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## DIMENSIONING OF TELECOMMUNICATION NETWORK BASED ON QUALITY OF SERVICES DEMAND AND DETAILED BEHAVIOUR OF USERS

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**Abstract:** *The aim of this paper is to be determined the network capacity (number of necessary internal switching lines) based on detailed users' behaviour and demanded quality of service parameters in an overall telecommunication system. We consider detailed conceptual and its corresponded analytical traffic model of telecommunication system with (virtual) circuit switching, in stationary state with generalized input flow, repeated calls, limited number of homogeneous terminals and losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing (absent called user) and abandoned conversation.*

*We propose an analytical - numerical solution for finding the number of internal switching lines and values of the some basic traffic parameters as a function of telecommunication system state. These parameters are requisite for maintenance demand level of network quality of service (QoS). Dependencies, based on the numerical-analytical results are shown graphically.*

*For proposed conceptual and its corresponding analytical model a network dimensioning task (NDT) is formulated, solvability of the NDT and the necessary conditions for analytical solution are researched as well. It is proposed a rule (algorithm) and computer program for calculation of the corresponded number of the internal switching lines, as well as corresponded values of traffic parameters, making the management of QoS easily.*

**Keywords:** *Telecommunication Network, Circuit Switching, Network Traffic, Terminal Traffic, Human Factors, Network Dimensioning.*

**ACM Classification Keywords:** *C.2.1 Network Architecture and Design (Circuit-switching networks); C.2.3 Network Operations (Network management); H.1.2 User/Machine Systems (Human factors).*

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### 1. Introduction

The purpose of the teletraffic theory is to find relation between quality of services and equipment cost [Iversen 2004]. This is very important for a good planning and controlling of telecommunication networks.

The Quality of service (QoS) concept is defined in the ITU-T Recommendation E-800 as: "The collective effect of service performance, which determines the degree of satisfaction of a user of the service".

QoS parameters are administratively specified in Service Level Agreement (SLA) between users and operators. These QoS parameters (from a contract of SLA) are reflecting on GoS parameters.

Network dimensioning is necessary for designing and control of network and its level of quality of services (QoS), in an advance determined level.

Based on a given set of QoS requirements, a set of GoS (Grade of service) parameters are selected and determined as functions of human behaviour characteristics.

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### 2. Conceptual Model

In this paper we consider detailed conceptual and its corresponded analytical traffic model [Poryazov 2005b] of telecommunication system with channel switching, in stationary state, with BPP (Bernoulli-Poisson-Pascal) input flow, repeated calls, limited number of homogeneous terminals and losses due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned conversation.

The conceptual model of the telecommunication system includes the paths of the calls, generated from (and occupying) the A – terminals in the proposed network traffic model and its environment (shown on Fig. 1).

The names of the virtual devices used are constructed according to the device position in the model.

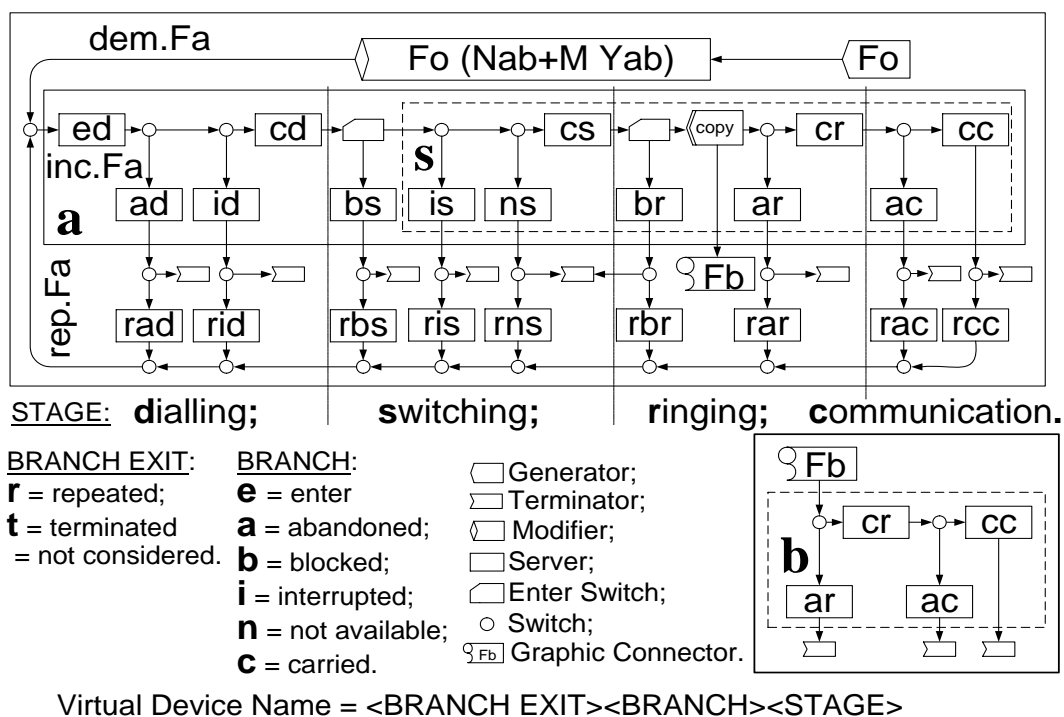


Fig. 1. Normalized conceptual model of the telecommunication system and its environment and the paths of the calls, occupying A-terminals (a-device), switching system (s-device) and B-terminals (b-device); base virtual device types, with their names and graphic notation.

