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DIRECTION FINDING ESTIMATORS OF CYCLOSTATIONARY SIGNALS IN ARRAY PROCESSING FOR MICROWAVE POWER TRANSMISSION

B. Shishkov, K. Hashimoto, H. Matsumoto, N. Shinohara, T. Mitani

A solar power satellite is paid attention to as a clean, inexhaustible large-scale base-load power supply. The following technology related to beam control is used: A pilot signal is sent from the power receiving site and after direction of arrival estimation the beam is directed back to the earth by same direction. A novel direction-finding algorithm based on linear prediction technique for exploiting cyclostationary statistical information (spatial and temporal) is explored. Many modulated communication signals exhibit a cyclostationarity (or periodic correlation) property, corresponding to the underlying periodicity arising from carrier frequencies or baud rates. The problem was solved by using both cyclic second-order statistics and cyclic higher-order statistics. By evaluating the corresponding cyclic statistics of the received data at certain cycle frequencies, we can extract the cyclic correlations of only signals with the same cycle frequency and null out the cyclic correlations of stationary additive noise and all other co-channel interferences with different cycle frequencies. Thus, the signal detection capability can be significantly improved. The proposed algorithms employ cyclic higher-order statistics of the array output and suppress additive Gaussian noise of unknown spectral content, even when the noise shares common

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cycle frequencies with the non-Gaussian signals of interest. The proposed method completely exploits temporal information (multiple lag τ), and also can correctly estimate direction of arrival of desired signals by suppressing undesired signals. Our approach was generalized over direction of arrival estimation of cyclostationary coherent signals. In this paper, we propose a new approach for exploiting cyclostationarity that seems to be more advanced in comparison with the other existing direction finding algorithms.

1. Introduction

Radio waves can benefit the welfare of humanity through other purposes than communications. Microwave Power Transmission (MPT) is one of the new technological frontiers. Solar Power Satellites (SPS) will provide a clean and limitless energy resource from space through this technique [20]. However to develop successfully this new and promising technology we have to rely on the latest achievements of communications theory.

In [14, 15] a beam-control system for MPT with spread-spectrum pilot signals is developed for the Space Solar Power Systems (SSPS). The spread-spectrum modulation is used to differentiate pilot signals sent from power receiving sites from other interference signals. The arrival direction is estimated from the phase difference between two antenna elements after dispreading the spread-spectrum modulation. In these two papers they propose a new system where a single frequency can be used for both monochromatic power transmission and as a carrier of the pilot signal. Antennas are shared for both power transmission and pilot-signal reception. See also [34, 39] about design and optimal beamforming of large antenna arrays for MPT.

The problem of beam-control in MPT is of paramount importance for this new and promising technology and in this paper we approach it in another way than that in [14, 15]. A novel direction-finding algorithm based on linear prediction technique for exploiting cyclostationary statistical information (spatial and temporal) is explored.

Due to the rapid increase in the number of users of mobile communications in recent years and limitations of the available frequency bands as a consequence, the applications of array beamforming techniques have gained attractive attention to enhance the desired signals and reduce the unavoidable presence of interference. By using adaptive beamforming one can modify the array outputs to enhance the desired signal reception and simultaneously suppress the undesired ones. At this the knowledge of the number of the Signals Of Interest (SOI) and their Direction Of Arrival (DOA) is of great importance.

Conventional array processing methods basically rely on the spatial properties (e.g., spatial delay) of the signals impinging on an array of sensors. The scenario that most conventional algorithms assume is that the sources under consideration are narrow band having the same center frequency, and that their temporal samples are uncorrelated. With this assumptions, in the now familiar subspace formulation (see, e.g., [22, 41, 42]), the data vector from each source spans a one-dimensional subspace and the DOA estimation problem then becomes one of finding the signal subspace.

Many algorithms, e.g., MUltiple SIgnal Classification (MUSIC) and Estimation of Signal Parameters via Rotational Invariance Techniques (ESPRIT) [31, 25] have been proposed to estimate the DOA by searching for the signal subspaces. One shortcoming of the above approaches is that they ignore the temporal properties of the SOI. Nevertheless, it is very difficult, in general, to combine efficiently the temporal and spatial information of the signals in determining the source DOA.

In this paper, we attempt to solve the problem of estimating the DOA of signals exhibiting cyclostationarity or periodic correlation and that generate spectral lines when they pass through certain nonlinear transformations [10]. By exploiting this special temporal property of the signals, we can null out or greatly reduce the effect of other co-channel jammers and background noise [9, 10].The cyclostationarity concept was first introduced into array signal processing by Gardner [8] and Schell et al. [26]. In their approaches, the correlation matrix estimate used in the general subspace algorithms is replaced by a Cyclic AutoCorrelation Matrix (CACM) estimate, based on Cyclic Second-Order Statistics (CSOS). The cyclic correlation function is evaluated at one lag-parameter τ and then used to estimate the DOA. But we don't know the optimal lag τ at which correlation function achieves it maximum. In reality this lag-parameter is rarely available. An alternative approach is to use larger temporal information or cyclic correlation function with $\tau = 0, \pm 1, \ldots, \pm(Q-1)$, but then cyclic MUSIC is not so computationally efficient. Another opportunity to exploit cyclostationarity is the family of Self-COherence REstoral (SCORE) methods proposed by Agee et al. [1]. Specifically, these methods are based on a property termed spectral correlation in which a signal is correlated with a frequency-shifted, possibly conjugated, version of itself and that arises as a consequence of the periodic fluctuations of the autocorrelation function. The presence of noise and interferences at the array input distorts the self-coherence of the SOI. The ultimate objective of SCORE methods is to restore the self-coherence of the SOI. Nevertheless SCORE methods suffer from diverse limitations – see [4]. The algorithm suggested in [51] unlike our method, doesn't regard such an important information, containing in Cyclic Cross-Correlation Function (CCCF) between sensor outputs. It seems that the algorithm suggested in [51] is based on Cyclic Autocorrelation Function (CACF) of sensors, containing insufficient spatial information and requires to turn to conjugate CACF. Our algorithm is based on Linear Prediction (LP) technique and exploiting all temporal information, is simpler and more convenient than other cyclic subspace methods known until now [8, 26, 51].

Since multipath propagation is often encountered in variety of communication systems due to various reflections, with the results that the signals are coherent and cyclic matrix becomes singular, DOA estimation of signals has received much attention in last decade [22, 27, 50]. We have developed the spatial smoothing technique [22, 17] for the cyclostationary coherent signals.

