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ИНСТИТУТ ПО МАТЕМАТИКА С ИЗЧИСЛИТЕЛЕН ЦЕНТЪР
INSTITUTE OF MATHEMATICS WITH COMPUTER CENTER

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ABSTRACTS

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Contents

Integer Programming Approach to a Linear Discriminant Problem

St. Balev [1] and N. Yanev [1]

Here we study the two-group discriminant problem, stated as: for a set of n positive (P) and negative (N) examples $x_i \in \mathbb{R}^m$ find a hyperplane with normal vector h and shift α s.t. $|x_i \in P : h\,x_i < \alpha| + |x_i \in N : h\,x_i > \alpha|$ is minimal. This problem is transformed to the following equivalent form: Find minimal set I of variables, such that the system $Ax = b$, $x \geq 0$ becomes infeasible when $x_i = 0$, $i \in I$ ($A \in \mathbb{R}^{m \times n}$, $b \in \mathbb{R}^m$, $x \in \mathbb{R}^n$). An algorithm is developed,which finds the optimal I after solving at most $m^{|I|}$ simple linear programs. A class of heuristics providing a fairly good approximation to the optimal solution is also developed. Some features of both exact and heuristic algorithms are considered.

Differential Inclusions Depending on Small Parameters

Tzanko Donchev [2]

Consider the differential inclusions, having the form:

$$\begin{pmatrix} \dot{x}(t) \\ \varepsilon \dot{y}(t) \end{pmatrix} \in F(t, x(t), y(t), \delta), \quad x(0) = x_0, \quad y(0) = y_0 \tag{1}$$

The real parameter $\varepsilon > 0$ represents the singular perturbation, while the parameter δ represents regular one. Here $x \in R^n$; $y \in R^m$; $F : I(:= [0,1]) \times R^n \times R^m \times R \Longrightarrow R^{n+m}$. The variable x is commonly called "slow" variable, while y is called "fast" variable. The solution set $Z(\varepsilon)$, $\varepsilon > 0$ of (1) consists of all absolutely continuous (AC) functions (x,y) satisfying (1) for a.e. $t \in I$. For $\varepsilon \longrightarrow +0$ is natural to mean all pairs (x, y) with $x \in AC$ and $y \in L^1(I, R^m)$ (the space of all Lebesgue integrable functions with values in R^m) satisfying the following "degenerate system" for a.e. $t \in I$.

$$\begin{pmatrix} \dot{x}(t) \\ 0 \end{pmatrix} \in F(t, x(t), y(t), \delta), \quad x(0) = x_0$$

We investigate the (semi)continuous dependence of the solution set $Z(\varepsilon, \delta)$ of (1). We present two results. The first is that the functions $Z(., \delta)$ and $Z(\varepsilon, .)$ are lower semicontinuous and the second that they are upper semicontinuous. The proof about upper semi-continuity of $Z(., \delta)$ is compicated as well as the generalized solution is used.

The most of the results are obtained in collaboration with Iordan Slavov.

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On the Existence for Maximum Equations

Ljubomir Georgiev [1] **and Vasil Angelov** [1]

The paper is devoted to the existence problem for maximum equations:

$$x'(t) = f(t, x(t), \max \|x(s)\| : g(t) \leq s \leq t), \quad t > 0$$

$$x(t) = \varphi(t), \quad t \leq 0,$$

where $x(t) \in c(R^1; R^n)$ is the unknown function. By means of a fixed point theorem, globally continuous and essentially bounded solutions are obtained.

The above mentioned theorems are applied to the following equation arising in the electrotechnics:

$$LI'(t) + MI_{\max} = V_h(I(t)), \; t > 0$$

$$I(t) = \varphi(t), \; t \leq 0.$$

In the above equation $I(t)$ is an unknown current function and

$$I_{\max} = \max |I(s)| : g(t) \leq s \leq t.$$

Variational Principles for Minimax Problems

Pando Gr. Georgiev [2]

We obtain the following extension of Ekeland's variational principle to minimax problems.

Theorem. Let E_1 and E_2 be Banach spaces, X and Y be closed subsets of E_1 and E_2 respectively, Y be convex, bounded with non-empty interior, $f : X \times Y \to R$ be a function with the following properties:

1) The functions $\{f(.,y) : y \in Y\}$ are equicontinuous,

2) $f(x,.)$ is continuous and concave for every $x \in X$,

3) $\inf\limits_{x \in X} \sup\limits_{y \in Y} f(x,y)$ is finite.