## 2.1. The Comprising Virtual Devices

The following important virtual devices on Fig.1 are shown and considered:

a = comprises all the A-terminals (calling) in the system (shown with continuous line box).

b = comprises all the B-terminals (called) in the system (box with dashed line).

ab = comprises all the terminals (calling and called) in the system (not shown on Fig.1);

s = virtual device corresponding to the switching system. It is shown with dashed line box into the a-device. *Ns* stand for the capacity (number of equivalent internal switching lines) of the switching system.

## 2.2. Stages and Branches in the Conceptual Model:

Service *stages*: dialing, switching, ringing and communication.

Every service stage has *branches*: enter, abandoned, blocked, interrupted, not available, carried (correspondingly to the modeled possible cases of ends of the calls' service in the branch considered).

Every branch has two *exits*: repeated, terminated (which show what happens with the calls after they leave the telecommunication system). Users may make a new bid (repeated call), or to stop attempts (terminated call).

## 2.3. Device Parameters and its Notations in the Conceptual Model:

Letter *F* stands for intensity of the flow [calls/sec.], *P* = probability for directing the calls of the external flow to the device considered, *T* = mean service time, in the device, of a served call [sec.], *Y* = intensity of the device traffic [Erl], *N* = number of service places (lines, servers) in the virtual device (capacity of the device). In the normalized models [Poryazov 2001], used in this paper, every virtual device, except switches, has no more than one entrance and/or one exit. Switches have one entrance and two exits. For characterizing the intensity of the flow, we are using the following notation: *inc.F* for incoming flow, *dem.F*, *ofr.F* and *rep.F* for demand, offered and repeated flows respectively (ITU E.600). The same characterization is used for traffic intensity (*Y*).

*Fo* is the intent intensity of calls of one idle terminal; *inc.Fa* = *Fa* is intensity of incoming flow of A-terminals and *M* is a constant, characterizing the BPP flow of demand calls (*dem.Fa*). If *M* = -1, the intensity of demand flow

corresponds to Bernoulli (Engset) flow model, if  $M = 0$  - to the Poisson (Erlang), and if  $M = +1$  - to the Pascal (Negative Binomial) flow model. In our analytical model every value of  $M$  in the interval  $[-1, +1]$  is allowed. The BPP-traffic model is very applicable [Iversen 2004], but in the numerical examples, presented here,  $M = 0$ , because the conclusions made are independent of the input flow model.

#### 2.4. The Main Assumptions of the Model:

For creating a simple analytical model, we make the following system of fourteen (A-1 – A-14) assumptions [Poryazov 2005b]:

A-1. (Closed System Structure) We consider a closed telecommunication system with functional structure shown in Fig. 1;

A-2. (Device Capacity) All base virtual devices in the model have unlimited capacity. Comprising devices are limited: ab-device contains all the active  $N_{ab} \in [2, \infty)$  terminals; switching system (s) has capacity of  $N_s$  calls (every internal switching line may carry only one call); every terminal has capacity of one call, common for both incoming and outgoing calls;

A-3. (A-Terminal Occupation) Every call, from the flow incoming in the telecommunication system ( $inc.F_a$ ), falls only on a free terminal. This terminal becomes a busy A-terminal;

A-4. (Stationarity) The system is in stationary state. This means that for every virtual device in the model (including comprising devices like switching system), the intensity of input flow  $F(0, t)$ , call holding time  $T(0, t)$  and traffic intensity  $Y(0, t)$  in the observed interval  $(0, t)$  converge to the correspondent finite numbers  $F$ ,  $T$  and  $Y$ , when  $t \rightarrow \infty$ . In this case we may apply the Theorem of Little (1961) and for every device:  $Y = FT$ ;

A-5. (Calls' Capacity) Every call occupies one place in a base virtual device, independently from the other devices (e.g. a call may occupy one internal switching line, if it find free one, independently from the state of the intent B-terminal (busy or free));

A-6. (Environment) The calls in the communication systems' environment (outside the blocks a and b in Fig. 1) don't occupy any telecommunication systems' device and therefore they don't create communication systems' load. (For example, unsuccessful calls, waiting for the next attempt, are in "the head of" the user only. The calls and devices in the environment form the intent and repeated calls flows). Calls leave the environment (and the model) in the instance they enter a Terminator virtual device;

A-7. (Parameters' independability) We consider probabilities for direction of calls to, and holding times in the base virtual devices as independent of each other and from intensity  $F_a = inc.F_a$  of incoming flow of calls. Values of these parameters are determined by users' behavior and technical characteristics of the communication system. (Obviously, this is not applicable to the devices of type Enter Switch, correspondingly to  $P_{bs}$  and  $P_{br}$ );

A-8. (Randomness) All variables in the analytical model may be random and we are working with their mean values, following the Theorem of Little.

A-9. (B-Terminal Occupation) Probabilities of direction of calls to, and duration of occupation of devices  $a_r$ ,  $c_r$ ,  $a_c$  and  $c_c$  are the same for A and B-calls;

A-10. (Channel Switching) Every call occupies simultaneously places in all the base virtual devices in the telecommunication system (comprised of devices a or b) it passed through, including the base device where it is in the moment of observation. Every call releases all its occupied places in all base virtual devices of the communication system, in the instant it leaves comprising devices a or b.

