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MULTIPLIERS ON A HILBERT SPACE OF FUNCTIONS ON $\mathbb{R}$

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Abstract. For a Hilbert space $H \subset L^1_{loc}(\mathbb{R})$ of functions on $\mathbb{R}$ we obtain a representation theorem for the multipliers M commuting with the shift operator S. This generalizes the classical result for multipliers in $L^2(\mathbb{R})$ as well as our previous result for multipliers in weighted space $L^2_\omega(\mathbb{R})$. Moreover, we obtain a description of the spectrum of S.

1. Introduction. Let $H \subset L^1_{loc}(\mathbb{R})$ be a Hilbert space of functions on $\mathbb{R}$ with values in $\mathbb{C}$. Denote by $\|\cdot\|$ (resp. $\langle\cdot,\cdot\rangle$) the norm (resp. the scalar product) on H. Let $C_c(\mathbb{R})$ be the set of continuous functions on $\mathbb{R}$ with compact support. For a compact K of $\mathbb{R}$ denote by $C_K(\mathbb{R})$ the subset of functions of $C_c(\mathbb{R})$ with support in K and denote by $\hat{f}$ or by $\mathcal{F}(f)$ the usual Fourier transform of $f \in L^2(\mathbb{R})$. Let S_x be the operator of translation by x defined on H by

$$(S_x f)(t) = f(t-x), \ a.e.\ t \in \mathbb{R}.$$

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Let S (resp. S^{-1}) be the translation by 1 (resp. -1). Introduce the set

$$\Omega = \Big\{ z \in \mathbb{C},\ -\ln \rho(S^{-1}) \le \operatorname{Im} z \le \ln \rho(S) \Big\},$$

where $\rho(A)$ is the spectral radius of A and let I be the interval $[-\ln \rho(S^{-1}), \ln \rho(S)]$. Assuming the identity map $i : H \longrightarrow L^1_{loc}(\mathbb{R})$ continuous, it follows from the closed graph theorem that if $S_x(H) \subset H$, for $x \in \mathbb{R}$, then the operator S_x is bounded from H into H. In this paper we suppose that H satisfies the following conditions:

(H1) $C_c(\mathbb{R}) \subset H \subset L^1_{loc}(\mathbb{R})$, with continuous inclusions, and $C_c(\mathbb{R})$ is dense in H.

(H2) For every $x \in \mathbb{R}$, $S_x(H) \subset H$ and $\sup_{x \in K} \|S_x\| < +\infty$, for every compact set $K \subset \mathbb{R}$.

(H3) For every $\alpha \in \mathbb{R}$ let T_α be the operator defined by

$$T_\alpha : H \ni f(x) \longrightarrow f(x)e^{i\alpha x},\ x \in \mathbb{R}.$$

We have $T_\alpha(H) \subset H$ and, moreover, $\sup_{\alpha \in \mathbb{R}} \|T_\alpha\| < +\infty$.

(H4) There exists $C > 0$ and $a \ge 0$ such that $\|S_x\| \le Ce^{a|x|}$, $\forall\, x \in \mathbb{R}$.

Set $|||f||| = \sup_{\alpha \in \mathbb{R}} \|T_\alpha f\|$, for $f \in H$. The norm $||| \cdot |||$ is equivalent to the norm of H and without loss of generality, we can consider below that T_α is an isometry on H for every $\alpha \in \mathbb{R}$. Obviously, the condition $(H3)$ holds for a very large class of Hilbert spaces.

We give some examples of Hilbert spaces satisfying our hypothesis.