Let $\varepsilon_1, \varepsilon_2, \lambda_1, \lambda_2 > 0$ be given and $x_0 \in X$ and $y_0 \in Y$ be such that:

4) $\sup\limits_{y \in Y} f(x_0, y) < \inf\limits_{x \in X} \sup\limits_{y \in Y} f(x,y) + \varepsilon_1$

5) $f(x_0, y_0) > \sup\limits_{y \in Y} f(x_0, y) - \varepsilon_2$.

Then there exist $\tilde{x} \in X$, $\tilde{y} \in Y$ such that

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6) $f_2(\tilde{x},\tilde{y}) = \sup_{y\in Y} f_2(\tilde{x},y) = \inf_{x\in X}\sup_{y\in Y} f_2(x,y)$, where
$f_2(x,y) = f(x,y) + \frac{\varepsilon_1}{\lambda_1}\|x-\tilde{x}\| - \frac{\varepsilon_2}{\lambda_2}\|y-\tilde{y}\|$, $\bar{\varepsilon}_1 = \varepsilon_1 + \frac{\varepsilon_2}{\lambda_2} diam Y$,
7) $\|\tilde{x}-x_0\| < \lambda_1$, $\|\tilde{y}-y_0\| < \lambda_2$.

An extension of the Deville-Godefroy-Zizler smooth variational principle is proved for minimax problems. Applications of the two variational principles to well posedness of minimax problems, to Mountain-pass type theorems and to Brondsted-Rockafellar's type theorem for saddle functions are obtained.

Duality of Planner Directed Networks

D. Ivanchev [1]

Some of the network optimization problems are easier to be solved if the graph is a planner one. If in addition it is an undirected one we can use the dual graph and reduce the problem on it. But what to do if the graph is directed? We define in an appropriate way duality of planner directed graphs and use this definition for finding the maximum flow in the network and to determine the most vital arcs and nodes.

Differential Inclusions with Upper Semicontinuous Right-Hand Side

R. P. Ivanov [2]

An existence theorem is proved for solutions of differential inclusions

$$\dot{x} \in F(t,x), \quad x(t_0) = x_0, \quad t \in [t_0, t_1]$$

with an upper semicontinuous and nonconvex right-hand side. The proof is based on an inner and directional continuous parameterization. This parameterization leads to a family of disturbed differential inclusions. The solution of the starting differential inclusion is obtained as an uniform limit of the solutions of disturbed systems. Some aspects of the existence of the above mentioned inner parameterization are discussed. A few examples are presented.

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An Iterative Method of Solving a Game with Locally Lipschitz Pay-off

Vsevolod Ivanov [1]

Let $\Gamma = \langle X, Y, f \rangle$, $X = [0,1]$, $Y = [0,1]$ be an antagonistic game over the unit square. The pay-off f is locally Lipschitz with respect to its two arguments, $f(.,y)$ is strictly generalized pseudo-concave for every $y \in Y$, and $f(x,.)$ is generalized pseudo-concave for every $x \in X$.

Let us denote by $\partial_x f(x,y)$ the partial generalized gradient of Clarke with respect to x of the function f in the point (x,y). We offer an algorithm, which solves the game by building a sequence of subgames of Γ:

1. Let x' be the middle of the interval X. We find y', so that the function $f(x',.)$ has a global minimal value at y'. Computation of $\partial_x f(x',y')$:

2.1. If $0 \in \partial_x f(x',y')$ we go to the end, (x',y') is a saddle point.

2.2. If $\partial_x f(x',y') \subset (0,\infty)$, then we take the set $x \in X : x \geq x'$ for the set of strategies of the first player.

2.3. If $\partial_x f(x',y') \subset (-\infty,0)$, then we take the set $x \in X : x \geq x'$ for the set of strategies of the first player.

3.1. If the length of the interval, which is the set of strategies of the first player, is greater than a given number ε, then we repeat steps 1 and 2.

3.2. Else we take an arbitrary point of this interval for approximation of the optimal strategy of the first player. We find an approximation of the optimal strategy of the second player.

Class of Quadratic Transportation Problems

Rumena Kaltinska [2]

The class of quadratic transportation problems

$$\text{minimize (maximize)} \quad F(X) = P(X)Q(X)$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1,2,\cdots,m,$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1,2,\cdots,n$$

$$x_{ij} \geq 0, \quad i = 1,2,\cdots,m, \quad j = 1,2,\cdots,n,$$

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where $P(X)$ and $Q(X)$ are affine functions of $X \in E^{m \times n}$, is considered.

