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CORRIGENDUM
for
WEIERSTRASS POINTS WITH FIRST NON-GAP FOUR ON A DOUBLE COVERING OF A HYPERELLIPTIC CURVE

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In the proof of Lemma 3.1 in [1] we need to show that we may take the two points p and q $\underline{\text{with } p \neq q}$ such that

$$p + q + (b-2)g_2^1(C') \sim 2(q_1 + \cdots + q_{b-1})$$

where $q_1, \ldots, q_{b-1}$ are points of C', but in the paper [1] we did not show that $p \neq q$. Moreover, we hadn't been able to prove this using the method of our paper [1]. So we must add some more assumption to Lemma 3.1 and rewrite the statements of our paper after Lemma 3.1. The following is the correct version of Lemma 3.1 in [1] with its proof:

Lemma A. *Let r be a positive integer. We set $t = 2n$ with a positive integer $n \leq r$. Let s be an odd integer with $1 \leq s \leq t-1$. Assume that*

$$r + \frac{s+1}{2} = n(b+1) + \zeta \text{ with } 0 \leqq \zeta \leqq \frac{s-1}{2}.$$

Since we have

$$r = n(b+1) + \zeta - \frac{s+1}{2} = nb + \left(n + \zeta - \frac{s+1}{2}\right)$$

with $n - \frac{s+1}{2} \leqq n + \zeta - \frac{s+1}{2} \leqq n-1$, *we can construct a hyperelliptic curve* C *of genus* r *in the way in front of Lemma* 3.1. *Then there exist points* $P_1, \ldots, P_t, Q_1, \ldots, Q_{\frac{s+1-t}{2}+r}$ *of* C *such that*

$$P_1 + P_2 + \cdots + P_t + \left(r - t + \frac{s+1}{2}\right) g_2^1(C) \sim 2\left(Q_1 + \cdots + Q_{\frac{s+1-t}{2}+r}\right)$$

where $P_1, \ldots, P_t$ *are distinct points,* $P_1, \ldots, P_n$ *are Weierstrass points and* $Q_1, \ldots,$ $Q_{\frac{s+1-t}{2}+r}$ *are points which are different from* P_1. *Moreover, we get* $h^0\left(\mathcal{O}_C\left(Q_1 + \cdots + Q_{\frac{s+1-t}{2}+r}\right)\right) = 1$.

Proof. Let p be a point on $C' = HC(F)$. For any point q on C' we have

$$p + q + (b-1)g_2^1(C') \sim 2(q_1 + \cdots + q_b)$$

where $q_1, \ldots, q_b$ are points on C'. In fact, we get

$$p + q + (b-1)g_2^1(C') \sim 2D$$

where D is a divisor of degree b, because of

$$\circ\left(p + q + (b-1)g_2^1(C')\right) = 2b.$$

Moreover, we get $h^0(D) \geqq b+1-b = 1$, which implies that D is linearly equivalent to some effective divisor $q_1 + \cdots + q_b$. Let p be a Weierstrass point on C' and q a point on C' distinct from p. Then we have

$$p + q + (b-1)g_2^1(C') \sim 2(q_1 + \cdots + q_b)$$

where $q_1, \ldots, q_b$ are points on C'. We may assume that $q_1, \ldots, q_b$ are different from p. Let $\tilde{\phi}^* p = P_1 + \cdots + P_n$ and $\tilde{\phi}^* q = P_{n+1} + \cdots + P_{2n}$. Since p is a Weierstrass point on C', $P_1, \ldots, P_n$ are also Weierstrass points on C. We obtain

$$P_1 + \cdots + P_t + \left(r - t + \frac{s+1}{2}\right) g_2^1(C) \sim$$

$$\tilde{\phi}^*\left(p + q + (b-1)g_2^1(C')\right) + \left(\left(r - t + \frac{s+1}{2}\right) - (nb - n)\right) g_2^1(C)$$

because of $\tilde{\phi}^* g_2^1(C') = n g_2^1(C)$. We have

$$\left(r - t + \frac{s+1}{2}\right) - (nb - n) = n(b+1) + \zeta - 2n - nb + n = \zeta \geqq 0.$$

Hence, we get

$$P_1 + \cdots + P_t + \left(r - t + \frac{s+1}{2}\right) g_2^1(C) \sim 2\left(Q_1 + \cdots + Q_{\frac{s+1-t}{2}+r}\right)$$

where $Q_1, \ldots, Q_{\frac{s+1-t}{2}+r}$ are points of C distinct from P_1 because of

$$\zeta \leqq \frac{s-1}{2} \leqq \frac{t-1-1}{2} = n - 1 \leqq r - 1.$$

In the same way as in the proof of Lemma 3.1 in [1] we may assume that

$$h^0\left(\mathcal{O}_C\left(Q_1 + \cdots + Q_{\frac{s+1-t}{2}+r}\right)\right) = 1. \qquad \square$$

We set

$$\mathcal{L} = \mathcal{O}_C(Q_1 + \cdots + Q_{\frac{s+1-t}{2}+r} - (r + \frac{s+1}{2})P_1).$$

Then by Lemma A we get

$$\mathcal{L}^{\otimes 2} \cong \mathcal{O}_C(P_1 + P_2 + \cdots + P_t - t g_2^1(C)) \cong \mathcal{O}_C(-\iota(P_1) - \cdots - \iota(P_t))$$

where ι is the hyperelliptic involution on C. By the same proof as in Theorem 3.2 in [1] we get the correct version of Theorem 3.2:

Theorem B. *Let the notation and the assumption be as in Lemma A. Let*

$$\pi : \tilde{C} = \mathrm{Spec}(\mathcal{O}_C \oplus \mathcal{L}) \longrightarrow C$$

be the canonical morphism. We set $\pi^{-1}(P_1) = \{\tilde{P}_1\}$. *If* $r \geq 5$, *then we get*

$$S(H(\tilde{P}_1)) = \{4, 2r + s, 2r + 2t - s, 4r + 2\}$$

By Theorem B we obtain the correct version of Main Theorem 3.3 in [1]:

Main Theorem C. *Let H be a 4-semigroup of genus $g(H) \geq 10$ with $g(H) \leq 3r(H) - 1$. In this case, by Proposition 2.7 we have*

$$S(H) = \{4, 2r + s, 2r + 2t - s, 4r + 2\}$$

where $r = r(H)$, $t = 2n$ with a positive integer $n \leq r$ and s is an odd integer with $1 \leq s \leq t - 1$. Assume that

$$r + \frac{s+1}{2} = n(b+1) + \zeta \text{ with } 0 \leqq \zeta \leqq \frac{s-1}{2}.$$

Then there exist a double covering $\pi : \tilde{C} \longrightarrow C$ of a hyperelliptic curve and its ramification point $\tilde{P} \in \tilde{C}$ such that $H(\tilde{P}) = H$.

In the forthcoming paper we will prove Main Theorem C without the condition where

$$r + \frac{s+1}{2} = n(b+1) + \zeta \text{ with } 0 \leqq \zeta \leqq \frac{s-1}{2},$$

using a method completely different from the above one.

REFERENCES

[1] J. Komeda, A. Ohbuchi. Weierstrass points with first non-gap four on a double covering of a hyperelliptic curve. *Serdica Math. J.* **30** (2004), 43–54.

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