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TOWARDS NEW TEACHING IN MATHEMATICS HOW TO REALIZE INQUIRY BASED AND PROBLEM ORIENTED MATHEMATICS LESSONS?

Dagmar Raab

Mathematics is exciting and brings fun. Is it possible to convince pupils that this could become reality?

Tasks are the most important "tools" available to mathematics teachers for their lesson planning. It is about the teachers initiating, accompanying and analysing the way pupils work on tasks. Teachers shouldn't have pupils simply produce answers, but should make them get involved with the respective questions.

As teacher help pupils to:

- make it possible to actively and productively work with problems;
- initiate problems and tasks;
- vary problems;
- recognize patterns;
- prepare strategies for solutions;
- find different paths toward solutions and then take them;
- link everyday knowledge and mathematical knowledge with one another in a meaningful way.

Selected and already tested examples will demonstrate how teachers and pupils can find a good way for new experiances in mathematical teaching and learning.

Where we stand. Mathematics is exciting and brings fun. Is it possible to convince pupils that this could become reality?

Actually the perception of mathematics by students and the public mostly is far away from fun and fascination. Let's have a look to an American survey, written by Alan SCHOENFELD in the year 1992. He describes typical student beliefs about the nature of mathematics:

- Mathematics problems have one and only one right answer.
- There is only one correct way to solve any mathematics problem – usually the rule the teacher has most recently demonstrated to the class.
- Ordinary students cannot expect to understand mathematics; they expect simply to memorize it, and apply what they have learned mechanically and without understanding.
- Mathematics is a solitary activity, done by individuals in isolation.
- Students who have understood the mathematics they have studied, will be able to solve any assigned problem in five minutes or less.

- The mathematics learned in school has little or nothing to do with the real world.
- Formal proof is irrelevant to processes of discovery or invention.

Why do students display such a "skewed" perception of mathematics? An important cause is certainly to be seen in the type of instruction they get. Heinz Klippert, a proponent of classroom reform, puts it in a nutshell:

"Knowledge is served up, ingurgitated, forgotten."

To change this traditional way of classroom instruction in Germany, the pilot study "Increasing Efficiency in Mathematics and Science Education" (abbreviated as SINUS) was launched in 1998, lasted until 2003, and was followed by the large scaled SINUS-Transfer project. Details about this project can be found at the SINUS server (www.sinus-transfer.eu) in German and English language. See also the literature at the end of this article.

The success of the ideas and realization of SINUS and SINUS-Transfer was recognized in numerous ways. 2004 could be read in the German weekly newspaper DIE ZEIT (50/2004):

"True-to-life problems rather than schematic arithmetic, individual learning rather than swatting up on formulas at one pace for all. Such a reform in teaching mathematics is what SINUS stands for. [...] SINUS has shown how to change instruction successfully."

Nevertheless, despite all progress, made SINUS is still far from reaching every single classroom.

Inquiry based and problem oriented access to mathematics. The role of teachers should be to "encourage students to see mathematics as a science, not as a canon, and to recognize that mathematics is really about patterns and not merely about numbers". (American National Research Council, 1989).

The most powerful "tools" available to mathematics teachers for their lesson planning are tasks. It is about the teachers initiating, accompanying and analysing the way pupils work on tasks. Teachers shouldn't have pupils simply produce answers, but should make them get involved with the respective questions.

The teacher should less work as an instructor than as a moderator. He or she can help students to make it possible to actively and productively work with problems. Students should get able to:

- initiate problems and tasks;
- vary problems;
- recognize patterns;
- prepare strategies for solutions.
- find different paths toward solutions and then take them.
- link everyday knowledge and mathematical knowledge with one another in a meaningful way.

Selected examples of rich learning tasks. The task in itself is not an important criterion for assessing the quality of teaching. More important is its integration into the whole teaching / learning environment. Variable approaches to solving tasks, discussions and interpretations may also turn "classical" tasks into interesting and useful elements of math and science lessons. There is no need to eliminate them as out-dated in general.