A-11. (Terminals' Homogeneity) All terminals are homogeneous, e.g. all relevant characteristics are equal for every terminal;

A-12. (A-Calls Directions) Every A-terminal directs uniformly all its calls only to the other terminals, not to itself;

A-13. (B-flow ordinariness) The flow directed to B-terminals ( $F_b$ ) is ordinary. (The importance of A-13 is limited only to the case when two or more calls may reach simultaneously a free B-terminal. A-13 may be acquitted from results like in (Burk 1956) and (Vere-Jones 1968);

A-14. (B-Blocking Probability for Repeated attempts) The mean probability ( $P_{br}$ ) of a call to find the same B-terminal busy at the first and at the all following repeated attempts is one and the same.

### 3. Analytical Model

#### 3.1. Some General Equations

For the proposed conceptual model we derived the following system of equations (Poryazov, Saranova 2005):

$$Yab = Fa[S_1 - S_2(1 - Pbs) Pbr - S_3 Pbs] \quad (3.1.1)$$

$$Fa = dem.Fa + rep.Fa \quad (3.1.2)$$

$$dem.Fa = Fo (Nab + M Yab) \quad (3.1.3)$$

$$rep.Fa = Fa [R_1 - R_2 Pbr (1 - Pbs) - R_3 Pbs] \quad (3.1.4)$$

$$Pbr = \begin{cases} \frac{Yab-1}{Nab-1} & \text{in case of } 1 \leq Yab \leq Nab, \\ 0 & \text{in case of } 0 \leq Yab < 1. \end{cases} \quad (3.1.5)$$

$$Ts = S_{1z} - S_{2z} Pbr \quad (3.1.6)$$

$$ofr.Fs = Fa (1 - Pad)(1 - Pid) \quad (3.1.7)$$

$$ofr.Ys = ofr.Fs Ts \quad (3.1.8)$$

$$Pbs = Erl_b (Ns, ofr.Ys) \quad (3.1.9)$$

$$crr.Ys = (1 - Pbs) ofr.Ys \quad (3.1.10)$$

The following notations are used:

$$S_1 = Ted + Pad Tad + (1 - Pad)[Pid Tid + (1 - Pid)[Tcd + Pis Tis + (1 - Pis)[Pns Tns + (1 - Pns)[Tcs + 2Tb]]]] \quad (3.1.11)$$

$$S_2 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)[2Tb - Tbr] \quad (3.1.12)$$

$$S_3 = (1 - Pad)(1 - Pid)[Pis Tis + (1 - Pis)[Pns Tns + (1 - Pns)[Tcs + 2Tb]]] - (1 - Pad)(1 - Pid)Tbs \quad (3.1.13)$$

$$S_{1z} = Pis Tis + (1 - Pis)[Pns Tns + (1 - Pns)(Tb + Tcs)] \quad (3.1.14)$$

$$S_{2z} = (1 - Pis)(1 - Pns)(Tb + Tcs) \quad (3.1.15)$$

$$R_1 = Pad Pr ad + (1 - Pad)(Pid Pr id + (1 - Pid)[Pis Pr is + (1 - Pis)(Pns Pr ns + (1 - Pns)Q]) \quad (3.1.16)$$

$$R_2 = (1 - Pad)(1 - Pid)(1 - Pis)(1 - Pns)(Q - Pr br) \quad (3.1.17)$$

$$R_3 = (1 - Pad)(1 - Pid)\{Pis Pr is + (1 - Pis)[Pns Pr ns + (1 - Pns)Q - Pr bs]\} \quad (3.1.18)$$

$$Q = Par Pr ar + (1 - Par)[Pac Pr ac + (1 - Pac) Pr cc] \quad (3.1.19)$$

An important assumption for proposed analytical model is:

$$\text{The intent intensity of calls of one idle terminal is } Fo \geq 0.$$

#### 3.2. General Blocking Probability

Based on the conceptual model we define general blocking probability as follows:

Definition: General blocking probability ( $Pbl$ ):

$$Pbl = \{Pbr \oplus Pbs, Pbr \in (0,1), Pbs \in (0,1):$$

$$(1 - Pad)(1 - Pid)[Pbs + (1 - Pbs)(1 - Pis)(1 - Pns)Pbr]\} \quad (3.2.1)$$

$Pad, Pid, Pis, Pbs, Pns, Pbr, Par, Pac$  and  $Pcc$  are known probabilities (see the conceptual model).

### 3.3. Probabilities of Blocking Switching ( $Pbs$ ) and of Finding B-Terminal Busy ( $Pbr$ ).

If  $Pbr \in [0,1], Pbs \in (0,1]$  then each duple ( $Pbr, Pbs$ ) defines a value of  $Pbl$  throw (3.2.1) and back, each value of  $Pbl$  defines a set of duples ( $Pbr, Pbs$ ).

As GoS - parameter we consider general blocking probability  $Pbl$  based on (3.2.1).

Analogously,  $adm.Pbl$  (administratively determined value of  $Pbl$  in SLA in advance) defines set of duples ( $adm.Pbr, adm.Pbs$ ) and back.

We consider general blocking probability ( $adm.Pbl$ ) as a main QoS parameter, administratively determined in advance in SLA.

## 4. Network Dimensioning Task

### 4.1. Formulation of a Network Dimensioning Task (NDT):

1. To be dimensioned a network (to be found necessary number of internal switching lines), when in advance level of QoS is administratively determined and the values of known parameters are dimensioned and/ or calculated.
2. To be found the values of the unknown parameters, describing the system state in the upper case. For example, a system parameter, describing macrostate of the system (through the value of  $Yab$ ), a terminal capacity of the system (the maximal number of active terminals  $Nab$ ), intensity of demanded and repeated call attempts (respectively  $dem.Fa$  and  $rep.Fa$ ), offered to the switching system traffic intensity ( $ofr.Ys$ ) and others.