In array signal processing there are, however, cases involving non-Gaussian random processes where second moment analysis does not provide all of the needed information. One example is when phase of communication signals is important (second moment quantities suppress phase information). Some types of modulated signals like Quaternary Phase-Shift Keyed (QPSK) and digital Quadrature-Amplitude Modulated (QAM) can not be processed adequately by using second-order statistics that are appropriate when the signals are Gaussian. In other words higher-order time products are needed. So, this field is currently an area of intense research and new results are constantly being reported [12, 13, 52]. By exploiting the higher-order temporal properties of communication signals, many algorithms for Direction Finding (DF) have been proposed for narrow-band non-Gaussian signals. However, conventional cumulants based algorithms [23] become very complicated and are computationally intensive when more than the third-order cumulant is used. The reference sources [32, 33, 40] are based on the fundamental properties of cyclostationarity concept and discussed the problem of estimating the DOA of cyclostationary signals by using Cyclic Higher-Order Statistics (CHOS), where cyclic MUSIC was generalized by using fourth-order cumulant for one lag τ.

In this paper, by utilizing a LP model of the sensor outputs, new DF HOS algorithms that exploit the non-Gaussian and cyclostationary nature of communication signals are explored.

2. Problem Statement

Several important problems in the signal processing field, among them direction finding with narrowband sensor arrays can be reduced to estimating the parameters in the following model (discrete time):

$$\mathbf{x}(n) = \mathbf{A}(\theta)\mathbf{s}(n) + \mathbf{i}(n), \quad n = 0, 1, 2, \ldots, N-1, \tag{1}$$

where $\mathbf{x}(n) \in \mathbf{C}^{M\times 1}$ is complex observation vector, $\mathbf{s}(n) \in \mathbf{C}^{L\times 1}$ is complex signal vector, $\mathbf{i}(n) \in \mathbf{C}^{M\times 1}$ is complex additive noise vector, and $\mathbf{A}(\theta) \in \mathbf{C}^{M\times L}$ $\left(\theta \in \mathbf{R}^{L\times 1}\right)$ is matrix in which the kth column depends from an unknown bounded parameter of the kth signal and any l columns having different parameters are independent as long as $l \leq M$. In DF, L is the number of signals and M is the number of sensors.

There are three main problems associated with fitting models of the form (1) to the data set

$$\{x(0), x(1), \ldots, x(N-1)\}.$$

1. Estimation of the number of signals L.
2. Estimation of the signal amplitudes $\{s_k(n)\}$.
3. Estimation of the vector parameter θ.

Methods for accomplishing this last task, and their performance, are the main topics to be dealt with in this paper.

The following notations will be used:
b, $\mathbf{b}$ and $\mathbf{B}$ stand for scalar, vector and matrix in that order. Similarly $\mathbf{B}^*$, $\mathbf{B}^T$, $\mathbf{B}^H$, $\mathrm{tr}(\mathbf{B})$ and $\det(\mathbf{B}) = |\mathbf{B}|$ represent the complex conjugate, transpose, complex conjugate transpose, trace and determinant of $\mathbf{B}$ respectively. By notation $E(\cdot)$ will denote the expectation operator.

3. Data Model and Cyclostationarity (Conventional and Cyclic MUSIC)

A random process $\{x(n), n = 0, 1, 2, \ldots\}$ is called wide-sense cyclostationary if its mean-value $m_x(n) = E\{x(n)\}$ and autocorrelation function $R_x(n, l) = E\{x(n)x^*(l)\}$ are periodic with some period say $1/\alpha$ (see e.g. [10]):

$$m_x(n + k/\alpha) = m_x(n) \tag{2}$$

$$R_x(n + k/\alpha, l + k/\alpha) = R_x(n, l), \tag{3}$$

where k is an integer and asterisk denotes complex conjugation. Such signals are common in communication applications, where the period arises from carrier frequencies or baud rates.

In MPT antennas are shared for both power transmission and pilot-signal reception. We consider a uniform linear array (ULA) with a big amount of elements in transmitting part [34], which shares a small amount of elements for receiving the pilot signals [14, 15]. The receiving part consists of M identical isotropic sensors with separation distance d and receiving signals from L narrowband "sources" $s_1(n), \ldots, s_L(n)$ that arrive at the array from directions $\theta_1, \theta_2, \ldots, \theta_L$ and measured clockwise from the normal of the array.

The narrowband model (1) can be presented as follows

$$(4) \quad \mathbf{x}(n) = \mathbf{A}(\theta)\,\mathbf{s}(n) + \mathbf{i}(n) = \sum_{k=1}^{L} \mathbf{a}(\theta_k)\, s_k(n) + \mathbf{i}(n), \quad n = 0, 1, \ldots, N-1,$$

where data vector $\mathbf{x}(n)$, the steering matrix $\mathbf{A}(\theta)$, signal vector $\mathbf{s}(n)$ and noise vector $\mathbf{i}(n)$are defined by

$$(5) \quad \begin{aligned} \mathbf{x}(n) &= [x_1(n), \ldots, x_M(n)]^T \\ \mathbf{A}(\theta) &= [\mathbf{a}(\theta_1), \ldots, \mathbf{a}(\theta_L)] \\ \theta &= [\theta_1, \theta_2, \ldots, \theta_L]^T \\ \mathbf{s}(n) &= [s_1(n), \ldots, s_L(n)]^T \\ \mathbf{i}(n) &= [i_1(n), \ldots, i_M(n)]^T \end{aligned}$$

and the array response vector (or steering vector, direction vector) $\mathbf{a}(\theta_k)$ is given by

$$(6) \quad \mathbf{a}(\theta_k) = \left[1, \exp\left(-j2\pi f_c \frac{d}{c}\sin\theta_k\right), \ldots, \exp\left(-j2\pi f_c (M-1)\frac{d}{c}\sin\theta_k\right)\right]^T,$$

where f_c is the carrier frequency and c is the speed of propagation. The signal $x_i(n)$ received by the ith array sensor can be expressed by the following way

$$(7) \quad x_i(n) = \sum_{k=1}^{L} s_k(n) \exp\left(-j2\pi f_c (i-1)\frac{d}{c}\sin\theta_k\right) + i_i(n).$$

It should be mentioned our narrowband model (7) is derived under the assumption that carrier frequency is fairly large compared to the bandwith of the modulating signal. The last can be treated as quasistatic during the time intervals

$$(8) \quad \tau_{ik} = (i-1)\frac{d}{c}\sin\theta_k$$

Generally in conventional algorithms the L SOI are assumed to be

$$\begin{array}{ll} \text{Uncorrelated} & r_{kl} = 0 \\ \text{Correlated} & 0 < |r_{kl}| < 1 \\ \text{Coherent} & |r_{kl}| = 1, \end{array} \tag{9}$$

where r_{kl} is correlation coefficient of the two jointly stationary signals $s_k(n)$ and $s_l(n)$. The noise $\mathbf{i}(n)$ is assumed to be Gaussian distributed, with zero-mean and uncorrelated both temporally and spatially with themselves and with other source signals.