Let's start with a task, given in the German TV show "The PISA test".

You have two cats. The big tomcat can get by on one can of cat food two days, the small kitten three days. If you buy 10 cans of cat food how many days can you feed your two cats?

Make your choice from a) 8 1/3, *b)* 12, *c)* 12 1/2, *d)* 14

The solution, given by the moderator, was astonishing: In his opinion an equation was needed: $\frac{1}{2}x + \frac{1}{3}x = 10$.

First he declared in detail how to find out the fractions $\frac{1}{2}$ and $\frac{1}{3}$ and then he used some rules everybody should have learned at school to solve this kind of equations. Finally, he could write down $x = 12$. Millions of people were reminded of their own math lessons and that many of them had hated to solve such "difficult" tasks at school.

But wouldn't it be much easier to solve the simple problem without using fractions and equations? Think about possibilities and, if possible, ask students without knowledge of fractions.

Open ended tasks. Hans Schupp characterizes the situation in mathematics lessons as follows:

In real life, questions are asked by those who do not know something and receive a response from those who know it. The reverse case applies at school.

Tasks specifically promoting independent approaches to learning are open-ended. This gives students a degree of independence or autonomy in working on them. Open-ended tasks do not have one single correct solution. They outline a situation that we can discuss in mathematical terms.

Below you find some possibilities how to generate open-ended learning situations, mostly to realize very easy, also if using "traditional" tasks:

- give incomplete information;
- give more than needed information;
- vary some information;
- invert a task (start from the result);
- enable (and allow!) students to find different solutions;
- let students argue and discuss, accept also wrong tracks.

Pictures can be a good starting point. They offer students attractive access to a specific subject.

An example given by the German National Educational Standards:

It's time for opening a barrel again!

No other data or comments are given.

Students are asked to formulate questions about this situation, e.g.

- What's the volume of this barrel?
- How many bottles of whiskey (beer, ...) could be filled in from this barrel?
- What's the weight of the empty barrel?
- What's the weight of the whiskey-filled barrel?

Of course, students are not expected to be able to find out the exact volume or weight of this barrel. However, mathematics lessons do (and should) confront students with general ideas about sizes and dimensions. That means that they should be able to estimate the average diameter of the barrel or the height of a man on this picture.

It's easy to find tasks by your own similar to the given one above. Just have a walk with your students through the town with a "mathematical view" and you will discover things like seen at the right and below:

But it doesn't have to be pictures. Ordinary, everyday selections of data (train schedules, restaurant menus, statistics, etc.) can be just as useful in posing and studying mathematical issues.

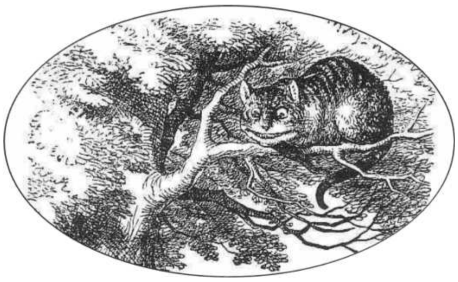
Detailed datasheets support the correct handling of an electrical or electronical device. But in every case you have first to find out the relevant information written down in the datasheet. In contrast to reality most textbooks provide exactly those information you need for solution of a task. How can you change the situation?

Take a datasheet (or let your students bring one with them), find out meaningful questions (or let your students find out). Working on solutions the students learn to read carefully, to select and use relevant information, but also to better understand the technique of the device.