Example 1. A weight ω on $\mathbb{R}$ is a non negative function on $\mathbb{R}$ such that

$$\sup_{x \in \mathbb{R}} \frac{\omega(x+y)}{\omega(x)} < +\infty,\ \forall\, y \in \mathbb{R}.$$

Denote by $L^2_\omega(\mathbb{R})$ the space of measurable functions on $\mathbb{R}$ such that

$$\int_{\mathbb{R}} |f(x)|^2 \omega(x)^2 dx < +\infty.$$

The space $L^2_\omega(\mathbb{R})$ equipped with the norm

$$\|f\| = \Big(\int_{\mathbb{R}} |f(x)|^2 \omega(x)^2 dx \Big)^{\frac{1}{2}}$$

is a Hilbert space satisfying our conditions (H1)–(H3). Moreover, we have the estimate

$$\|S_t\| \le Ce^{m|t|},\ \forall\, t \in \mathbb{R}, \tag{1.1}$$

where $C > 0$ and $m \ge 0$ are constants. This follows from the fact that ω is equivalent to the special weight ω_0 constructed in [1]. The details of the construction of ω_0 are given in [6], [1]. Below after Theorem 2 we give some examples of weights.

Definition 1. *A bounded operator M on H is called a multiplier if*

$$MS_x = S_x M,\ \forall\, x \in \mathbb{R}.$$

Denote by $\mathcal{M}$ the algebra of the multipliers. Our aim is to obtain a representation theorem for multipliers on H and to characterize the spectrum of S. These two problems are closely related. In [6] we have obtained a representation theorem for multipliers on $L^2_\omega(\mathbb{R})$. Here we generalize our result for multipliers on a Hilbert space and shift operators satisfying the conditions (H1)–(H4). Our proof is shorter than that in [6]. The main improvement is based on an application of the link between the spectrum $\sigma(S_t)$ of a element of the group $(S_t)_{t\in\mathbb{R}}$ and the spectrum $\sigma(A)$ of the generator A of this group. In general, in the setup we deal with the spectral mapping theorem

$$\sigma(S_t) \setminus \{0\} = e^{\sigma(tA)}$$

is not true. To establish the crucial estimate in Theorem 4 we use the general results (see [3] and [5]) for the characterization of the spectrum of S_t by the behavior of the resolvent of A. This idea has been used in [8] for $L^2_\omega(\mathbb{R})$ but one point in our argument needs a more precise proof and in this paper we do this in the general case.

Denote by $(f)_a$ the function

$$\mathbb{R} \ni x \longrightarrow f(x)e^{ax}.$$

We prove the following

Theorem 1. *For every $M \in \mathcal{M}$, and for every*

$$a \in I = [-\ln \rho(S^{-1}), \ln \rho(S)],$$

we have

1) $(Mf)_a \in L^2(\mathbb{R}),\ \forall\, f \in C_c(\mathbb{R})$.
2) *There exists $\mu_{(a)} \in L^\infty(\mathbb{R})$ such that*

$$\int_{\mathbb{R}} (Mf)(x)e^{ax}e^{-itx}dx = \mu_{(a)}(t)\int_{\mathbb{R}} f(x)e^{ax}e^{-itx}dx, \ a.e.$$

i.e.

$$\widehat{(Mf)_a} = \mu_{(a)}\widehat{(f)_a}.$$

3) *If* $\overset{\circ}{I} \neq \emptyset$ *then the function* $\mu(z) = \mu_{(\operatorname{Im} z)}(\operatorname{Re} z)$ *is holomorphic on* $\overset{\circ}{\Omega}$.

Definition 2. *Given* $M \in \mathcal{M}$, *if* $\overset{\circ}{\Omega} \neq \emptyset$, *we call symbol of* M *the function* μ *defined by*

$$\mu(z) = \mu_{(\operatorname{Im} z)}(\operatorname{Re} z), \ \forall\, z \in \overset{\circ}{\Omega}.$$

Moreover, if $a = -\ln\rho(S^{-1})$ *or* $a = \ln\rho(S)$, *the symbol* μ *is defined for* $z = x+ia$ *by the same formula for almost all* $x \in \mathbb{R}$.

Denote by $\sigma(A)$ the spectrum of the operator A. From Theorem 1 we deduce the following interesting spectral result.