Parameters in the Network Dimensioning Task:

Administrative determined parameters:

$$adm.Pbl \text{ and } M \quad (4.1.1)$$

Known parameters:

$$Fo, Tb, S_1, S_2, S_3, R_1, R_2, R_3, S_{1z}, S_{2z} \quad (4.1.2)$$

Aim: To determine the number of switching lines  $Ns$ ; and the following unknown parameters:

$$Yab, Fa, dem.Fa, rep.Fa, ofr.Fs, Ts, ofr.Ys \quad (4.1.3)$$

Condition:

$$Pbl(Pbr, Pbs) \leq adm.Pbl \quad (4.1.4)$$

### 4.2. Solvability of the NDT:

The traffic intensity  $Yab$  characterizes the macrostate of the system. In Poryazov, Saranova (2005) is shown that

$$Yab = \frac{F_0(S_1 - S_3Pbs) - (F_0(S_1 - S_3Pbs) + F_0S_2(1 - Pbs))Pbr + F_0S_2(1 - Pbs)Pbr^2}{F_0(S_1 - S_3Pbs) - (F_0M(S_1 - S_3Pbs) + F_0S_2(1 - Pbs) - 1 + R_1 - R_3Pbs)Pbr + (1 - Pbs)(F_0MS_2 + R_2)Pbr^2} \quad (4.2.1).$$

Theorem 1: If  $Pbr \neq 0$  and  $Fo \neq 0$ , then analytical presentation (4.2.4) of  $Yab$  in the NDT exist.

Proof: Considering the system equations (3.1.1) - (3.1.10) when  $Pbr \neq 0$  and  $Fo \neq 0$  from (3.1.5) and (3.1.3) follows

$$dem.Fa = \frac{Fo}{Pbr} [Pbr - 1 + (M Pbr + 1)Yab] \quad (4.2.2)$$

From (3.1.2) and (3.1.4) follows

$$dem.Fa = Fa \{1 - R_1 + R_2 Pbr + (R_3 - R_2 Pbr) Pbs\} \quad (4.2.3)$$

Then (4.2.2), (4.2.3) and (3.1.2) gives

$$Yab = \frac{F_0(1 - Pbr)\{S_1 - S_2 Pbr - (S_3 - S_2 Pbr) Pbs\}}{F_0(1 + MPbr)\{S_1 - S_2 Pbr - (S_3 - S_2 Pbr) Pbs\} - Pbr\{1 - R_1 + R_2 Pbr + (R_3 - R_2 Pbr) Pbs\}} \quad (4.2.4)$$

(4.2.4) is new simplified expression of the (4.2.1).

If  $F_0 = 0$ , then obviously  $Fa = 0$ ,  $dem.Fa = 0$  and  $rep.Fa = 0$ .

Therefore, when  $Pbr \neq 0$  in NDT, on the base of administrative determined values of parameters  $Pbs$ ,  $Pbr$ ,  $M$  and the known parameters (4.1.2), traffic intensity  $Yab$  is derivable. The other system parameters in the NDT are depending on the system state (respectively on  $Yab$ ).

We will prove that the values of unknown parameters (4.1.3) in the NDT can be derived (evaluated) through  $Yab$  and known parameters (4.1.2) in correspondence of determined conditions.

Theorem 2: If

$$Pbr \neq 0 \text{ and } Pbs \neq \frac{S_1 - S_2 Pbr}{S_3 - S_2 Pbr} \quad (4.2.5)$$

in the NDT, then for each unknown parameter of (4.1.3), an analytical expression for its evaluation exists.

Proof: Using the system (3.1.1) – (3.1.10) and (4.2.1) by  $(S_1 - S_2 Pbr) - (S_3 - S_2 Pbr) Pbs \neq 0$ , follows

$$Fa = \frac{Yab}{S_1 - S_2 Pbr - (S_3 - S_2 Pbr) Pbs} \quad (4.2.6)$$

For  $dem.Fa$  from (3.1.3) and (3.1.5) is received (4.2.2).

It is resulted from (3.1.4) and (4.2.5):

$$rep.Fa = \frac{Yab \{R_1 - R_2 Pbr - (R_3 - R_2 Pbr) Pbs\}}{S_1 - S_2 Pbr - (S_3 - S_2 Pbr) Pbs} \quad (4.2.7)$$

From (3.1.6) and (4.2.5) follows:

$$ofr.Fs = \frac{Yab (1 - Pad)(1 - Pid)}{S_1 - S_2 Pbr - (S_3 - S_2 Pbr) Pbs} \quad (4.2.8)$$

The parameter  $Ts$  can be calculated from (3.1.7), and from (3.1.3) and (3.1.5) follows:

$$ofr.Ys = \frac{(1 - Pad)(1 - Pid)(S_{1z} - S_{2z} Pbr) Yab}{S_1 - S_2 Pbr - (S_3 - S_2 Pbr) Pbs} \quad (4.2.9)$$

Therefore, the values of the unknown parameters (4.1.3) in the NDT can be expressed and calculated by the conditions of Theorem 1 and Theorem 2.

For the network dimensioning, when the level of QoS is determined administratively in advance (for example blocking probability  $Pbs$ ), Erlangs'B - formula may be used:

$$Pbs = Erl\_b(Ns, ofr.Ys) \quad (4.2.10)$$

$$Erl\_b(Ns, ofr.Ys) = \frac{(ofr.Ys)^{Ns}}{Ns! \sum_{j=0}^{Ns} \frac{(ofr.Ys)^j}{j!}} \quad (4.2.11)$$

The number of switching lines  $Ns$  and the values of  $ofr.Ys$  are calculated by the conditions of Theorem 1 and Theorem 2.