The characteristics of the problems are studied and an algorithm is proposed. The known Quadratic Programming methods are not workable here, since the objective function is not convex (concave). It is proved that the optimal solutions lie on an edge of the constraint set. The algorithm reduces the solving of the problem to solving an one-parametric linear transportation problem combined with a line search on the edges of the constraint set. The objective function and the right sides of the restrictions depend on a parameter and the latter is needed to be changed only in the interval [0,1] during the solving. The line search is quite elementary since it uses explicit formulas.

Software based on the algorithm is briefly discussed.

Family of Interactive Optimization Systems

Rumena Kaltinska [1] and Tzvetan Petrov [1]

A *Family* of *Interactive Systems* intended to solve different classes of optimization problems (Linear, Fractional, Quadratic and Nonlinear Programming, Unconstrained and Box-constrained Optimization, Least-Squares Optimization, Transportation problems, etc.) on IBM PC is considered. The *Systems* support an unified user friendly interface environment, which is convenient for users with different professional and mathematical background and facilitates their efforts: once accustomed to a particular *System* they need minimal efforts to start working with another *System* from the *FAMILY*.

The interface of the *Systems* is properly oriented towards the classes of problems and included numerical methods. It is menu-driven and provides an easy-to-use Borland-like computer environment. Hot-key context sensitive help about the key definitions, options, classes of problems, methods and their parameters, etc., is supported.

All customary means to generate, edit, save, list or print problems are provided. Nonlinear functions are to be defined through their formulas. The computational process can be optionally visualized and controlled by the user: choose/change the method, change the values of its parameters, trace the computations, interrupt/continue the process, etc. In the nonlinear case, three-dimensional graphics and level curves maps of the functions are also supported in order functions extrema, as well as a proper initial points for the numerical methods, to be visually found. Numerical results can be optionally saved and are easily retrieved for further processing.

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Generic Well Posedness of SupInf Problems

P. S. Kenderov [1]

We consider two notions of well posedness for problems of the type $\sup_X \inf_Y f(x,y)$ and give conditions under which the majority (in the Baire category sense) of bounded functions f defined in $X \times Y$ give rise to problems which are well posed. As a corollary we get that the problem $\sup_X f(x)$ is well posed for the majority of bounded lsc real valued functions f if, and only if, X contains a dense completely metrizable subset.
This is a joint work with Prof. Roberto Lucchetti, Department of Mathematics, University of Milano.

An Approach for Generating Efficient Points through Reducing a Set of Normalized Coefficients

Leonid Kirilov [2]

An approach for sampling the efficient set is presented. It is based on the reducing of the set of normalized weighted coefficients. Additionally, the reduced set is moved according to the DM's preferences. Using so formed set of weights, a number of $k+1$ (k - number of the objective functions) points are generated by one iteration. As main advantages of the proposed approach, simplicity and flexibility can be pointed out its. A comparison with other related approaches is done.

Multidimensional Constructive Function Theory Based on the Elliptic Equations Solutions

Ognyan Kounchev [1]

All parts of the multivariate constructive theory of functions (Moment Problems - best approximation, Approximation Theory, Spline Theory) are built by using solutions of higher order elliptic equations. This is an alternative to the usual approach based on polynomials of several variables.

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Convex Curves in the Plane

Mariana Nedelcheva [1]

Denote by E the Euclidean plane and by $\mathcal{K}$ the class of the non-empty, convex and compact subsets of E. For a real φ put $\overrightarrow{e}_{\varphi} = (\cos\varphi, \sin\varphi)$. The concept of a convex curve is usually defined as the boundary of a convex compact subset of E with non-empty interior, see e. g. [2], but this definition seems to be restrictive in some applications. Each subset $\Gamma \subset E$ homeomorphic to a nondegenerate interval is said to be an arc. An arc with a linear order on it is said to be an oriented arc.

