# INQUIRY-BASED LEARNING OF REAL FINANCIAL SITUATIONS IN MIDDLE SCHOOL MATH CLASSES 

Iordanka Gortcheva<br>Institute of Mathematics and Informatics at the Bulgarian Academy of Sciences gorcheva@yahoo.com


#### Abstract

Studying the finances related topic of interest in middle school math classes lays the fundamentals of students' financial literacy and skills. Since banks already issue children's and teenagers' debit cards, word problems which require sticking to pre-defined financial limits are of special importance. They prepare the students to make adequate financial decisions and build a bridge between school mathematics and real world.


Key words: Word problems, Checking account, Principal, Interest rate, Negative interest rate

Money should be the solution and not the problem.
Gustave Flaubert (1821-1880)

## 1. Introduction

Recently many banking institutions started issuing children's debit cards affiliated with their parents' checking accounts. Knowledge of the basic mechanisms of running such accounts thus becomes part of the financial literacy of parents and children alike.

## 2. Problem formulation

Inquiry-oriented word problems with realistic numerical data teach the students not only to manage their pocket money but also to spend it responsibly [1]. Therefore, as a source of information such problems should be periodically updated. The problem situation that follows uses annual percentage yield (APY) which is about thirty to fifty times less than the levels of the bank interest used in textbooks [2-4]. Such a close-to-real-world value allows the students to work in more practical environment which requires not only knowledge of percentage, simple and compound interest, but also of taxes and bank fees.

Problem situation. Mr. Beem has a checking account at $\mathbf{0 . 1 \%}$ APY. It costs him a $\$ 3.25$ maintenance fee which is automatically withdrawn at the end of the month. Every day of the financial year (which for the bank is 360 days) the money in the account is subject to simple interest. The interest is added to the account at the end of the financial year after being charged $8 \%$ tax.

In the beginning of 2016 there were $\$ 1,000$ in Mr. Beem's checking account.
a) Calculate the account balance at the end of each financial month of 2016 and the percentage of Mr. Beem's profit or loss at the end of the year compared to its beginning. Show your results in a table.
b) Calculate the account balance at the end of each year from 2016 to 2027 if no money is added to the account and every year on January $1^{\text {st }}$ the bank raises its maintenance fee by $15 \%$ compared to the previous year. Show your results in a table.
c) Under the conditions above, when will Mr. Beem not be able to maintain his checking account anymore?

## 3. Inquiry-based learning

In textbook writing, numerical data included in word problems is often chosen for the sake of calculation simplicity. With the means of information technologies (IT), this is no longer necessary. Spreadsheets decrease the burden of calculations, help the students focus on the algorithm, and provide a neat representation of the results as suggested in [5].

The formulation of the Problem situation shows the students not only the idea of simple interest depending on the daily value of the principal but also requires them to evaluate Mr. Beem's checking account as a kind of financial instrument. This helps them learn how to calculate profit and loss, taxes and fees, etc. (Table 1).

Table 1. The principal in Mr. Beem's account throughout financial year 2016
(all amounts of money are in dollars)


| 1 | 3.25 |
| :---: | :---: |
| 2 | 3.25 |
| 3 | 3.25 |
| 4 | 3.25 |
| 5 | 3.25 |
| 6 | 3.25 |
| 7 | 3.25 |
| 8 | 3.25 |
| 9 | 3.25 |
| 10 | 3.25 |
| 11 | 3.25 |
| 12 | 3.25 |


| End Value: |
| ---: |
| Percentage: |



| $1,000.00$ | 0.08 |
| :---: | :---: |
| 996.75 | 0.08 |
| 993.50 | 0.08 |
| 990.25 | 0.08 |
| 987.00 | 0.08 |
| 983.75 | 0.08 |
| 980.50 | 0.08 |
| 977.25 | 0.08 |
| 974.00 | 0.08 |
| 970.75 | 0.08 |
| 967.50 | 0.08 |
| 964.25 | 0.08 |
| 961.88 | 0.88 |
| $-3.81 \%$ |  |


| Principal <br> for the Month | Interest <br> for the Month |
| :---: | :---: |


| $10,000.00$ | 0.83 |
| :---: | :---: |
| $9,996.75$ | 0.83 |
| $9,993.50$ | 0.83 |
| $9,990.25$ | 0.83 |
| $9,987.00$ | 0.83 |
| $9,983.75$ | 0.83 |
| $9,980.50$ | 0.83 |
| $9,977.25$ | 0.83 |
| $9,974.00$ | 0.83 |
| $9,970.75$ | 0.83 |
| $9,967.50$ | 0.83 |
| $9,964.25$ | 0.83 |
| $9,970.16$ | 9.16 |
| $-0.30 \%$ |  |

Using a spreadsheet for mathematical modelling of the situation provides a convenient tool for inquiry-based learning. By changing only one entry of the table at
a time, the students can see how Mr. Beem's loss of money can be turned into profit (Table 2). The moral of the story concerns the students' own bank accounts. They should stay informed about the bank fees and be aware of their checking balances.