In cyclic algorithms it is assumed that there are $L_\alpha \leq L$ SOI sharing the same cycle frequency α, where α is either known or estimated from the data. So $\mathbf{s}(n)$ in (4) contains only the L_α signals that have cycle frequency α, and all of the remaining signals (of which there are $L - L_\alpha$) and the noise are lumped into $\mathbf{i}(n)$. The signals $s_k(n)$ are mutually not cyclically correlated, and only L_α sources are self cyclically correlated with cycle frequency α. This condition is weaker than the one requiring that the sources be mutually uncorrelated. The additive noises $i_i(n)$ are assumed to be cyclically uncorrelated with themselves and with other source signals at the same cycle α. The cycle frequency α can be determined from the carrier frequency and baud rate. In this paper suppose that it is known.

The CACF and CCCF for discrete time processing will be expressed as follows [9, 10]:

$$R^{\alpha}_{x_i x_i}(\tau) = \left\langle x_i(n) x_i^*(n-\tau) e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty} \tag{10}$$

$$R^{\alpha}_{x_i x_{mi}}(\tau) = \left\langle x_i(n) x_m^*(n-\tau) e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty}, \tag{11}$$

where $i, m = 1, 2, \ldots, M; n = 0, 1, \ldots, N-1; \tau = 0, \pm 1, \ldots, \pm(Q-1)$ and notation $\langle\cdot\rangle_{N\to\infty}$ denotes discrete-time averaging

$$\langle z(n)\rangle_{N\to\infty} = \lim_{N\to\infty} \frac{1}{N} \sum_n \{z(n)\}. \tag{12}$$

The CACF and CCCF are components of cyclic autocorrelation matrix (CACM),

$$\mathbf{R_{xx}}^{\alpha}(\tau) = \left\langle \mathbf{x}(n)\, \mathbf{x}^H(n-\tau)\, e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty} = \mathbf{A}(\theta)\, \mathbf{R_{ss}}^{\alpha}(\tau)\, \mathbf{A}^H(\theta) \tag{13}$$

The CACM is evaluated at one lag-parameter τ and then used to estimate the DOA. In the signal subspace fitting (SSF) interpretation of the cyclic MUSIC, it is noted that

$$\mathbf{R_{xx}}^{\alpha}(\tau) = \mathbf{A}(\theta) \mathbf{R_{ss}}^{\alpha}(\tau)\, \mathbf{A}^H(\theta) \tag{14}$$

has the same column space as $\mathbf{A}(\theta)$ and its left null space is orthogonal to $\mathbf{A}(\theta)$. As a result: 1) $\mathbf{R_{ii}}$ need not be known or estimated; 2) the number L_α of DOA's estimate is never larger than the number L of DOA's for MUSIC to estimate (and $L_\alpha \langle\langle L$ in some cases); 3) the need for postprocessing and classification of DOA's is inherently reduced by signal selectivity. For a finite number of time samples the algorithm can be implemented by estimating CACM

$$\hat{\mathbf{R}}^{\alpha}_{\mathbf{xx}}(\tau) = \left\langle \mathbf{x}(n)\mathbf{x}^H\left(n-\tau\right)e^{-j2\pi\alpha(n-\tau/2)}\right\rangle_N = \frac{1}{N}\sum_{n=0}^{N-1}\mathbf{x}(n)\mathbf{x}^H\left(n-\tau\right)e^{-j2\pi\alpha(n-\tau/2)} \tag{15}$$

at one lag-parameter τ and then used to estimate the DOA. The CACM $\mathbf{R_{xx}}^{\alpha}(\tau)$ has rank L_α, the number of desired signals. In particular, if only one signal exhibits spectral correlation at the chosen α, then $\mathbf{R_{xx}}^{\alpha}(\tau)$ is a rank-one matrix

$$\mathbf{R_{xx}}^{\alpha}(\tau) = \mathbf{a}(\theta_1)\mathbf{a}^H(\theta_1)\mathbf{R}^{\alpha}_{s_1 s_1}(\tau), \quad \text{for} \quad L_\alpha = 1. \tag{16}$$

In general the CACM has contributions for $L_\alpha\rangle 1$ signals that exhibit spectral correlation at the chosen α. To view this geometrically, the columns of $M - L_\alpha$ eigenvectors of $\mathbf{R_{xx}}^{\alpha}(\tau)$ that correspond to its zero eigenvalues span a null space $\mathbf{E}_{N,\alpha}$ of $\mathbf{H}$. Since the L_α direction vectors $\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \ldots, \mathbf{a}(\theta_{L_\alpha})$ are themselves linearly independent, the L_α dimensional "signal" subspace $\mathbf{S}$ spanned by this actual direction vectors is orthogonal to the subspace $\mathbf{E}_{N,\alpha}$. Thus $\mathbf{E}_{N,\alpha}$ is the "noise" subspace. More over, since $\mathbf{H}$ can always be written as the direct sum of any finite dimensional subspace and its orthogonal complement [42], we have

$$\mathbf{H} = \mathbf{S} \oplus \mathbf{E}_{N,\alpha}. \tag{17}$$

Since none of the signals $s_k(n)$ are perfectly correlated with each other, then $\mathbf{R_{ss}}^{\alpha}(\tau)$ has full rank equal to L_α. Further more since the columns of $\mathbf{A}(\theta)$ are linearly independent, then (14) and (17) imply that the null space is orthogonal to the direction vectors of the desired signals,

$$\mathbf{E}^H_{N,\alpha}\mathbf{a}(\theta_k) = 0, \quad k = 1, \ldots, L_\alpha. \tag{18}$$

This fact can be used to form a measure of orthogonality $P_{CM}(\theta)$ (also referred to as the spatial spectrum) similar to that used by MUSIC and other algorithms:

$$P_{CM}(\theta) = \frac{\left\|\mathbf{a}(\theta)\right\|^2}{\left\|\mathbf{E}^H_{N,\alpha}\mathbf{a}(\theta)\right\|^2}. \tag{19}$$

The DF algorithm must search over θ for the L_α highest peaks in $P_{CM}(\theta)$ and is referred to as the Cyclic MUSIC algorithm (modified and simplified MUSIC algorithm) [8, 9, 10, 26, 28, 30, 31].

