_0})}$. It is clear that $B_R(T)$ is a complete metric space.

Consider the map $\Phi[\psi]$ defined by the right-hand side of the Duhamel's integral representation (1.3). Then, for $\psi \in B_R(T)$, using (2.1), (2.2) and (2.6), we can write

$$\|\Phi[\psi]\|_{L^{q_0}([0,T],L^{r_0})} \leq \|U(\cdot)\psi_0\|_{L^{q_0}([0,T],L^{r_0})} + C_1\|\psi\|^{\alpha_c+1}_{L^{q_0}([0,T],L^{r_0})}. \tag{2.14}$$

From the fact that $\|U(\cdot)\psi_0\|_{L^{q_0}([0,T],L^{r_0})} \to 0$ as $T \to 0$, we can choose T in such a way that $\|U(\cdot)\psi_0\|_{L^{q_0}([0,T],L^{r_0})} \leq \frac{R}{2}$. Now, we can take $R \leq (2C_1)^{-\frac{1}{\alpha_c}}$, which implies $\Phi[\psi] \in B_R(T)$. On the other hand, from the relation

$$(2.15) \qquad (\Phi[\psi] - \Phi[\chi])(t) = -i \int_0^t U(t-s)\left(N(\psi) - N(\chi)\right)(s)ds,$$

and (2.6) it follows that

$$(2.16) \qquad \|\Phi[\psi] - \Phi[\chi]\|_{L^{q_0}([0,T],L^{r_0}} \leq 2R^{\alpha_c}C_2\|\psi - \chi\|_{L^{q_0}([0,T],L^{r_0})},$$

for every $\psi, \chi \in B_R(T)$. If we choose $R < \min\left\{(2C_1)^{-\frac{1}{\alpha_c}}, (2C_2)^{-\frac{1}{\alpha_c}}\right\}$, then we finally obtain that the map $\Phi[\psi]$ is a strict contraction on the ball $B_R(T)$. Thus, $\Phi[\psi]$ has a fixed point, which is the unique solution of (1.1) in $C^0([0,T],L^2)) \cap L^{q_0}([0,T],L^{r_0})$. To prove (iii), let us denote by T^* the supremum of all $T > 0$ for which such a solution exists. Observe now that if ψ_0 is sufficiently small, then (2.14) holds regardless of the value of T. Thus, we may accomplish the fixed point procedure in the ball $B_R(\infty)$, providing $T^* = \infty$.

Further, we claim that if $T^* < \infty$, then $\|\psi\|_{L^q([0,T^*),L^r)} = \infty$ for every $r > r_0$. Indeed, on the contrary, let us assume that $T^* < \infty$ and $\|\psi\|_{L^{q_0}([0,T^*),L^{r_0})} < \infty$. For any $t \in [0,T^*)$ let $\tau \in [0, T^* - t)$. Using Duhamel's formula (1.3), we can write

$$(2.17) \qquad \psi(t+\tau) = U(\tau)\psi(t) - i\int_t^{t+\tau} U(t+\tau-s)N(\psi)(s)ds.$$

From (2.17) and the estimate (2.6) in Lemma 2.3 we obtain

$$(2.18) \qquad \|U(\cdot)\psi(t)\|_{L^{q_0}([0,T^*-t),L^{r_0})} \leq C\Big(\|\psi(t)\|_{L^{q_0}([t,T^*),L^{r_0})} + \|\psi\|^{\alpha_c+1}_{L^{q_0}([t,T^*),L^{r_0})}\Big).$$

Observing now that $\|U(\cdot)\psi\|_{L^{q_0}([0,T],L^{r_0})} \to 0$ as $T \to 0$ and taking t close enough to T^*, it follows that $\|U(\cdot)\psi(t)\|_{L^{q_0}([0,T^*-t),L^{r_0})}$ can be made small enough and the assumptions in **(iii)** are fulfilled. Therefore, ψ can be extended beyond T^*, which contradicts the maximality. Let (q,r) be a Schrödinger-admissible pair with $r \geq r_0$. Then, from Hölder inequality, for $T < T^*$, we can write

$$(2.19) \qquad \|\psi\|_{L^{q_0}([0,T],L^{r_0})} \leq \|\psi\|^{1-\theta}_{L^\infty([0,T],L^2)}\|\psi\|^{\theta}_{L^q([0,T],L^r)}, \quad \theta \in (0,1).$$

Letting $T \to T^*$, we obtain that $\|\psi\|_{L^q([0,T],L^r)} = \infty$, which proves the statement (iv).