Table 2. An appropriate principal can turn financial loss to profit (all amounts of money are in dollars)

| $2016$ <br> Month | 2016 Monthly Fee | Principal for the Month | Interest for the Month | Principal for the Month | Interest for the Month |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 3.25 | 100,000.00 | 8.33 | 1,000,000.00 | 83.33 |
| 2 | 3.25 | 99,996.75 | 8.33 | 999,996.75 | 83.33 |
| 3 | 3.25 | 99,993.50 | 8.33 | 999,993.50 | 83.33 |
| 4 | 3.25 | 99,990.25 | 8.33 | 999,990.25 | 83.33 |
| 5 | 3.25 | 99,987.00 | 8.33 | 999,987.00 | 83.33 |
| 6 | 3.25 | 99,983.75 | 8.33 | 999,983.75 | 83.33 |
| 7 | 3.25 | 99,980.50 | 8.33 | 999,980.50 | 83.33 |
| 8 | 3.25 | 99,977.25 | 8.33 | 999,977.25 | 83.33 |
| 9 | 3.25 | 99,974.00 | 8.33 | 999,974.00 | 83.33 |
| 10 | 3.25 | 99,970.75 | 8.33 | 999,970.75 | 83.33 |
| 11 | 3.25 | 99,967.50 | 8.33 | 999,967.50 | 83.33 |
| 12 | 3.25 | 99,964.25 | 8.33 | 999,964.25 | 83.33 |
|  | End Value: | 100,052.96 | 91.96 | 1,000,880.96 | 919.96 |
|  | Percentage: | 0.05\% |  | 0.09\% |  |

Table 3. The status of Mr. Beem's account through several successive years (all amounts of money are in dollars)

| Financial <br> Year | Monthly <br> Fee |
| :---: | :---: |
| 2016 | 3.25 |
| 2017 | 3.74 |
| 2018 | 4.30 |
| 2019 | 4.95 |
| 2020 | 5.69 |
| 2021 | 6.54 |
| 2022 | 7.52 |
| 2023 | 8.65 |
| 2024 | 9.95 |
| 2025 | 11.44 |
| 2026 | 13.16 |
| 2027 | 15.13 |


| Balance on <br> January $1^{\text {st }}$ | Annual <br> Change |
| :---: | :---: |
| $1,000.00$ | $-3.81 \%$ |
| 961.88 | $-4.57 \%$ |
| 917.89 | $-5.53 \%$ |
| 867.11 | $-6.75 \%$ |
| 808.60 | $-8.34 \%$ |
| 741.16 | $-10.50 \%$ |
| 663.34 | $-13.52 \%$ |
| 573.66 | $-18.01 \%$ |
| 470.34 | $-25.28 \%$ |
| 351.44 | $-38.95 \%$ |
| 214.55 | $-73.48 \%$ |
| 56.89 | $-79.73 \%$ |


| Balance on <br> January $1^{\text {st }}$ | Annual <br> Change |
| :---: | :---: |
| $10,000.00$ | $-0.30 \%$ |
| $9,970.16$ | $-0.36 \%$ |
| $9,934.45$ | $-0.43 \%$ |
| $9,891.98$ | $-0.51 \%$ |
| $9,841.75$ | $-0.60 \%$ |
| $9,782.64$ | $-0.71 \%$ |
| $9,713.11$ | $-0.84 \%$ |
| $9,631.78$ | $-0.99 \%$ |
| $9,536.80$ | $-1.16 \%$ |
| $9,426.24$ | $-1.37 \%$ |
| $9,297.70$ | $-1.61 \%$ |
| $9,148.37$ | $-1.89 \%$ |


| Balance on <br> January $1^{\text {st }}$ | Annual <br> Change |
| :---: | :---: | | $1,000,000.00$ | $0.0881 \%$ |
| :---: | :---: |
| $1,000,880.96$ | $0.0875 \%$ |
| $1,001,756.87$ | $0.0868 \%$ |
| $1,002,626.89$ | $0.0861 \%$ |
| $1,003,490.01$ | $0.0852 \%$ |
| $1,004,345.01$ | $0.0842 \%$ |
| $1,005,190.48$ | $0.0830 \%$ |
| $1,006,024.97$ | $0.0817 \%$ |
| $1,006,846.66$ | $0.0801 \%$ |
| $1,007,653.63$ | $0.0784 \%$ |
| $1,008,443.45$ | $0.0763 \%$ |
| $1,009,213.35$ | $0.0740 \%$ |

Inquiry-based learning through the spreadsheet shows that an amount of money which is a thousand times greater than Mr. Beem's initial balance makes the account profitable despite the annual exponential growth of the maintenance fee. Even more, the percentage of profit is reasonably close to the bank's APY (Table 3).