4. Linear Prediction Method for DOA Estimation

Let suppose that one of the sensor outputs is predicted as a linear combination of the remaining (M-1) sensor outputs at any instant and cyclic frequency α, and the predictor coefficients are selected so as to minimize the prediction error. Letting the component $x_M(n)$ of $\widetilde{\mathbf{x}}_M$

$$\widetilde{\mathbf{x}}_M = \left(x_M(n), x_{M-1}(n), \ldots, x_1(n)\right)^T$$

stand for the M sensor output at any time "n" and $\hat{x}_M(n)$ the predictor for $x_M(n)$, we have

$$\hat{x}_M(n) = -\sum_{i=1}^{M-1} a_i x_{M-i}(n), \tag{20}$$

where by $\mathbf{x}_M = \left(x_1(n), \ldots, x_M(n)\right)^T$ is denoted array sensors' vector and by $\widetilde{\mathbf{x}}_M$- its reversal version. This gives the error as

$$\varepsilon_M(n) = x_M(n) - \hat{x}_M(n) = \sum_{i=0}^{M-1} a_i x_{M-i}(n), \quad a_0 = 1 \tag{21}$$

and LP error $\varepsilon_M(n)$ is assumed to be spatially and temporally white Gaussian noise. The equation (21) can be written as follows

$$x_M(n) + \sum_{i=1}^{M-1} a_i x_{M-i}(n) = \varepsilon_M(n). \tag{22}$$

The prediction error filter with coefficients $\mathbf{a} = (a_1, a_2, \ldots, a_{M-1})^T$ minimizes the prediction error variance σ_ε^2 or the mean-square error (MSE) – $\sigma_\varepsilon^2 = E\left\{\left|\varepsilon_M(n)\right|^2\right\}$, if the filter coefficients are chosen such that $E\left\{x_{M-i}(n)\,\varepsilon_M^*(n)\right\} = 0$, $i = 1, 2, \ldots, M-1$, that is, if the error is orthogonal to the $\widetilde{\mathbf{x}}_M$(principal of orthogonality). Note that our LP model is spatially oriented.

By multiplying the equation (22) by $x_M^*(n-\tau)$ and taking expectations, it is not difficult to show that CSOS satisfy the above difference equation

$$R^\alpha_{x_M x_M}(\tau) + \sum_{i=1}^{M-1} a_i R^\alpha_{x_{M-i} x_M}(\tau) = \sigma^2_{\varepsilon_M}\delta(\tau), \tag{23}$$

where $\delta(\cdot)$ is discrete unit impulse equal 0 for $\tau \neq 0$. They use equation (23) for $\tau = 0$ in conventional algorithms to determine mean-square error $\sigma^2_{\varepsilon_M}$. We will disregard its right side's influence in our cyclic algorithms. If CSOS of the process $\{x_M(n)\}$ are known or estimated for various lag $\tau, (\tau = 0, \pm 1, \ldots, \pm(Q-1))$ it is possible to obtain linear equations to solve for the coefficients $\{a_i\}$. In matrix notations this leads to

(24)

$$\begin{bmatrix} R_{x_{M-1}x_M}(-Q+1) \cdots R_{x_1 x_M}(-Q+1) \\ R_{x_{M-1}x_M}(-Q+2) \cdots R_{x_1 x_M}(-Q+2) \\ \vdots \\ R_{x_{M-1}x_M}(0) \cdots\cdots\cdots R_{x_1 x_M}(0) \\ \vdots \\ R_{x_{M-1}x_M}(+Q-2) \cdots R_{x_1 x_M}(+Q-2) \\ R_{x_{M-1}x_M}(+Q-1) \cdots R_{x_1 x_M}(+Q-1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_{M-1} \end{bmatrix} = - \begin{bmatrix} R_{x_M x_M}(-Q+1) \\ R_{x_M x_M}(-Q+2) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ R_{x_M x_M}(+Q-1) \end{bmatrix},$$

where

$$R^{\alpha}_{x_{M-i}x_M}(\tau) = \left\langle x_{M-i}(n) x^*_M(n-\tau) e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty} \tag{25}$$

$$= \left\langle \begin{matrix} \left(\sum_{k=1}^{L} s_k(n) \exp\left(-j2\pi f_c (M-i-1)\dfrac{d}{c}\sin\theta_k\right) + i_{M-i}(n) \right) \cdot \\ \left(\sum_{l=1}^{L} s^*_l(n-\tau) \exp\left(j2\pi f_c (M-1)\dfrac{d}{c}\sin\theta_l\right) + i^*_M(n-\tau) \right) e^{-j2\pi\alpha(n-\tau/2)} \end{matrix} \right\rangle_{N\to\infty}$$

$$i = 0, 1, \ldots, M-1; \quad \tau = 0 \pm 1, \ldots, \pm(Q-1)$$

or their estimates

$$\hat{R}^{\alpha}_{x_{M-i}x_M}(\tau) = \frac{1}{N} \sum_{n=0}^{N-1} x_{M-i}(n)\, x^*_M(n-\tau)\, e^{-j2\pi\alpha(n-\tau/2)} \tag{26}$$

$$i = 0, 1, \ldots, M-1; \quad \tau = 0 \pm 1, \ldots, \pm(Q-1).$$

Since the signals $s_k(n)$ are mutually not cyclically correlated, and since only L_α sources are self cyclically correlated with cycle frequency α, the double summation reduces to a single sum. Further the additive noises do not have cycle frequency α and therefore make no contribution to $R^{\alpha}_{x_{M-i}x_M}(\tau)$ and the last reduces to:

$$R^{\alpha}_{x_{M-i}x_M}(\tau) = \sum_{k=1}^{L_\alpha} R^{\alpha}_{s_k s_k}(\tau) \exp\left(j2\pi f_c i \frac{d}{c} \sin\theta_k\right) \tag{27}$$

$$i = 0, 1, \ldots, M-1; \quad \tau = 0 \pm 1, \ldots, \pm(Q-1).$$

Clearly the contributions from the interferences and noise to the CACF and CCCF (25)–(27) vanish, by selecting the cyclic frequency αappropriately. It should be noticed that CACF and CCCF will be different for the different lag parameter τ [3, 10]. The equation (24) can be rewritten more compactly as follows

$$\mathbf{R}\mathbf{a} = \mathbf{y} \tag{28}$$

where the matrix $\mathbf{R}$ is the pseudodata Cyclic Cross-Correlation Matrix (CCCM) – $(2Q-1) \times (M-1)$; $\mathbf{a} = (a_1, a_2, \ldots, a_{M-1})^T$ is the LP vector-coefficient to be estimated; $\mathbf{y} = -\left[R_{x_M x_M}(-Q+1), \cdots, R_{x_M x_M}(0), \cdots, R_{x_M x_M}(Q-1)\right]^T$ is CACM $-1\times(2Q-1)$. As discussed below, finding LP vector-coefficient's estimate or solving equation (28) on the base of CSOS estimates depends upon a number of factors including the relative size of Q and M and the rank of matrix $\mathbf{R}$.