Think about good questions relating to a washing machine, described by the following datasheet (excerpt):

TECHNICAL SPECIFICATIONS		
DIMENSIONS	Height	85 cm (33.5")
	Width	60 cm (23.6")
	Depth	57 cm (22.4")
POWER SUPPLY VOLTAGE		220-240 V/50 Hz
TOTAL POWER ABSORBED		2750 W (13A)
WATER PRESSURE	Minimum (hot)	5 psi (3.5 N/cm²)
	Minimum (cold)	7 psi (4.8 N/cm²)
	Maximum	110 psi (76 N/cm²)
MAXIMUM RECOMMENDED LOAD	Cotton, linen	4.5 kg (10 lb)
	Synthetics, delicate fabrics	2 kg (4.5 lb)
	Wool	1 kg (2.2 lb)
SPIN SPEED	Maximum	800 rpm (FL850-FL850 AL) 1000 rpm (FL1085-FL1085 AL)

But what about the "pure" mathematics? Not in every case it is adequate to use those "realistic" problems. Of course students have to gain a fundament of mathematical knowledge and core strategies for problem solving.



Experiments with triangle and circle. *"Would you tell me, please, which way I ought to go from here?"*

"That depends a good deal on where you want to go," said the Cat.

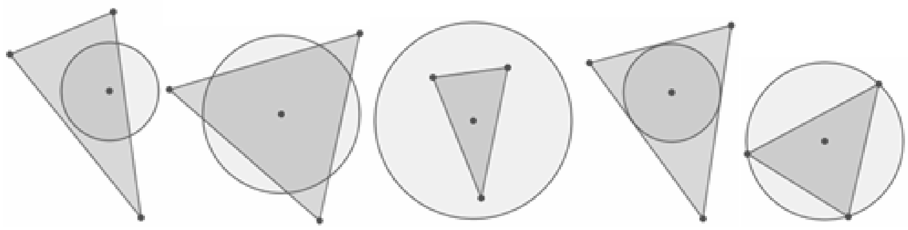
(Dialogue between Alice and the Cheshire-Cat in "Alice's Adventures in Wonderland" by L. Carrol)

Two simple figures, a triangle and a circle, are the starting point for mathematical explorations.

Let's send the students on a mathematical expedition. The modern device of dynamic geometry will enrich this excursion in many instances. Our aim is to transmit some sense of wonder about the mathematical diversity that can be developed from the basic forms of triangle and circle.

The task invites participants to select interesting aspects and to study them in-depth or to develop them freely into other directions. The subject is also well-suited to raise awareness about mathematical methods. In this sense, new knowledge can be derived from familiar things; the solution strategies can be applied to many more advanced topics.

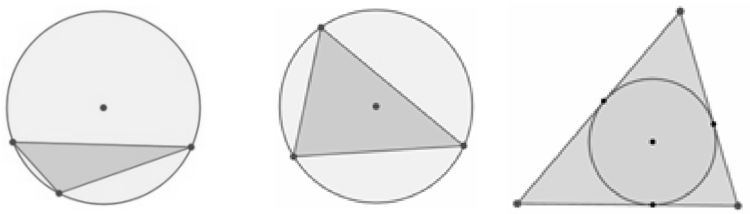
A dynamic worksheet makes these two figures come alive:



Triangle and circle can be varied in manifold ways. At first the students study various possibilities of the relative positions. How many common points are possible between the two figures? Do the special cases of isosceles, equilateral and rectangular triangles offer interesting additional aspects?

Already at this stage many starting points for further investigations emerged. Many possibilities, from a more exact analysis of the intersecting and/or tangential points to surface analysis and proportions, seem to exist.

Special cases: circumcircle and inscribed circle. The two instances of "close-fit" superimposition have emerged as special cases. Which conditions must triangle and circle fulfil to ensure that the triangle with its vertices fits exactly on the circle perimeter or that the circle touches the inside of every triangle side in one point?



In this way an intuitive access to the subject “circumcircle and inscribed circle of a triangle” is possible.

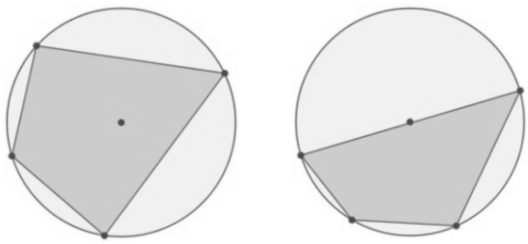
At the same time various facets up to the Thales Circle are revealed, which can be researched step by step.