Theorem 2. *We have*

$$\sigma(S) = \Big\{z \in \mathbb{C} : \ \frac{1}{\rho(S^{-1})} \leq |z| \leq \rho(S)\Big\}.$$

To prove this characterization of the spectrum of S we exploit the existence of a symbol for every multiplier. Notice that in general S is not a normal operator and there are no spectral calculus which could characterize the spectrum of S. On the other hand, Theorem 2 has been used in [9] to obtain spectral mapping theorems for a class of multipliers. Now we give some examples of weights.

Example 2. The function $\omega(x) = e^x$ is a weight. For the associated weighted space $L^2_\omega(\mathbb{R})$ we obtain $\sigma(S) = \{z \in \mathbb{C}, \ |z| = e\}$.

Example 3. The functions of the form $\omega(x) = 1 + |x|^\alpha$, for $\alpha \in \mathbb{R}$ are weights and we get $\sigma(S) = \{z \in \mathbb{C}, \ |z| = 1\}$.

Example 4. Let $\omega(x) = e^{a|x|^b}$ with $a > 0$ and $0 < b < 1$. Then in $L^2_\omega(\mathbb{R})$ we have

$$\sigma(S) = \{z \in \mathbb{C}, \ e^{-a} \leq |z| \leq e^a\}.$$

Example 5. Functions like

$$e^{\frac{|x|}{\ln(2+|x|)}}, \quad e^{|x|}(1+|x|^2)^n, \ \text{for } n > 0$$

also are weights.

The weights in the Examples 4 and 5 are used to illustrate Beurling algebra theory (cf. [10]).

2. Proof of Theorem 1. For $\phi \in C_c(\mathbb{R})$ denote by M_ϕ the operator of convolution by ϕ on H. We have

$$(M_\phi f)(x) = \int_{\mathbb{R}} f(x-y)\phi(y)dy, \; \forall\, f \in H.$$

It is clear that M_ϕ is a multiplier on H for every $\phi \in C_c(\mathbb{R})$.

In [7] we proved the following

Theorem 3. *For every $M \in \mathcal{M}$, there exists a sequence $(\phi_n)_{n\in\mathbb{N}} \subset C_c(\mathbb{R})$ such that:*

i) $M = \lim_{n\to\infty} M_{\phi_n}$ with respect to the strong operator topology.

ii) We have $\|M_{\phi_n}\| \leq C\|M\|$, where C is a constant independent of M and n.

The main difficulty to establish Theorem 1 is the proof of an estimate for $\widehat{\phi_n}(z)$ for $z \in \Omega$ by the norm of M_{ϕ_n}.

Theorem 4. *For every $\phi \in C_c(\mathbb{R})$ and every $\alpha \in \Omega$ we have*

$$\Big| \int_{\mathbb{R}} \phi(x) e^{-i\alpha x} dx \Big| \leq \|M_\phi\|.$$

Theorem 1 is deduced from Theorem 3 and Theorem 4 following exactly the same arguments as in Section 3 of [6] and Section 3 of [7]. The function $\mu_{(a)}$ introduced in Theorem 1 is obtained as the limit of $(\widehat{(\phi_n)_a})_{n\in\mathbb{N}}$ with respect to the weak topology of $L^2(\mathbb{R})$. The reader could consult [6] and [7] for more details. Here we give a proof of Theorem 4 by using the link between the spectrum of S and the spectrum of the generator A of the group $(S_t)_{t\in\mathbb{R}}$.

P r o o f o f T h e o r e m 4. Let $\lambda \in \mathbb{C}$ be such that $e^\lambda \in \sigma(S)$. First we show that there exists a sequence $(n_k)_{k\in\mathbb{N}}$ of integers and a sequence $(f_{n_k})_{k\in\mathbb{N}}$ of functions of H such that

$$\Big\| \Big(e^{tA} - e^{(\lambda + 2\pi i n_k)t} \Big) f_{n_k} \Big\| \longrightarrow 0, \; n_k \to \infty, \;\; \|f_{n_k}\| = 1, \; \forall\, k \in \mathbb{N}. \tag{2.1}$$

Let A be the generator of the group $(S_t)_{t\in\mathbb{R}}$. We have to deal with two cases:

(i) $\lambda \in \sigma(A)$,

(ii) $\lambda \notin \sigma(A)$.