Let $(2Q-1) \geq (M-1)$. If the columns of $\mathbf{R}$ are linearly independent ($\mathbf{R}$ has full rank), then the matrix $\mathbf{R}^H\mathbf{R}$ is invertible and the least square (LS) solution of equation (28) is [42]

$$\hat{\mathbf{a}}_{LS} = \left(\hat{\mathbf{R}}^H \hat{\mathbf{R}}\right)^{-1} \hat{\mathbf{R}}^H \hat{\mathbf{y}}. \tag{29}$$

The matrix $\mathbf{R}^+ = \left(\mathbf{R}^H\mathbf{R}\right)^{-1}\mathbf{R}^H$ is known as the Moore-Penrose pseudoinverse of the matrix $\mathbf{R}$ for the overdetermined problem. Now let's take singular value decomposition (SVD) of $\mathbf{R} = \mathbf{R}^{\alpha}$

$$\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{V}^H, \tag{30}$$

where $\mathbf{U}$ is the $(2Q-1) \times (2Q-1)$ unitary matrix of left singular vectors and $\mathbf{V}$ is the $(M-1) \times (M-1)$ unitary matrix of right singular vectors. The matrix $\mathbf{\Lambda}$ is the $(2Q-1) \times (M-1)$ matrix of non-negative real singular values, which is written here in block partitioned form as

$$\mathbf{\Lambda} = \begin{bmatrix} \mathbf{S_1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \tag{31}$$

where $\mathbf{S_1}$ is a diagonal matrix of the nonzero singular values

$$\mathbf{S_1} = \mathrm{diag}\left[\lambda_1, \lambda_2, \ldots, \lambda_{L_\alpha}\right] \tag{32}$$

and L_α is the rank of $\mathbf{R}$ under cyclostationarity. Rank($\mathbf{R}$) $< M-1$. The Moore-Penrose pseudoinverse is defined as

$$\mathbf{R}^{+} = \mathbf{V}\mathbf{\Lambda}^{+}\mathbf{U}^{H}, \tag{33}$$

where the positions of $\mathbf{U}$ and $\mathbf{V}$ have been interchanged and where $\mathbf{\Lambda}^{+}$, the pseudoinverse of $\mathbf{\Lambda}$, is $(M-1)\times(2Q-1)$ matrix.

$$\mathbf{\Lambda}^{+} = \begin{bmatrix} \mathbf{S_1}^{-1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{34}$$

The expression (29) where $\mathbf{R}^{+}$ is defined by (33) is the unique minimum norm (MN) least square solution of the equation (28) [42]

$$\begin{aligned} \hat{\mathbf{a}}_{MNLS} &= \mathbf{V}\mathbf{\Lambda}^{+}\mathbf{U}^{H} \\ &= \sum_{i=1}^{L_\alpha} \frac{1}{\hat{\lambda}_i}\left(\hat{\mathbf{u}}_i^{H}\hat{\mathbf{y}}\right)\hat{\mathbf{v}}_i, \end{aligned} \tag{35}$$

where L_α is the rank of $\mathbf{R}$ and $\mathbf{u}_i$ and $\mathbf{v}_i$ are the singular vectors of $\mathbf{U}$ and $\mathbf{V}$ respectively, that can be written also in block partitioned form. The second form of $\hat{a}_{MNLS}$ in (35) is the alternative one and easiest to use from computational viewpoint than in (29). The signal $x_M(n)$ at the Mth sensor output, so described, presents by itself autoregressive (AR) process of order $(M-1)$ or Gauss-Markov process. The transfer function of LP filter will be "all poles type model"

$$G(z) = \frac{1}{H(z)} = \frac{1}{1 + a_1 z^{-1} + \cdots + a_{M-1} z^{-(M-1)}}. \tag{36}$$

Turn to array response vector (6) let introduce λ as the associated carrier wavelength and D as the normalized distance between the reference element and the second sensor $D_1 = D = d/(\lambda/2)$. Then substituting $\omega_k = \pi \sin\theta_k$ array response vector will be modified as follows

$$\mathbf{a}(\omega_k) = \left[1, \exp(-jD_1\omega_k), \ldots, \exp(-jD_{M-1}\omega_k)\right]^{T}. \tag{37}$$

The power spectral density $S_{x_M}(\omega)$ of the output process $x_M(n)$ is related to the system transfer function as follows

$$S_{x_M}(\omega) = \frac{\sigma_{\varepsilon_M}^2}{\left|1 + a_1 z^{-1} + \cdots + a_{M-1} z^{-(M-1)}\right|^2}, \tag{38}$$

where $z = e^{-j\omega}$. After the parameters $\{a_i\}$ have been estimated their DOA can be found by searching for the positions of peaks of the spectrum (38).

In our algorithm, we can choose between that based on $\hat{R}^{\alpha}_{x_{M-i}x_M}(\tau)$ or its conjugate counterpart $\hat{R}^{\alpha}_{x_{M-i}x^{\bullet}_M}(\tau)$. Really we use cyclic information of all sensors but in shorter way in comparison with the cyclic subspace methods, such as the cyclic MUSIC and ESPRIT algorithms, where the cyclic correlation matrix of all sensors is required [8]. At this we exploit the whole temporal information (multiple lags τ) containing in cyclic correlation functions avoiding some drawbacks of existing cyclic algorithms. Our algorithm is simpler and more convenient than other cyclic subspace methods known until now [8, 51]. We will call this algorithm Linear Prediction – Signal Subspace Fitting (LP-SSF).

5. DOA Estimation of Coherent Signals

Let now suppose that the correlation coefficient of two jointly stationary signals $s_k(n)$ and $s_l(n)$ is $|r_{kl}| = 1$, $s_k(n), s_l(n)$ are coherent – see (9).

This case arises in a variety of communication systems due to various reflections in multipath propagation, with the result that the cyclic matrix becomes singular and previous cyclic techniques perform poorly. In that case at any instant for the basic model (1),(4) signals $s_1(n), \ldots, s_L(n)$ are phase delayed, amplitude weighted replicas of one of them – say the first and hence

$$s_k(n) = \mu_k s_1(n), \mu_1 = 1, \text{ for } \; k = 1, 2, \ldots, L_\alpha, \ldots, L \tag{39}$$

where μ_k represent the complex attenuation of the kth signal with respect to the first signal $s_1(n)$ with $\mu_k = \rho_k \exp(j\varphi_k)$. We also assume that only L_α sources are coherent with the cycle frequency α. Then equation (4) will be modified

$$\mathbf{x}(n) = s_1(n) \sum_{k=1}^{L} \mu_k \mathbf{a}(\theta_k) + \mathbf{i}(n) = s_1(n)\,\mathbf{b} + \mathbf{i}(n) \tag{40}$$

where $\mathbf{b} = \sum_{k=1}^{L} \mu_k \mathbf{a}(\theta_k)$. Let turn to CACM (14) and take eigenvalue decomposition (EVD)

$$\mathbf{R_{xx}}^{\alpha}(\tau) = \mathbf{A}(\theta)\mathbf{R_{ss}}^{\alpha}(\tau)\mathbf{A}^H(\theta) = \mathbf{U\Lambda V}^H \tag{41}$$

where $\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_M]$ and

$$\mathbf{\Lambda} = \operatorname{diag}\left[\lambda_1, \lambda_2, \ldots, \lambda_{L_\alpha}, \lambda_{L_{\alpha+1}}, \ldots, \lambda_M\right] \tag{42}$$

$$\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_{L_\alpha} \geq \lambda_{L_\alpha+1} = \cdots = \lambda_M = 0.$$