Remark 1-2:  $ofr.Ys$ , being evaluated on the base of the Theorem 1 and Theorem 2 for  $adm.Pbs$  and  $adm.Pbr$ , is resulted in a fixed value. Then  $Pbs = Pbs(Ns, ofr.Ys)$  is a function of  $Ns$  only and  $Pbs = Pbs(Ns)$ .

Theorem 3: The function  $Pbs = Pbs(Ns, ofr.Ys)$ , defined through (4.2.11) in the NDT is strictly monotone decreasing according to  $Ns \geq 1$ , when  $ofr.Ys > 0$  is a fixed value.

Proof: It can be proved that  $Pbs(Ns+1, ofr.Ys) < Pbs(Ns, ofr.Ys)$ . Obviously (see (4.2.11))  $Pbs(0, ofr.Ys) = 1$ . Using the recursion Erlangs'B - formula [Iversen 2004]:

$$Pbs(Ns, ofr.Ys) = \frac{ofr.Ys Pbs(Ns-1, ofr.Ys)}{Ns + Pbs(Ns-1, ofr.Ys)}. \quad (4.2.12)$$

But  $Pbs(Ns, ofr.Ys) > 0$  when  $ofr.Ys > 0$  and  $Ns \geq 1$ ,  $ofr.Ys Pbs(Ns, ofr.Ys) + Ns + 1 > 0$  and

$$\begin{aligned} Pbs(Ns+1, ofr.Ys) - Pbs(Ns, ofr.Ys) &= Pbs(Ns, ofr.Ys) \frac{ofr.Ys[1 - Pbs(Ns, ofr.Ys)] - (Ns+1)}{ofr.Ys Pbs(Ns, ofr.Ys) + (Ns+1)} = \\ &= Pbs(Ns, ofr.Ys) \frac{crr.Ys - (Ns+1)}{ofr.Ys Pbs(Ns, ofr.Ys) + (Ns+1)} \end{aligned}$$

Because  $crr.Ys \leq Ns$  follows  $crr.Ys - (Ns+1) < 0$ .

Therefore,  $Pbs(Ns+1, ofr.Ys) - Pbs(Ns, ofr.Ys) < 0$  and the function  $Pbs = Pbs(Ns, ofr.Ys)$ , defined through (4.2.10) is strictly monotone decreasing, when  $ofr.Ys > 0$  is fixed value.

## 5. Analytical Solution

Based on the Assumption A-8 we are working with mean values of the parameters. Various techniques for analyzing complex teletraffic systems require a formulation of the Erlang function that is continuous in the parameter  $Ns$ . This is done via the integral representation [Berezner 1998].

Theorem 4: There is only one solution in the NDT through the equation

$$Erl\_b(Ns, ofr.Ys) = adm.Pbs, \quad (5.1.1)$$

according to the number of switching lines  $Ns$ .

$Adm.Pbs \in (0; 1]$  is in advance administratively determined value of blocking probability, providing of QoS.

Proof: Existence: It was proved, that the function  $Pbs = Pbs(Ns, ofr.Ys)$ , defined through (4.2.10) in the NDT, is strictly monotone decreasing, when  $ofr.Ys > 0$  is fixed value. The absolute maximum is 1 and 0 is absolute minimum of the function. There is only one solution for equation (4.2.11) for  $adm.Pbs \in (0; 1]$ , relying of the Intermediate Value Theorem (Dirschmidt, H. Yorg, 1992).

Uniqueness: Admitting that there are two different solutions  $Ns' \neq Ns''$  of the equation (3.1.1) – (3.1.19) for  $adm.Pbs \in (0; 1]$ , therefore they are simultaneously fulfilled  $Pbs(Ns', ofr.Ys) = adm.Pbs$  and  $Pbs(Ns'', ofr.Ys) = adm.Pbs$ , is contradicting to Theorem 3.

It is proved that only one solution of  $Ns$  exists, fulfilling the equation (4.2.11) and corresponding to the determined administratively in advance value of the blocking probability  $adm.Pbs \in (0; 1]$ .

## 6. Algorithm for Calculating the Values of the Parameters in the NDT:

1. From SLA and ITU-Recommendation are specified and determined administratively blocking probability  $adm.Pbl$ , respectively  $adm.Pbs \in (0; 1]$  and  $adm.Pbr \in [0; 1]$ :

$$\forall adm.Pbl \Rightarrow \exists (adm.Pbr, adm.Pbs): \quad (6.1.1)$$

$$adm.Pbr \oplus adm.Pbs = adm.Pbl : adm.Pbr \in [0,1], adm.Pbs \in (0,1]$$

2. The unknown parameters (4.1.3) in the NDT are evaluated on the base of Theorem 1 and Theorem 2, known parameters (4.1.2), especially  $adm.ofr.Ys(adm.Pbr, adm.Pbs)$ .