## 4. Dealing with negative interest rate

The positive or negative annual percentage change of Mr. Beem's account balance designates the profit or loss from using it as a financial instrument. The Problem formulation can be developed even further: the numbers in it can be changed to introduce the students to a recent trend in European finances related to negative percentage rate, an idea already in use by the European Central Bank [6] and several national banks across Europe [7-9].

Let us suppose that since the beginning of 2016 Mr. Beem's bank had been applying negative interest rates to the money deposited and his account was subject to $-0.1 \%$ interest rate.

The spreadsheet used so far gives a pretty good idea how this negative value will affect Mr. Beem's account. Since it will be the bank which will receive revenue, the solution presumes that Mr. Beem should not pay tax on the interest (Table 4):

Table 4. A yearly effect of the negative interest rate
(all amounts of money are in dollars)


| 1 | 3.25 |
| :---: | :---: |
| 2 | 3.25 |
| 3 | 3.25 |
| 4 | 3.25 |
| 5 | 3.25 |
| 6 | 3.25 |
| 7 | 3.25 |
| 8 | 3.25 |
| 9 | 3.25 |
| 10 | 3.25 |
| 11 | 3.25 |
| 12 | 3.25 |


| End Value: |
| ---: |
| Percentage: |



| $1,000.00$ | -0.08 |
| :---: | :---: |
| 996.75 | -0.08 |
| 993.50 | -0.08 |
| 990.25 | -0.08 |
| 987.00 | -0.08 |
| 983.75 | -0.08 |
| 980.50 | -0.08 |
| 977.25 | -0.08 |
| 974.00 | -0.08 |
| 970.75 | -0.08 |
| 967.50 | -0.08 |
| 964.25 | -0.08 |


| 960.04 | -0.96 |
| :---: | :---: |
| $-4.00 \%$ |  |


| Principal <br> for the Month | Interest <br> for the Month |
| :---: | :---: |


| $1,000,000.00$ | -83.33 |
| :---: | :---: |
| $999,996.75$ | -83.33 |
| $999,993.50$ | -83.33 |
| $999,990.25$ | -83.33 |
| $999,987.00$ | -83.33 |
| $999,983.75$ | -83.33 |
| $999,980.50$ | -83.33 |
| $999,977.25$ | -83.33 |
| $999,974.00$ | -83.33 |
| $999,970.75$ | -83.33 |
| $999,967.50$ | -83.33 |
| $999,964.25$ | -83.33 |


| $998,961.04$ | -999.96 |
| :---: | :---: |
| $-0.10 \%$ |  |

## 5. Discussions

Situations like the one described above are still hypothetical for the national bank system. Their discussion aims to provide the middle school teachers with an additional resource for developing their students' mathematical creativity. They help students mature financially, develop adaptiveness to real-life financial environment, and devise rational financial strategies.

Some students can interpret the negative interest rate as a kind of maintenance fee which the banks charge to keep money safe. However, as Table 4 shows, the amount of the interest depends on the amount of the principal, while the amount of maintenance fee does not.

Table 4 conveys another message to middle school students concerning the business applications of their knowledge. They need to decide what would be more profitable in this situation: to keep money in bank accounts or cash in deposit boxes, paying the respective bank fees for the service, or invest it. These types of decisions broaden their interests beyond the mere calculation of percentages and introduce them to the ideas of monetary policy. The level of students' mathematical knowledge and skills should not be an obstacle to inquiry-based learning: according to [10], "In the beginning there are never rules or formulas."

## 6. Concluding remarks

Inquiry-based learning develops middle school students' critical outlook on reallife situations such as those related to finances and banking. Together with personal financial skills, it ensures their responsible societal attitude to the financial system thus contributing to its stability.

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# ИЗСЛЕДОВАТЕЛСКИ ПОДХОД ПРИ ИЗУЧАВАНЕ НА РЕАЛНИ ФИНАНСОВИ СИТУАЦИИ В ЧАСОВЕТЕ ПО МАТЕМАТИКА В ПРОГИМНАЗИАЛЕН КУРС 

## Йорданка Горчева

Институт по математика и информатика към БАН
gorcheva@yahoo.com
Резюме: Темата за лихва в часовете по математика в прогимназиален курс е основополагаща за финансовата грамотност и умения на учениците. Тъй като банките у нас вече издават и детски дебитни карти, текстови задачи за лихва, предполагащи съобразяване с определени финансови ограничения, стават все поактуални. Те подготвят учениците за вземане на адекватни финансови решения и изграждат мост между училищната математика и реалния живот.