As the $\mathbf{R_{xx}}^{\alpha}(\tau)$ has the same column space as $\mathbf{A}(\theta)$ the last equation is equivalent to

$$\mathbf{a}(\theta_k)\mathbf{u}_i = 0, \text{ for } \; i = L_\alpha + 1, \ldots, M. \tag{43}$$

When the source signals are coherent, due to the new signal presentation (40) the Cyclic Signal Correlation Matrix (CSCM) $\mathbf{R_{ss}}^{\alpha}(\tau)$ in (41) becomes singular, i.e. *rank* $(\mathbf{R_{ss}}^{\alpha}(\tau)) = 1$. Therefore the relations (42), (43) can not be applied directly, so that it will be impossible to estimate any true arrival angle θ_k by using the cyclic MUSIC or LP-SSF. The crucial role that plays the matrix $\mathbf{R_{ss}}^{\alpha}(\tau)$ requires to introduce preprocessing scheme [22] which guarantees full rank condition for the equivalent $\mathbf{R_{ss}}^{\alpha}(\tau)$. We consider a new cyclic DOA estimation by utilizing the Spatial Smoothing (SS) technique (see [22, 50, 52] and references there), which uses a spatial averaging technique to decorrelate the coherent signals. This preprocessing spatial smoothing scheme starts by dividing the total array with M sensors into P overlapping sub-arrays of size $m, (m > L_\alpha + 1)$, with sensors $\{1, b2, b, \ldots, m\}$, forming the first sub-array, sensors $\{2, 3, \ldots, m+1\}$ forming the second sub-array, etc up to the last sub-array formed by sensors $\{M-m+1, M-m+2, \ldots, M\}$. This is the forward spatial smoothing scheme.

Let $\mathbf{x}_p(n) = [x_p(n), x_{p+1}(n), \ldots, x_{p+m-1}(n)]^T$ stand for the output of the pth forward sub-array for *p=1,2,...,P*, where *P=M-m+1*. Then it can be expressed by virtue of this preprocessing as follows

$$\mathbf{x}_p(n) = \mathbf{A}_m(\theta)\mathbf{B}^{p-1}\mathbf{s}(n) + \mathbf{i}_p(n),\ p = 1, 2, \ldots, P \tag{44}$$

where $\mathbf{B}^{p-1}$ denotes the $(p-1)$ power of the $L \times L$ diagonal matrix [22] and

$$\mathbf{B} = \operatorname{diag}[\psi_1, \psi_2, \ldots, \psi_L]$$

$$\psi_k = \exp(-j\omega_k);\ \omega_k = \pi\sin\theta_k;\ k = 1, 2, \ldots, L$$

$$\mathbf{A}_m(\theta) = [\mathbf{a}_m(\theta_1), \ldots, \mathbf{a}_m(\theta_L)]$$

$$\mathbf{a}_m(\theta_k) = [1, \exp(-jD_1\omega_k), \ldots, \exp(-jD_{m-1}\omega_k)]^T \quad \text{(see (37))}$$

$$\mathbf{i}_p(n) = [i_p(n), \ldots, i_{p+m-1}(n)]^T.$$

From (39) and (44), we can obtain the CACM for the pth sub-array

$$\begin{aligned} \mathbf{R}^{\alpha}_{\mathbf{x}_P\mathbf{x}_P}(\tau) &= \left\langle \mathbf{x}_p(n)\mathbf{x}_p^H(n-\tau)e^{-j2\pi\alpha(n-\tau/2)} \right\rangle_{N\to\infty} \\ &= \mathbf{A}_m(\theta)\mathbf{B}^{p-1}\mathbf{R_{ss}}^{\alpha}(\tau)(\mathbf{B}^{p-1})^H\mathbf{A}_m^H(\theta) \end{aligned} \tag{45}$$

where $\mathbf{R_{ss}}^{\alpha}(\tau) = \mu R^{\alpha}_{s_1s_1}(\tau)\mu^H, R^{\alpha}_{s_1s_1}(\tau)$ is the CACF of the signal $s_1(n), \mu = [\mu_1, \mu_2, \ldots, \mu_L]^T$ and $\mu_k = 0$ for $k = L_\alpha + 1, \ldots, L$. Hence the SS CACM is given by [22]

(46)
$$\bar{\mathbf{R}}^{\alpha}_{\mathbf{xx}}(\tau) = \frac{1}{P}\sum_{p=1}^{P}\mathbf{R}^{\alpha}_{\mathbf{x}_P\mathbf{x}_P}(\tau) = \mathbf{A}_m(\theta)\bar{\mathbf{R}}^{\alpha}_{\mathbf{ss}}(\tau)\mathbf{A}^H_m(\theta),$$

where

(47)
$$\bar{\mathbf{R}}^{\alpha}_{\mathbf{ss}}(\tau) = \frac{1}{P}\sum_{p=1}^{P}\mathbf{B}^{p-1}\mu R^{\alpha}_{s_1s_1}(\tau)\mu^H(\mathbf{B}^{p-1})^H = R^{\alpha}_{s_1s_1}(\tau)\frac{1}{P}\mathbf{C}\mathbf{C}^H$$

and

(48)
$$\mathbf{C} = \begin{bmatrix} \mu_1 & & & & \\ & \mu_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & \mu_{L_\alpha} \end{bmatrix} \begin{bmatrix} 1 & \psi_1 & \psi_1^2 & \cdots & \psi_1^{P-1} \\ 1 & \psi_2 & \psi_2^2 & \cdots & \psi_2^{P-1} \\ & & & & \\ 1 & \psi_{L_\alpha} & \psi_{L_\alpha}^2 & \cdots & \psi_{L_\alpha}^{P-1} \end{bmatrix} = \mathbf{DV}.$$

Clearly the rank of $\bar{\mathbf{R}}^{\alpha}_{\mathbf{ss}}(\tau)$ is equal to the rank of $\mathbf{C}$. Since $\mathbf{C}$=$\mathbf{DV}$ and the square matrix $\boldsymbol{D}$ is of full rank, the rank of $\boldsymbol{C}$ is the same as that of $\boldsymbol{V}$. Now the rank of the $L_\alpha \times P$ Vandermonde matrix $\boldsymbol{V}$ is rank$(V) = \min(L_\alpha, P)$ and hence rank$(\mathbf{V}) = L_\alpha$ if and only if $P \geq L_\alpha$. Thus, if $P = M - m + 1 \geq L_\alpha$ or equivalently $M \geq m + L_\alpha - 1$, the smoothed CACM $\bar{\mathbf{R}}^{\alpha}_{\mathbf{ss}}(\tau)$ is nonsingular and the equations (42), (43) hold. However, in this case the number of sensor elements M must be at least $m + L_\alpha - 1$, and recalling that size m of each sub-array must be also at least $L_\alpha + 1$. It follows that the minimum number of sensors needed is $2L_\alpha$.