Definition 1 *We say that the oriented arc Γ_+ is a convex arc (with parameter ε) if there exists $\varepsilon > 0$ such that for arbitrary four points $c_1 < c_2 \le c_3 < c_4$ of Γ_+ the condition $0 \le \langle \overrightarrow{c_1c_2}, \overrightarrow{c_3c_4} \rangle \le \pi - \varepsilon$ is satisfied (here $\langle \cdot, \cdot \rangle$ denotes the oriented angle).*

Theorem 2 *Let Γ_+ be a bounded ε-convex arc. Then its closure $\bar{\Gamma}_+$ is a bounded ε-convex arc. For $c_1, c_2 \in \bar{\Gamma}_+$, $c_1 < c_2$, one can choose the real $\varphi(c_1, c_2)$ in such a way that $\overrightarrow{c_1c_2}/\|\overrightarrow{c_1c_2}\| = \overrightarrow{e}_{\varphi(c_1,c_2)}$ and the interval $[\varphi_a, \varphi_b]$, $\varphi_a = \inf\{\varphi(c_1, c_2) \mid c_1 < c_2, c_1, c_2 \in \bar{\Gamma}_+\}$, $\varphi_b = \sup\{\varphi(c_1, c_2) \mid c_1 < c_2, c_1, c_2 \in \bar{\Gamma}_+\}$, is of length not greater than $\pi - \varepsilon$. If $c \in \bar{\Gamma}_+$ is neither initial nor an end point put $\varphi_-(c) = \sup\{\varphi(c_1, c) \mid c_1 < c, c_1 \in \bar{\Gamma}_+\}$, $\varphi_+(c) = \inf\{\varphi(c, c_2) \mid c < c_2, c_2 \in \bar{\Gamma}_+\}$. If c is an initial or end point of $\bar{\Gamma}_+$ we put $\varphi_-(c) = \varphi_+(c)$.*

*For $\varphi \in [\varphi_a, \varphi_b]$ put $c_-(\varphi) = \inf\{c \in \bar{\Gamma}_+ \mid \varphi_+(c) \ge \varphi\}$, $c_+(\varphi) = \sup\{c \in \bar{\Gamma}_+ \mid \varphi_-(c) \le \varphi\}$ (*inf *and* sup *are taken with respect to the order of $\bar{\Gamma}_+$). Denote by s the natural parameter on $\bar{\Gamma}_+$ and introduce the functions: $s_- : [\varphi_a, \varphi_b] \longrightarrow R$, $s_-(\varphi) = s(c_-(\varphi))$, $s_+ : [\varphi_a, \varphi_b] \longrightarrow R$, $s_+(\varphi) = s(c_+(\varphi))$.*

Let $\varphi_0 \in [\varphi_a, \varphi_b]$. Then

a) *The following integral representation has place:*

$$\begin{aligned} c_+(\phi) &= c_+(\varphi_0) + \int_{\varphi_0}^{\phi} \overrightarrow{e}_{\varphi}\, ds_+(\varphi), \quad \phi \in [\varphi_a, \varphi_b], \\ c_-(\phi) &= c_-(\varphi_0) + \int_{\varphi_0}^{\phi} \overrightarrow{e}_{\varphi}\, ds_-(\varphi), \quad \phi \in [\varphi_a, \varphi_b] \end{aligned} \tag{1}$$

(the integrals are in the sense of Riemann-Stiltjes).

b) *The representation of Γ_+ with the natural parameter s is*

$$c = \begin{cases} c_-(\varphi), & s = s_-(\varphi) \\ c_+(\varphi), & s = s_+(\varphi) \\ c_+(\varphi)\frac{s_+(\varphi)-s}{s_+(\varphi)-s_-(\varphi)} + c_+(\varphi)\frac{s-s_-(\varphi)}{s_+(\varphi)-s_-(\varphi)}, & s_-(\varphi) < s < s_+(\varphi) \end{cases} \tag{2}$$

Theorem 2 remains true in the more general case of a convex curve, where

Definition 3 *We say that the compact oriented curve Γ_+ is a convex curve if each point of Γ_+ possesses a neighborhood being a convex arc.*

Using (1) and (2) we generalize some results of the work[3] (to be published later). The used here representation (1) is in a relation with results of M. Kallay [4] and R. Vitale [5].