A conscious decision has been made at this point to pursue the special cases of circumcircles and inscribed circles. Experimentation should not become an end to itself. A topic becomes truly interesting only when you can successfully apply the acquired knowledge.

After having studied the properties of the circumcircle and inscribed circle of an triangle (possibly also using dynamic worksheets) we can go one step forward:

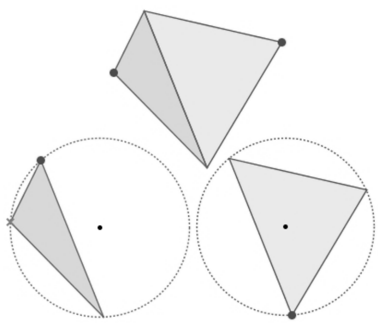
Do all quadrangles have a circumcircle?

Again the students start with a brief research phase, in which they can orientate themselves, gather first impressions, express assumptions and discuss.



A quadrangle with circumcircle serves as starting point. The vertices can be moved freely on the circle perimeter, and the radius of the circumcircle is variable too. Though special cases such as squares, rectangles, kites and trapeziums will quickly emerge, there is no coherent system, and neither can special angles or parallelisms be observed. The position of the circle midpoint seems to be arbitrary too.

In everyday class situations, tasks that require the broaching of new terrain, and accordingly more complex analyses, present considerable obstacles for students. At the outset there is bewilderment: “I don’t know where to start. We haven’t done any of that yet . . . ”



At this point it is worthwhile to stop and take a good look at the treasures that have already been unveiled.

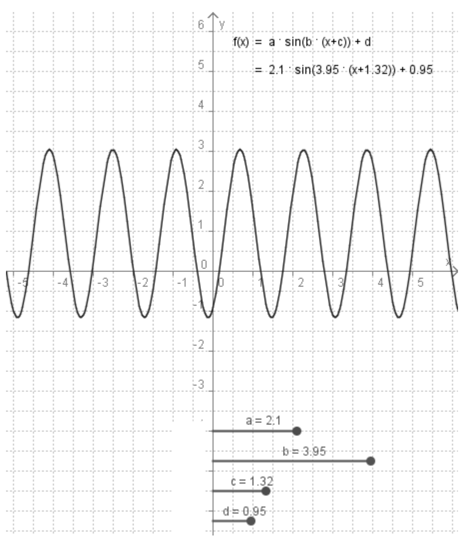
How does the new situation differ from the old one? In fact, only one angle has been added. And upon close inspection, a quadrangle can be represented as a combination of two triangles.

A dynamic worksheet helps to recover many familiar elements in the new situation. The two triangles can be deduced from the quadrangle and separately explored.

By overlaying the two triangle circumcircles, it is easily possible to verify the theoretical solution: If the circle midpoints of both partial triangles coincide, the quadrangle has a circumcircle. The following statement amounts to the same: If the perpendicular bisectors of the quadrangle intersect in one point, the quadrangle has a circumcircle.

A more detailed description of this unit is available, see literature "Round and Angular – A Theme with Variations".

There are manifold opportunities for inquiry based mathematics also at the upper secondary level. Some ideas about variations of the sinus-function are described in the last part.



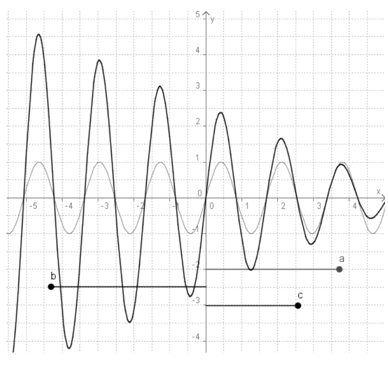
Again using dynamic worksheets students can observe how the graph of the function $f(x) = a\sin[b(x+c)] + d$ is influenced through variation of the parameters a, b, c, d.