However, the choice of the statistically significant lag parameter τ is very crucial [3, 10]. As the cyclic correlation function is dependent on the lag τ, if the cyclic correlation of one source is zero or insignificant for a given τ, then the signal will not be resolved. For exploiting the cyclic statistical information sufficiently, we choose again τ as $\tau = 0 \pm 1, \ldots, \pm(Q-1)$. Then second temporal averaging was applied to $\bar{\mathbf{R}}^{\alpha}_{\mathbf{xx}}(\tau)$

(49)
$$\bar{\bar{\mathbf{R}}}_{\mathbf{xx}}{}^{\alpha}(\tau) = \frac{1}{2Q-1}\sum_{\tau}\bar{\mathbf{R}}^{\alpha}_{\mathbf{xx}}(\tau)$$

and therefore the effectiveness of algorithm was improved.

6. Linear Prediction Modeling for Signal Selective DOA Estimation Based on Cyclic Higher-Order Statistics

In communications, however, there are cases involving non-Gaussian random processes and second moment analysis performs poorly. Since computing the entire density function for the random process is usually impractical or impossible, a compromise is to deal with the Higher-Order Statistics (HOS). This approach not only provides tractable analysis methods in the signal domain, but also leads to frequency-domain methods through suitable extensions of Fourier analysis for stationary random processes. More over, for certain signals (e.g., QAM) while the Second-Order Statistics (SOS) are time-invariant, the higher-order cumulants display the cyclostationary property.

In this paper novel DF algorithms that exploit the non-Gaussian and cyclostationary nature of communication signals are proposed.

Will be used again the narrowband model (4), (7) under the following assumptions:

[A1] $s_k(n)$ is non-Gaussian, mth-order cyclostationary with a common cycle frequency and with absolutely summable cumulants $\forall m$ and non-zero cumulants of order m.

[A2] $i_i(n)$ in (4), (7) are zero-mean, either stationary or Gaussian, and independent of the source signals, or non-Gaussian with different cyclostationarity to the source signals.

6.1. Cyclic Moments and Cumulants

We next provide a brief introduction to Cyclic Higher-Order Statistics (CHOS) and establish notation along with some useful properties [32]. For a more complete treatment of CHOS, see [40].

A process $\{x(n), n = 0, 1, \ldots\}$ is said to exhibit mth-order cyclostationarity when its time-varying cumulants up to order m are periodic (almost periodic) functions of time. The mth-order cyclic cumulant with cycle frequency α of $\{x(n), n = 0, 1, \ldots\}$ is the Fourier series coefficient of its time-varying cumulant

$$C^{\alpha}_{mx}(\tau_1, \tau_2, \ldots, \tau_{m-1}) = cum\left\{x(n), x^{(\bullet)}(n+\tau_1), \ldots, x^{(\bullet)}(n+\tau_{m-1})\right\} \tag{50}$$

and is given by [32, 33, 40]

$$C^{\alpha}_{mx}(\tau_1, \tau_2, \ldots, \tau_{m-1}) = \lim_{N\to\infty} \frac{1}{N} \sum_{n=0}^{N-1} C_{mx}(n; \tau_1, \tau_2, \ldots, \tau_{m-1})\, e^{-j2\pi\alpha n}, \tag{51}$$

where ${}^{(\bullet)_r}(r = 0, 1, \ldots, m-1)$ is either a conjugate manipulation or nothing, that is, ${}^{(\bullet)_r}$ is an optional conjugation of the rth lag factor $x(n+\tau_r)$ [21, 40].

Specifically for $m = 3, 4$ when $E\{x(n)\} = 0$ we have for the complex processes

[19, 32, 33, 40]

(52) $$C_{3x}^{\alpha}(\tau_1,\tau_2) = \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1} E\left\{x(n)\,x(n+\tau_1)\,x^{\bullet}(n+\tau_2)\right\} e^{-j2\pi\alpha n}$$

(53)
$$\begin{aligned} C_{4x}^{\alpha}(\tau_1,\tau_2,\tau_3) = \lim_{N\to\infty}\frac{1}{N}\sum_{n=0}^{N-1}&[E\left\{x(n)\,x(n+\tau_1)\,x^{\bullet}(n+\tau_2)\,x^{\bullet}(n+\tau_3)\right\} e^{-j2\pi\alpha n}\\ &-E\left\{x(n)\,x^{\bullet}(n+\tau_2)\right\} E\left\{x(n+\tau_1)\,x^{\bullet}(n+\tau_3)\right\}\\ &-E\left\{x(n)\,x^{\bullet}(n+\tau_1)\right\} E\left\{x(n+\tau_2)\,x^{\bullet}(n+\tau_3)\right\}\\ &-E\left\{x(n)\,x^{\bullet}(n+\tau_3)\right\} E\left\{x(n+\tau_1)\,x^{\bullet}(n+\tau_2)\right\}]e^{-j2\pi\alpha n}. \end{aligned}$$

The third- and fourth-order cumulants and corresponding spectral quantities have been used in a number of applications dealing with non-Gaussian processes. Cumulants and spectra of order higher than the fourth are difficult to compute reliably and so far have found limited [practical use. It should be noted, however, that when the random process has a density that is symmetric about the mean, the third-order cumulant and the bispectrum are identically zero and it is necessary to consider fourth-order quantities to perform any higher-order statistical analysis.

For stationary process $\{x(n)\}$, the cyclic cumulant is time invariant, and hence $C_{mx}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{m-1}) = 0$, for all $m,\alpha \neq 0$, whereas for Gaussian (cyclostationary or not) $\{x(n)\}, C_{mx}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{m-1}) = 0, m \geq 3$, for all α. Consequently, CHOS can distinguish between stationary/cyclostationary and Gaussian/non-Gaussian processes. With the finite data and under absolute cumulant summability (i.e. mixing) the estimate

(54) $$\hat{C}_{mx}^{\alpha}(\tau_1,\tau_2,\ldots,\tau_{m-1}) = \frac{1}{N}\sum_{n=0}^{N-1}\left[x(n)\,x^{\bullet}(n+\tau_1)\cdots x^{\bullet}(n+\tau_{m-1})\right] e^{-j2\pi\alpha n}$$

is consistent and asymptotically normal. In this paper will be assumed that the cycle frequencies α in $s_k(n)$ and thus, in $C_{ms,k}^{\alpha}$ are known and that $s_k(n)$ and $s_l(n)$ are mutually not cyclically correlated. For the applications of cumulants in coherent signals environment see for example [12, 13]. Will be used all the properties of cumulants (see [2, 19, 21, 32]).