In the case (i) we have $\lambda \in \sigma_p(A) \cup \sigma_c(A) \cup \sigma_r(A)$, where $\sigma_p(A)$ is the point spectrum, $\sigma_c(A)$ is the continuous spectrum and $\sigma_r(A)$ is the residual spectrum of A. If we have

$$\lambda \in \sigma_p(A) \cup \sigma_c(A),$$

it is easy to see that there exists a sequence $(f_m)_{m\in\mathbb{N}} \subset H$ such that

$$\|(A-\lambda)f_m\| \underset{m\to+\infty}{\longrightarrow} 0,\ \|f_m\| = 1,\ \forall\, m \in \mathbb{N}.$$

Then the equality

$$(e^{At} - e^{\lambda t})f_m = \Big(\int_0^t e^{\lambda(t-s)} e^{As} ds\Big)(A-\lambda)f_m,$$

yields

$$\|(e^{At} - e^{\lambda t})f_m\| \underset{m\to+\infty}{\longrightarrow} 0,\ \forall\, t \in \mathbb{R}$$

and we obtain (2.1). If $\lambda \notin \sigma_p(A) \cup \sigma_c(A)$, we have $\lambda \in \sigma_r(A)$ and

$$\overline{Ran(A-\lambda I)} \neq H,$$

where $Ran(A-\lambda I)$ denotes the range of the operator $A - \lambda I$. Therefore there exists $h \in D(A^*)$, $\|h\| = 1$, such that

$$\langle f, (A^* - \overline{\lambda})h\rangle = 0,\ \forall\, f \in D(A).$$

This implies $(A^* - \overline{\lambda})h = 0$ and we take $f = h$. Then

$$\langle (e^{At} - e^{\lambda t})f, f\rangle = \langle f, (e^{A^*t} - e^{\overline{\lambda}t})f\rangle = \left\langle f, \left(\int_0^t e^{\overline{\lambda}(t-s)} e^{A^*s} ds\right)(A^* - \overline{\lambda})f\right\rangle = 0.$$

In this case we set $n_k = k$ and

$$f_k = f,\ \forall\, k \in \mathbb{N}$$

and we get again (2.1).

The case (ii) is more difficult since if $\lambda \notin \sigma(A)$, we have $e^{\lambda} \in \sigma(e^A) \setminus e^{\sigma(A)}$.

Taking into account the results about the spectrum of a semi-group in Hilbert space [5] satisfying the condition $(H4)$ (see also [3] for the contraction semi-groups), we deduce that there exists a sequence of integers n_k, such that $|n_k| \to \infty$ and

$$\|(A - (\lambda + 2\pi i n_k)I)^{-1}\| \geq k,\ \forall\, k \in \mathbb{N}.$$

Let $(g_{n_k})_{k\in\mathbb{N}}$ be a sequence such that

$$\|g_{n_k}\| = 1,\ \left\|\Big((A-(\lambda+2\pi i n_k)I)^{-1}\Big)g_{n_k}\right\| \geq k/2,\ \forall\, k\in\mathbb{N}.$$

We define

$$f_{n_k} = \frac{\Big((A-(\lambda+2\pi i n_k)I)^{-1}\Big)g_{n_k}}{\left\|\Big((A-(\lambda+2\pi i n_k)I)^{-1}\Big)g_{n_k}\right\|}.$$

Then we obtain

$$\Big(e^{tA}-e^{(\lambda+2\pi i n_k)t}\Big)f_{n_k} = \int_0^t e^{(\lambda+2\pi i n_k)(t-s)}e^{sA}ds\Big(A-(\lambda+2\pi i n_k)\Big)f_{n_k}$$

and for every t we deduce

$$\lim_{k\to+\infty}\left\|\Big(e^{tA}-e^{(\lambda+2\pi i n_k)t}\Big)f_{n_k}\right\| = 0.$$

Thus is established (2.1) for every λ such that $e^{\lambda}\in\sigma(S)$.