3. On the basis of each calculated value  $adm.ofr.Ys$ , we evaluate

$$\tilde{Ns} \in R_+ : \{\forall (adm.ofr.Ys, \tilde{Ns}) : Pbs(adm.ofr.Ys, \tilde{Ns}) = adm.Pbs\} \quad (6.1.2)$$

4. If  $adm.Ns = \sup \tilde{Ns}$ , then

$$Ns = [adm.Ns] + 1 : Pbl \leq adm.Pbl. \quad (6.1.3)$$

5. For finding of the number of internal switching lines  $Ns$ , a computer program is created on the base of the recursion Erlangs'B – formula (4.2.11) [Iversen 2004]. From numerical point of view, the following linear form is the most stable:

$$I(Ns, ofr.Ys) = 1 + \frac{Ns}{ofr.Ys} I(Ns - 1, ofr.Ys), \quad I(0, ofr.Ys) = 1, \quad (6.1.4)$$

where  $I(Ns, ofr.Ys) = 1 / Pbs(Ns, ofr.Ys)$ . This recursion formula is exact, and for large values of  $(Ns, ofr.Ys)$  there are no round of errors.

6. The received results for numerical inversion of the Erlang's formula (for finding the number of switching lines  $Ns$ ) were confirmed with results of others commercial computer programs.

Therefore, it is proved that if  $Pbr \neq 0$  and  $Pbs \neq (S_1 - S_2 Pbr) / (S_3 - S_2 Pbr)$ , then the NDT is solvable and there is proposed algorithm for its solution.

When  $Pbr = 0$  the network loading is rather low and it is not of great practical interest, but in this case a mathematical research is made also.

## 7. Numerical Results

Among the easy computable QoS - parameters in the system (resulted from QoS- strategy of the network operators) is blocking probability  $Pbl$  in *pie-form model* [Poryazov 2000]. The sum of the loss probabilities due to abandoned and interrupted dialing, blocked and interrupted switching, not available intent terminal, blocked and abandoned ringing and abandoned conversation in *pie-form model* is 1.

For finding of the main teletraffic characteristics in proposed conceptual and its corresponding analytical model, the so called *normal-form model* (see Fig. 1) is used for presentation of blocking switching probability ( $Pbs$ ) and probability of finding B-terminal busy ( $Pbr$ ).

Based on the conceptual and its corresponding analytical model (3.1.1) - (3.1.19), defined general blocking probability  $Pbl$  is presented in *pie-form model* in (3.1.20) as function of the  $Pbr$  and  $Pbs$  (which are presented in *normal-form model* in the same equation).

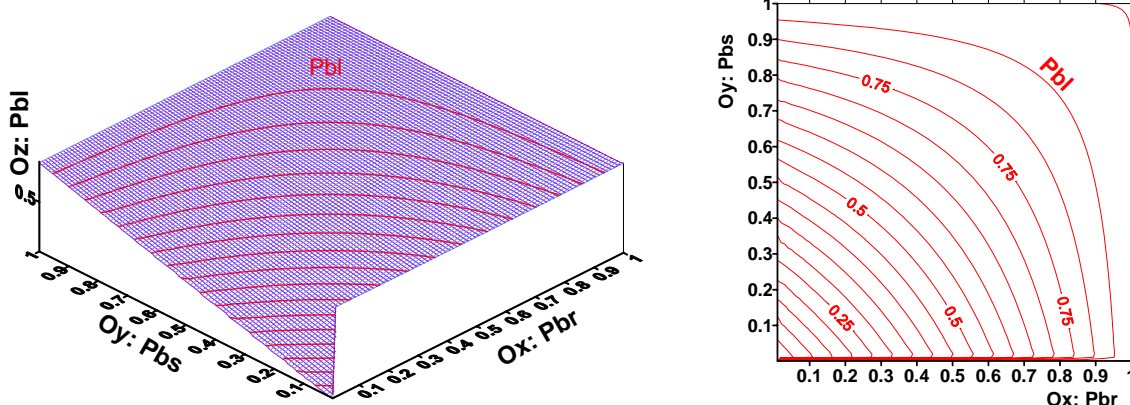


Fig. 2. General blocking probability  $Pbl$  in *pie-form model* is presented as function of probability of finding B-terminal busy  $Pbr$  and probability of blocking switching  $Pbs$  in *normal-form*  $Pbr$  and  $Pbs$  in 3D and contour - map presentation.

On the Fig. 2 blocking probability  $Pbl$  is shown in *pie-form model*, depending on probability of finding B-terminal busy  $Pbr$  (Ox – axis) and probability of blocking switching  $Pbs$  (Oy – axis) in *normal-form model*.  $Pbl$  increases



when  $Pbr$  and  $Pbs$  increase. Therefore, when  $adm.Pbl$  is predetermined as level of QoS administratively then  $adm.Pbr$  and  $adm.Pbs$  can be determined (evaluated) correspondingly.

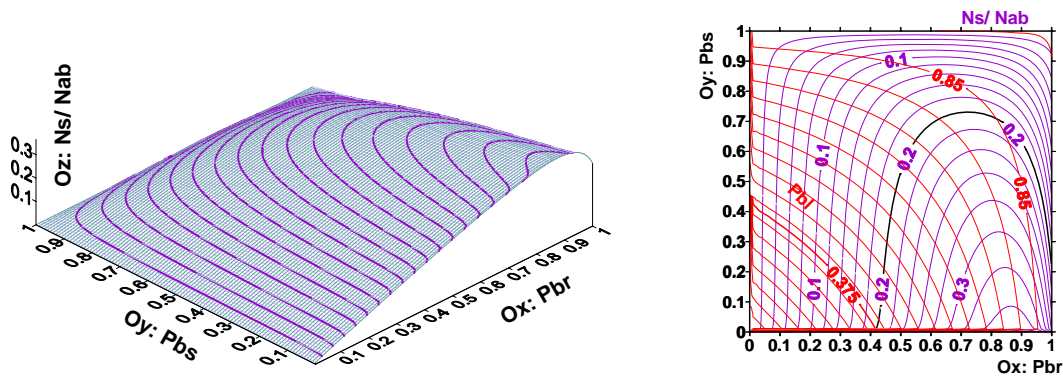


Fig. 3. The number of equivalent internal switching lines  $Ns$  (as percentage of number of active terminals  $Nab$ , where  $Nab=7000$  terminals) and general blocking probability  $Pbl$  are shown as functions of  $Pbr$  (Ox – axis) and  $Pbs$  (Oy – axis) in *normal-form model*, as well.