6.2. CHOS DOA Estimation Exploiting Linear Prediction Model

Let suppose that the received data $x_M(n)$ is predicted as a linear combination of the remaining $(M-1)$ sensor outputs by using again linear difference equation (22). By multiplying this equation by appropriate delayed versions of the random process $\{x_M(n)\}$ and taking expectations, it is not difficult to show that the third- and fourth-order cumulants satisfy this difference equation and one can be written

$$C^{\alpha}_{x_M x_M x^{\bullet}_M}(\tau_1,\tau_2) + \sum_{i=1}^{M-1} a_i C^{\alpha}_{x_{M-i} x_M x^{\bullet}_M}(\tau_1,\tau_2) = 0 \tag{55}$$

$$C^{\alpha}_{x_M x_M x^{\bullet}_M x^{\bullet}_M}(\tau_1,\tau_2,\tau_3) + \sum_{i=1}^{M-1} a_i C^{\alpha}_{x_{M-i} x_M x^{\bullet}_M x^{\bullet}_M}(\tau_1,\tau_2,\tau_3) = 0 \tag{56}$$

$i = 1, 2, \ldots, M-1$.

Recall the narrowband signal model (4), (7) and consider the third-order and fourth-order cyclic cumulants of the Mth sensor output and corresponding cross cumulants, which under assumptions [A1],[A2] and properties of cumulants [2, 19, 21, 32] are respectively

$$C^{\alpha}_{x_{M-i} x_M x^{\bullet}_M}(\tau_1,\tau_2) = \sum_{k=1}^{L_\alpha} C^{\alpha}_{3s_k}(\tau_1,\tau_2)\, e^{-j2\pi f_c (M-i-1)\frac{d}{c}\sin\theta_k} \tag{57}$$

$$C^{\alpha}_{x_{M-i} x_M x^{\bullet}_M x^{\bullet}_M}(\tau_1,\tau_2,\tau_3) = \sum_{k=1}^{L_\alpha} C^{\alpha}_{4s_k}(\tau_1,\tau_2,\tau_3)\, e^{j2\pi f_c i\frac{d}{c}\sin\theta_k} \tag{58}$$

$i = 1, 2, \ldots, M-1$, where $C^{\alpha}_{3s_k}(\tau_1,\tau_2)$ and $C^{\alpha}_{4s_k}(\tau_1,\tau_2,\tau_3)$ are third-order and fourth-order cyclic cumulant slice of the signal $s_k(n)$.

If CHOS of the process $\{x_M(n)\}$ are known or estimated for various lag $\tau_r\,(r = 0, 1, 2, 3)$ it is possible in general to solve equations (55), (56) and find the coefficients $\{a_i\}$.

Letting for simplicity $\tau_1 = \tau_2 = \tau_3 = 0 \pm 1, \cdots, \pm(Q-1)$ yields

$$\begin{bmatrix} C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M}(-Q{+}1,-Q{+}1) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M}(-Q{+}1,-Q{+}1) \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M}(-Q{+}2,-Q{+}2) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M}(-Q{+}2,-Q{+}2) \\ \vdots & \ddots & \vdots \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M}(0,0) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M}(0,0) \\ \vdots & \ddots & \vdots \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M}(+Q{-}2,+Q{-}2) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M}(+Q{-}2,+Q{-}2) \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M}(+Q{-}1,+Q{-}1) & \vdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M}(+Q{-}1,+Q{-}1) \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_{M-1} \end{bmatrix} = \tag{59}$$

$$= - \begin{bmatrix} C^{\alpha}_{x_M x_M x^{\bullet}_M}(-Q+1,-Q+1) \\ C^{\alpha}_{x_M x_M x^{\bullet}_M}(-Q+2,-Q+2) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ C^{\alpha}_{x_M x_M x^{\bullet}_M}(+Q-1,+Q-1) \end{bmatrix}$$

(60)

$$\begin{bmatrix} C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M x^{\bullet}_M}(-Q{+}1,-Q{+}1,-Q{+}1) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M x^{\bullet}_M}(-Q{+}1,-Q{+}1,-Q{+}1) \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M x^{\bullet}_M}(-Q{+}2,-Q{+}2,-Q{+}2) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M x^{\bullet}_M}(-Q{+}2,-Q{+}2,-Q{+}2) \\ \vdots & \ddots & \vdots \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M x^{\bullet}_M}(0,0,0) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M x^{\bullet}_M}(0,0,0) \\ \vdots & \ddots & \vdots \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M x^{\bullet}_M}(+Q{-}2,+Q{-}2,+Q{-}2) & \cdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M x^{\bullet}_M}(+Q{-}2,+Q{-}2,+Q{-}2) \\ C^{\alpha}_{x_{M-1}x_M x^{\bullet}_M x^{\bullet}_M}(+Q{-}1,+Q{-}1,+Q{-}1) & \vdots & C^{\alpha}_{x_1 x_M x^{\bullet}_M x^{\bullet}_M}(+Q{-}1,+Q{-}1,+Q{-}1) \end{bmatrix} \times$$

$$\begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ a_{M-1} \end{bmatrix} = - \begin{bmatrix} C^{\alpha}_{x_M x_M x^{\bullet}_M x^{\bullet}_M}(-Q+1,-Q+1,-Q+1) \\ C^{\alpha}_{x_M x_M x^{\bullet}_M x^{\bullet}_M}(-Q+2,-Q+2,-Q+2) \\ \vdots \\ \vdots \\ \vdots \\ \vdots \\ C^{\alpha}_{x_M x_M x^{\bullet}_M x^{\bullet}_M}(+Q-1,+Q-1,+Q-1) \end{bmatrix}$$

Clearly the contributions from the interferences and the noise to the cross-cumulants for $i = 1, 2, \ldots, M-1$ (see equations (57)-(60) vanish, by selecting the cycle frequency α appropriately. Equations (59), (60) can be rewritten more compactly as follows

$$\mathbf{Ca} = \mathbf{y}, \tag{61}$$

where $\mathbf{C}$ is the pseudodata Cyclic Cross-Cumulant Matrix (CCCM) – $(2Q-1) \times (M-1)$; $\mathbf{a} = (a_1, \ldots, a_{M-1})^T$ is the LP vector-coefficient to be estimated; $\mathbf{y}$ is Cyclic Auto-Cumulant Matrix (CACM) – $(2Q-1) \times 1$. Further the problem of finding minimum norm least square solution for the LP vector-coefficient is completely referred to section 4. This algorithm will be called again LP-SSF.

7. Conclusions

Most of existing conventional approaches to the direction of arrival problem ignore the temporal characteristics of the signals [25, 31]. In cyclic MUSIC and ESPRIT [8] the temporal information of signals does not exploit completely. We have shown that cyclostationarity of the signals, a situation common in many communications problems, can be exploited to considerable advantages.

The new LP-SSF method in this research effectively combines temporal and spatial properties in either CSOS and CHOS and it compares very favorably with earlier methods with respect to performance, ease of implementation, and applicability, to both correlated and coherent signals.

It seems that this approach can be useful in various problems of the control of beam in MPT, where the accurate determination of DOA of pilot signal is of paramount importance.

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