To prove **(v)**, consider a sequence $\psi_0^k \in L^2(\mathbb{R}^n)$, such that $\psi_0^k \to \psi_0 \in L^2(\mathbb{R}^n)$ as $k \to \infty$. Thus, for k large enough, $\|U(\cdot)\psi_0^k\|_{L^{q_0}([0,T],L^{r_0})} < \varepsilon$. We can use the Duhamel's formula (1.3) to construct a sequence of solutions $\psi^k \in L^{q_0}([0,T],L^{r_0}(\mathbb{R}^n))$ to (1.1) with initial data ψ_0^k. Applying the proof of **(iii)**, we obtain that $\psi^k \to \psi$ in $C^0([0,T],L^2(\mathbb{R}^n)) \cap L^{q_0}([0,T],L^{r_0}(\mathbb{R}^n))$ as $k \to \infty$, and in fact in every $L^q([0,T],L^r(\mathbb{R}^n)))$ for (q,r) be an admissible pair. Thus, the proof of the Theorem is completed.

We begin the proof of Theorem 1.2 by recalling some definitions from the scattering theory for Schrödinger equation. Let $v(t) = U(t)\psi_\pm$ be a solution to the free Schrödinger equation with initial data $\psi_\pm \in L^2(\mathbb{R}^n)$ (called the asymptotic state). If there exists a solution of (1.1), which behaves asymptotically as v when $t \to \pm\infty$, then the map $\Omega_\pm : L^2(\mathbb{R}^n) \mapsto L^2(\mathbb{R}^n)$ is called the wave operator for positive or negative times. In other words, a global strong L^2-solution ψ to the nonlinear equation (1.1) with an initial data ψ_0 scatters in $L^2(\mathbb{R}^n)$ to a solution $v(t) = U(t)\psi_\pm$ if we have

$$\lim_{t\to\pm\infty} \|\psi(t) - U(t)\psi_\pm\|_{L^2} = \lim_{t\to\pm\infty} \|U(-t)\psi(t) - \psi_\pm\|_{L^2} = 0. \tag{2.20}$$

The arguments for proving the existence of the wave operator are standard and follows the exposition in [3]. We prove only the (+) case since the (−) case can be proved similarly. Let $\psi_0 \in L^2(\mathbb{R}^n)$ with $\|U(\cdot)\psi_0\|_{L^{q_0}([0,T],L^{r_0})} < \varepsilon$ and $\psi \in B_{2\varepsilon}$. Then, for $t > t_0$, using Duhamel's integral formula (1.3), we have

$$U(-t)\psi(t) = U(-t_0)\psi(t_0) - i\int_{t_0}^{t} U(-s)N(\psi)(s)ds. \tag{2.21}$$

Therefore, the estimate (2.6) yields

$$\|U(-t)\psi(t) - U(-t_0)\psi(t_0)\|_{L^2} \le C\,\|\psi\|^{\alpha_c+1}_{L^{q_0}([t_0,t],L^{r_0})} \to 0, \tag{2.22}$$

as $t_0 \to \infty$. Since $U(-t_0)\psi(t_0) \in L^2$, the proof of part (ii) is completed.

To prove (i), assume that $\psi_+ \in L^2(\mathbb{R}^n)$, $\psi \in B_{2\varepsilon}$ and consider the map

$$\Phi_+[\psi](t) = U(t)\psi_+ + i\int_t^{+\infty} U(t-s)N(\psi)(s)ds, \quad t > T, \tag{2.23}$$

for some $T = T(\psi_+)$ large enough. Then, using the same arguments as in the proof of part **(iii)** of Theorem 1.1, we find that Φ_+ is a contraction on $B_{2\varepsilon}$ and has a unique fixed point if $\|\psi_+\|_{L^2} < \varepsilon$. Using the global well-posedness result established in Theorem 1.1 for small data, one can then extend this solution uniquely for any $t \in [0,\infty)$, and in particular ψ takes some value $\psi_0 = \psi(0) \in L^2$ at time $t = 0$. This ensures the existence of the wave operator Ω_+, defined by

$$\Omega_+\psi_+ = \psi_0 = \psi_+ + i\int_0^{+\infty} U(-s)N(\psi)(s)ds. \tag{2.24}$$

To prove (iii), we use the following observations. Since the wave operators $\Omega_\pm$ are isometric in the space $B_{2\varepsilon}$, it is clear that the scattering operator $S : \phi_- \mapsto \phi_+$ is well defined as a map from $B_{2\varepsilon}$ onto itself and is isometric in the L^2 norm, i.e. $\|S\psi\|_{L^2} = \|\psi\|_{L^2}$. This completes the proof of the Theorem 1.2.

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ГЛОБАЛНИ РЕЗУЛТАТИ ЗА РЕШЕНИЕТО НА УРАВНЕНИЕТО НА ШРЬОДИНГЕР С КРИТИЧНА МАСА И НЕЛИНЕЙНОСТ ОТ КОНВОЛЮЦИОНЕН ТИП В $\mathbb{R}^N$

Георги Венков, Христо Генев

Разглеждаме един клас от L^2-критични нелинейни уравнения на Шрьодингер в $\mathbb{R}^{1+n}$ с конволюционна нелинейност от тип Хартри. Целта ни е да установим локалното и глобално съществуване на решенията, както и коректност на задачата на Коши в достатъчно малка околност на нулата в пространството $L^2(\mathbb{R}^n)$. Като естествено следствие на глобалните резултати ние доказваме съществуване на оператор на разсейване за малки начални условия.