Now consider

$$\begin{aligned}\hat{\phi}(-i\lambda) &= \int_{\mathbb{R}}\left\langle\phi(t)\Big(e^{(\lambda+2\pi i n_k)t}-e^{tA}\Big)f_{n_k}, e^{2\pi i n_k t}f_{n_k}\right\rangle dt\\ &\quad+\int_{\mathbb{R}}\langle\phi(t)e^{tA}f_{n_k}, e^{2\pi i n_k t}f_{n_k}\rangle\,dt\\ &= J_{n_k}+\int_{\mathbb{R}}\langle\phi(t)e^{tA}f_{n_k}, e^{2\pi i n_k t}f_{n_k}\rangle\,dt,\end{aligned}$$

where $J_{n_k}\to 0$ as $n_k\to\infty$. On the other hand, we have

$$\begin{aligned}I_{n_k} &= \int_{\mathbb{R}}\langle\phi(t)e^{tA}f_{n_k}, e^{2\pi i n_k t}f_{n_k}\rangle\,dt = \left\langle\left[\int_{\mathbb{R}}\phi(t)e^{-2\pi i n_k t}f_{n_k}(.-t)dt\right], f_{n_k}\right\rangle\\ &= \left\langle\int_{\mathbb{R}}\phi(.-y)e^{-2\pi i n_k(.-y)}f_{n_k}(y)dy, f_{n_k}\right\rangle = \left\langle\Big(M_\phi(f_{n_k}e^{2\pi i n_k\cdot})\Big), e^{2\pi i n_k\cdot}f_{n_k}\right\rangle\end{aligned}$$

and $|I_{n_k}|\leq\|M_\phi\|$. Consequently, we deduce that

$$|\hat{\phi}(-i\lambda)|\leq\|M_\phi\|.$$

Next a similar argument yields

$$|\hat{\phi}(-i\lambda - a)| \leq \|M_\phi\|, \; \forall\, a \in \mathbb{R}. \tag{2.2}$$

In fact, if for $t \in \mathbb{R}$ there exists a sequence $(h_n)_{n\in\mathbb{N}} \subset H$ such that $(e^{tA} - e^{\lambda t})h_n \to 0$ as $n \to \infty$ with $\|h_n\| = 1$, we consider

$$\int_{\mathbb{R}} \left\langle (\phi(t)(e^{\lambda t} - e^{At}))h_n, e^{-iat}h_n \right\rangle dt = \hat{\phi}(-i\lambda - a) - \left\langle \int_{\mathbb{R}} \phi(t)e^{iat}e^{tA}h_n dt, h_n \right\rangle.$$

The term on the left goes to 0 as $n \to \infty$, so it is sufficient to show that the second term on the right is bounded by $\|M_\phi\|$. We have

$$\left(\int_{\mathbb{R}} \phi(t)e^{iat}e^{tA}h_n dt\right)(x) = \int_{\mathbb{R}} \phi(t)e^{iat}h_n(x-t)dt$$

$$= \int_{\mathbb{R}} \phi(x-y)e^{ia(x-y)}h_n(y)dy = e^{iax}[M_\phi(e^{-ai\cdot}h_n)](x), \; a.e.$$

and we obtain

$$|\hat{\phi}(-i\lambda - a)| \leq \|M_\phi\|.$$

Next consider the second case when we have a sequence $(f_{n_k})_{k\in\mathbb{N}}$ with the properties above. Multiplying by $e^{i(2\pi n_k - a)t}f_{n_k}$, we obtain

$$\hat{\phi}(-i\lambda - a) = \int_{\mathbb{R}} \left\langle \phi(t)e^{tA}f_{n_k}, e^{i(2\pi n_k - a)t}f_{n_k} \right\rangle dt + I_{n_k},$$

where $I_{n_k} \to 0$ as $n_k \to \infty$. To examine the integral on the right, we apply the same argument as above, using the fact that $(2\pi n_k - a) \in \mathbb{R}$. This completes the proof of (2.2). The property (2.2) implies that if for some $\lambda_0 \in \mathbb{C}$ we have

$$|\hat{\phi}(\lambda_0)| \leq \|M_\phi\|,$$

then

$$|\hat{\phi}(\lambda)| \leq \|M_\phi\|, \; \forall\, \lambda \in \mathbb{C}, \; s.t. \; \operatorname{Im}\lambda = \operatorname{Im}\lambda_0.$$