Conclusions of the numerical experiments:

According  $Pbr$ ,  $Pbs$  and  $Pbl$ :

If  $Pbr \in [0;1]$  and  $Pbs \in [2 \times 10^{-9}; 0.999917]$  then

1.  $Pbl \in [0; 0.900896]$ .
  2.  $0.000143 \leq \frac{Ns}{Nab} \leq 0.387857$ ,  $Ns \in [1; 2715]$ , when  $Nab = 7000$ ;
  3.  $0.77 \times 10^{-5} \leq \frac{ofr.Ys}{Nab} \leq 1.728311$ ,  $ofr.Ys \in [0.473782; 12098.18]$ , when  $Nab = 7000$ .
- $ofr.Ys$  may exceed  $Nab$  by 73% approximately. This is "unproductiveness occupying of resources".
4. Absolute maximum for  $ofr.Ys$ :

Maximum  $ofr.Ys = 12098.18$  and this value is about 4.9 times greater than switching system capacity  $Ns = 2715$ .

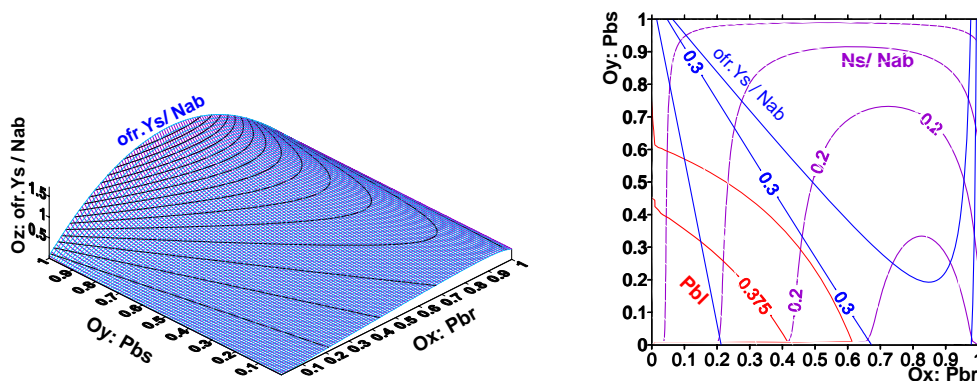


Fig. 4. The offered traffic  $ofr.Ys$ , the number of internal switching lines  $Ns$  and general blocking probability  $Pbl$  are presented as function of probability of finding B-terminal busy  $Pbr$  and probability of blocking switching  $Pbs$  in *normal – form model* in 3D and contour-map presentation.

Absolute maximum for  $Ns$ :

$Ns = 2715$  when  $Nab=7000$  terminals,  $Ns = 38.79\%$  of  $Nab$ . This is possible if  $Pbl = 0.900882 \approx 90\%$  (maximum theoretical value of  $Pbl$ ),  $Pbr = 0.876623 \approx 87.7\%$  and  $Pbs = 9.28 \times 10^{-9}$ ,  $Yab = 6136.487 \text{ Erl} \approx 87.66\%$  of  $Nab$  and  $ofr.Ys = 2452.021 \text{ Erl} \approx 35.0289\%$  of  $Nab$ .

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## 8. Conclusions

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1. Detailed normalized conceptual model, of an overall (virtual) circuit switching telecommunication system (like PSTN and GSM) is used. The model is relatively close to the real-life communication systems with homogeneous terminals.
  2. General blocking probability  $Pbl$  as GoS parameter and  $adm.Pbl$  as QoS – parameter in *pie - form model* are formulated. The offered traffic  $ofr.Ys$ , the number of internal switching lines  $Ns$  and general blocking probability  $Pbl$  are derived as functions of probability of finding B-terminal busy  $Pbr$  and probability of blocking switching  $Pbs$  in *normal – form model*.
  3. The network dimensioning task (NDT) is formulated on the base of preassigned values of QoS parameter  $adm.Pbl$  and its corresponding GoS - parameters -  $adm.Pbr$  and  $adm.Pbs$ ; The NDT is formulated on condition that  $Pbl \leq adm.Pbl$ .
  4. The conditions for existence and uniqueness of a solution of the NDT are researched and an analytical solution of the NDT is found;
  5. An algorithm and a computer program for a calculation the values of the offered ( $ofr.Ys$ ), carried ( $crr.Ys$ ) traffic and the number of equivalent switching lines  $Ns$ , are proposed. The results of numerical solution are derived and graphically shown;
  6. The received results, in NDT, make the network dimensioning, based on QoS requirements easily;
  7. The described approach is applicable directly for every (virtual) circuit switching telecommunication system (like GSM and PSTN) and may help considerably for ISDN, BISDN and most of core and access networks dimensioning. For packet switching systems, like Internet, proposed approach may be used as a comparison basis especially when they work in circuit switching mode.
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## Bibliography