The students also learn about the connection to harmonic vibrations, especially how to vary the amplitude.

In the next step they can learn to explore and analyze damped harmonic vibrations in various experiments.

Is it possible to find out the mathematical formula in an experimental way as well?

After the students have found out the connection of damping and amplitude they can think of the needed variation of the mathematical function. Now the amplitude itself has become a function, depending on time. So they have to substitute the parameter a in the former function $f(t) = a\sin[b(t+c)] + d$ by a function $a(t)$ (unknown at this stage of exploration).



Let's use the trial and error method, starting as simple as possible.

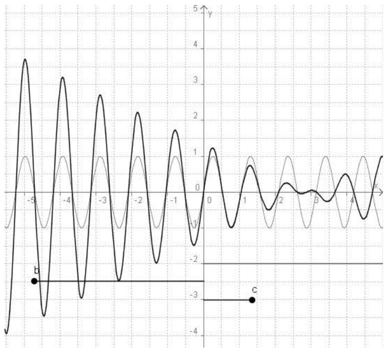
The students are asked to make experiments using the dynamic worksheet seen on the right:

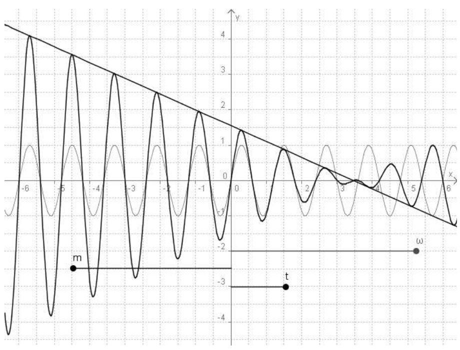
At the first view this function seems to be a good solution ... until some students find out something strange:

There should be very intensive discussions about the mathematics behind as well as about possible physical phenomena.

If you make use of a ruler, the solution easily can be found:

There are various ways, starting from the mathematics or the physics, to find an exponential function as an proper solution.





This variety of examples will show how easy it is to start with inquiry-based mathematics in every day classroom teaching. Now it is up to you to try out and to develop your own ideas, preferable in common with your colleagues.

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Dagmar Raab
Lehrstuhl für Mathematik und ihre Didaktik
Didaktik der Informatik
Universität Bayreuth, Deutschland
e-mail: dagmar.raab@uni-bayreuth.de

КЪМ НОВ СТИЛ НА ПРЕПОДАВАНЕ НА МАТЕМАТИКА ИЛИ КАК ДА ОСЪЩЕСТВЕИМ ИЗСЛЕДОВАТЕЛСКИЯ И ПРОБЛЕМНО-ОРИЕНТИРАНИЯ ПОДХОД В ЧАСОВЕТЕ ПО МАТЕМАТИКА?

Дагмар Рааб

Математиката е вълнуваща и забавна. Можем ли да убедим учениците, че това може да стане действителност.
Задачите са най-важните инструменти за учителите по математика, когато планират уроците си. Планът трябва да съдържа идеи как да се очертае и как да се жалонира пътят, по който учениците ще стигнат до решението на дадена задача. Учителите не трябва да очакват от учениците си просто да кажат кой е отговорът на задачата, а да ги увлекат в процеса на решаване с подходящи въпроси. Ролята на учителя е да помогне на учениците

- да бъдат активни и резултатни при решаването на задачи;
- самите те да поставят задачи;
- да модифицират задачи;
- да откриват закономерности;
- да изготвят стратегии за решаване на задачи;
- да откриват и изследват различни начини за решаване на задачи;
- да намират смислена връзка между математическите си знания и проблеми от ежедневието.

В доклада са представени избрани и вече експериментирани примери за това как учители и ученици могат да намерят подходящ път към нов тип преживявания в преподаването и изучаването на училищната математика.