There exists $\alpha_0 \in \sigma(S)$ such that $|\alpha_0| = \rho(S)$. Then we obtain that

$$|\widehat{\phi}(z)| \leq \|M_\phi\|,$$

for every z such that $\operatorname{Im} z = \ln \rho(S)$. In the same way there exists $\eta \in \sigma(S^{-1})$ such that $|\eta| = \rho(S^{-1})$ and $\alpha_1 = \dfrac{1}{\eta} \in \sigma(S)$. Then applying the above argument to α_1, we get

$$|\widehat{\phi}(z)| \le \|M_\phi\|,$$

for every z such that $\operatorname{Im} z = -\ln \rho(S^{-1})$. Since $\phi \in C_c(\mathbb{R})$ we have

$$|\hat{\phi}(z)| \le C\|\phi\|_\infty e^{k|\operatorname{Im} z|} \le K\|\phi\|_\infty, \ \forall\, z \in \Omega,$$

where $C > 0$, $k > 0$ and $K > 0$ are constants. An application of the Phragmen-Lindelöff theorem for the holomorphic function $\widehat{\phi}(z)$ yields

$$|\widehat{\phi}(\alpha)| \le \|M_\phi\|$$

for all $\alpha \in \Omega$. □

Now we pass to the proof of Theorem 2. It is based on Theorem 1 combined with the arguments in [9] to cover our more general case. For the convenience of the reader we give the details.

P r o o f o f T h e o r e m 2. Let $\alpha \in \mathbb{C}$ be such that $e^\alpha \notin \sigma(S)$. Then it is clear that $T = (S - e^\alpha I)^{-1}$ is a multiplier. Let $a \in [-\ln \rho(S^{-1}), \ln \rho(S)]$. Then there exists $\nu_{(a)} \in L^\infty(\mathbb{R})$ such that

$$\widehat{(Tf)_a} = \nu_{(a)} \widehat{(f)_a}, \ \forall\, f \in C_c(\mathbb{R}), \ a.e.$$

For $g \in C_c(\mathbb{R})$, the function $(S - e^\alpha I)g$ is also in $C_c(\mathbb{R})$. Replacing f by $(S - e^\alpha I)g$, for $g \in C_c(\mathbb{R})$ we get

$$\widehat{(g)_a}(x) = \nu_{(a)}(x)\mathcal{F}\Big([(S - e^\alpha I)g]_a\Big)(x), \ \forall\, g \in C_c(\mathbb{R}), \ a.e.$$

and

$$\widehat{(g)_a}(x) = \nu_{(a)}(x)\widehat{g_{(a)}}(x)[e^{a-ix} - e^\alpha], \ \forall\, g \in C_c(\mathbb{R}), \ a.e.$$

Choosing a suitable $g \in C_c(\mathbb{R})$, we have

$$\nu_{(a)}(x)(e^{a-ix} - e^\alpha) = 1, \ a.e.$$

On the other hand, $\nu_{(a)} \in L^\infty(\mathbb{R})$. Thus we obtain that $\operatorname{Re} \alpha \neq a$ and we conclude that

$$e^{a+ib} \in \sigma(S), \ \forall\, b \in \mathbb{R}.$$

Since S is invertible, it is obvious that

$$\sigma(S) \subset \{z \in \mathbb{C},\ \frac{1}{\rho(S^{-1})} \leq |z| \leq \rho(S)\},$$

Consequently, we obtain

$$\sigma(S) = \{z \in \mathbb{C},\ \frac{1}{\rho(S^{-1})} \leq |z| \leq \rho(S)\}$$

and this completes the proof. □

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