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- Berezner S.A., 1998, - On the inverse of Erlang's Function- J. Appl. Prob.35, 246- 252
- Dirschmidt, Hans Yorg, 1992 - Mathematische Grundlagen der Elektrotechnik-Braunschweig, Wiesbaden, pp.55
- Engset, T., 1918. The Probability Calculation to Determine the Number of Switches in Automatic Telephone Exchanges. English translation by Mr. Eliot Jensen, *Elektronikk*, juni 1991, pp 1-5, ISSN 0085-7130. (Thore Olaus Engset (1865-1943). "Die Wahrscheinlichkeitsrechnung zur Bestimmung der Wählerzahl in automatischen Fernsprechämtern", *Elektrotechnische zeitschrift*, 1918, Heft 31.)
- ITU E.501. ITU-T Recommendation E.501: Estimation of traffic offered in the network. (Previously CCITT - Recommendation, revised 26. May 1997)
- ITU E.600, ITU-T Recommendation E.600: Terms and Definitions of Traffic Engineering (Melbourne, 1988; revised at Helsinki, 1993).
- ITU E.800. ITU-T Recommendation E.800: Terms and Definitions related to Quality of Service and Network Performance, including Dependability. (Helsinki, March 1-12, 1993, revised August 12, 1994).
- Iversen V. B., 2004. *Teletraffic Engineering and Network Planning*, Technical University of Denmark, pp.125
- Little J. D. C., 1961. A Proof of the Queueing Formula  $L=\lambda W$ . *Operations Research*, 9, 1961, 383-387.
- Poryazov S. A, Saranova E. T., 2002. On the Minimal Traffic Measurements for Determining the Number of Used Terminals in Telecommunication Systems with Channel Switching. In: "Modeling And Simulation Environment for Satellite and Terrestrial Communication Networks - Proceedings of the European COST Telecommunications Symposium", Kluwer Academic Publishers, 2002, pp. 135-144;
- Poryazov S. A, Saranova E. T. 2005. Some General Terminal and Network Teletraffic Equations in Virtual Circuit Switching Systems. Symposium "Modeling and Simulation Tools for Emerging Telecommunications Networks: Needs, Trends, Challenges, Solutions", Munich, Germany, 8 - 9 September 2005, Institut für Technische Informatik, Universität der Bundeswehr München (in printing in LNCS, Springer).
- Poryazov S. A. 2005a. Can Terminal Teletraffic Theory Be Liberated from the Main Illusions? In: Proceedings of the International Workshop "Distributed Computer and Communication Networks", Sofia, Bulgaria 24-25 April, 2005, Editors: V. Vishnevski and Hr. Daskalova, Technosphaera publisher, Moscow, Russia, 2005, ISBN 5-94836-048-2, pp. 126-138.; COST-285 TD/285/05/04; COST 290 TD(05)009.
- Poryazov S. A. 2005b. What is Offered Traffic in a Real Telecommunication Network? COST 285 TD/285/05/05; 19th International Teletraffic Congress, Beijing, China, August 29- September 2, 2005, accepted paper No 32-104A.

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- Poryazov S. 1991. Determination of the Probability of Finding B-Subscriber Busy in Telephone Systems with Decentralized Control. Comptes Rendus de l'Academie Bulgare des Sciences – Sofia, 1991, Tome 44, No.3, pp. 37-39.
- Poryazov S. A. 2000. Потребительский и терминальный телетрафик- классификация и обозначения , Conference "Telecom'2000" - Varna, Bulgaria, October 2000 – pp. 58-59.
- Poryazov S. A. 2001. On the Two Basic Structures of the Teletraffic Models. Conference "Telecom'2001" - Varna, Bulgaria, 10-12 October 2001 – pp. 435-450.
- Poryazov, S. A. 2004. The B-Terminal Busy Probability Prediction. IJ Information Theories & Applications, Vol.11/2004, Number 4, pp. 409-415;
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## IMPLICATIONS OF RECENT TRENDS IN TELECOMMUNICATIONS ON MODELING AND SIMULATION FOR THE TELECOMMUNICATION INDUSTRY

Gerta Köster, Stoyan Poryazov

***Abstract:** With this paper we would like to trigger a discussion on future needs of modeling and simulation techniques and tools for the telecommunication industry. We claim that the telecommunication market has undergone severe changes that affect the need for and type of simulations in industrial research. We suggest some approaches how to address these new challenges. We believe that there is need for intensive research in the area.*

***Keywords:** Telecommunications, Market Evolution, Modeling and Simulation Challenges.*

***ACM Classification Keywords:** D.2.2 Design Tools and Techniques; K.6.1 Project and People Management.*

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### Introduction

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Models and simulations in telecommunications fulfill a number of tasks. We build models and run simulations to check the design of emerging products, we model the environment in which such a product is employed and we evaluate and optimize the performance of telecommunication equipment.

In the telecommunications industry the ultimate goals behind these tasks is to save costs and to make money by better or faster design and efficient development support.

We claim that the world of telecommunication has lately undergone severe changes that affect the need for and type of simulations in industrial research. In the course of this paper, we will outline the main trends we observe and discuss the implications on modeling and simulations.

1. Fractalization of the "old" incumbent telecommunication market.
2. Shift of operators' interest from providing a network to providing services and applications.
3. Shortening of development cycles.
4. Telecommunication and telecommunication problems pervade other sectors of private and business life.

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### Evolution of the Telecommunication Market

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2.1. Fractal markets: We observe that a large part of the market has become fractal as opposed to monolithic in the past. The large state owned companies have been replaced by private enterprises. This is true for both, the Western industries, where the telecommunication business has been privatized and the former Soviet block where telecommunication has been denationalized.