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[2] I. M. Yaglom, V. G. Boltyanskij. *Convex Figures*. Moscow, Leningrad, 1951 (In Russian)

[3] M. D. Nedelcheva. *Characterization of convex subsets in the plane through their local approximation properties*. Serdica 11, no 2 (1985), 165–170

[4] M. Kallay. *Reconstruction of a plane convex body from the curvature of its boundary*. Israel J. Math. 17(1974), 149–161

[5] R. A. Vitale. *Support functions of plane convex sets* (Report). Claremont Graduate School 1979

Differential Inclusions with Finite Number of Impulses in Fixed Moments

V. A. Plotnikov [1], R. P. Ivanov [2], N. M. Kitanov [3]

The work considers the following differential inclusions with finite number of multi-valued impulses in fixed moments:

$$\dot{x} \in F(t,x), \quad t \neq \tau_i,$$

$$\Delta x|_{t=\tau_i} \in I(t,x), \quad i = 1,2,\ldots,k < \infty,$$

where $F(t,x)$ is a Caratheodory function and $x(\tau_i+0) = x(\tau_i-0)+\Delta x$. The paper presents an existence theorem for the Cauchy problem and a theorem about the connectivity of the integral funnel.

The work considers the existence and uniqueness of the so called R-solutions of differential inclusions for the multi-valued impulse Cauchy problem.

The paper presents sufficient conditions about continuous dependence of the solutions set from the initial conditions and from the right-hand side. The proof is based on the Filippov theorem.

A Numerical Realization of One Implicit Runge-Kutta Scheme

N. Pulova [4]

A numerical realization of Runge-Kutta implicit method for solving of nonlinear systems of differential equations

$$\frac{dy}{dx} = f(x,y), \; y(x_0) = y_0, \; x \geq x_0,$$

where $y, f \in R^n$ is presented. Especially, the following midpoint rule with stepsize h

$$y_1 = y_0 + kh$$

$$k = f\left(x_0 + \frac{h}{2}, \; y_0 + k\frac{h}{2}\right), \; k \in R^n$$

is used. In order to obtain the coefficient k, an iterative Newton method is applied.

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At first, the discreet approximation $y_0, y_1, \ldots, y_m$ is calculated. Then, starting from the final point (x_m, y_m) with the negative stepsize $-h$ the point $(x_0, \tilde{y}_0)$ is get too. The interest is fixed on estimating the distance between y_0 and $\tilde{y}_0$.

Using steps with different size, a test for several systems (including stiff ones) was made.

Optimization Problems and the Banach–Mazur Game

Julian Revalski [1]

The Banach-Mazur game involves two players, α and β, who alternatively choose (β starts) non-empty open subsets $U_1 \supset V_1 \supset U_2 \supset V_2 \ldots$ of a topological space X. The resulting sequence $\{U_n, V_n\}$ is called a play and α wins this play if $\bigcap_{n=1}^{\infty} V_n \neq \emptyset$. Otherwise β wins. Let $C(X)$ denote the Banach space of all bounded and continuous real-valued functions in X equipped with the usual sup-norm. It is shown that the existence of a winning strategy for the player α in the Banach-Mazur game in X is equivalent to the fact that the set $\{f \in C(X) :$ the function f attains its minimum in $X\}$ contains a dense G_δ-subset of $C(X)$. Also, strengthened forms of the winning strategies in the above game are shown to be equivalent to other generic properties of optimization problems in X.

Set-Valued Maps in Singularly Perturbed Optimal Control Problems

I. Slavov [2]

In a series of papers we study the continuity properties of some set-valued maps arising in singularly optimal control problems. These are the set of solutions of the control system, the attainable set and the set of admissible controls considered as functions of the singular parameter $\varepsilon > 0$. As a result we are able to explain more clear known effects like the discontinuity of the optimal value at $\varepsilon = 0^+$ in the Mayer's problem for linear systems, to derive new results concerning differentiability of the optimal value at $\varepsilon = 0^+$ etc. We pay more attention to the behavior of the set of solutions as $\varepsilon \to 0^+$ for more general differential systems, namely for differential inclusions. One recent result in this direction is for functional-differential inclusions with a singular perturbation. We apply it in investigation of the well-posedness of the Lagrange's problem for a nonlinear singularly perturbed integro-differential system with a time delay.

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A Differential Game Described by a Hyperbolic System

Diko Souroujon [1]

An antagonistic differential game of hyperbolic type with a separable linear vector pay-off function is considered. The main result is the description of all ε– Slater saddle points consisting of program strategies, ε– Slater maximins and minimaxes for each $\varepsilon \in IR^N >$ for this game. To this purpose, the considered differential game is reduced to find the optimal program strategies of two multicriterial problems of hyperbolic type. The application of approximation enables us to relate these problems to a problem of optimal program control, described by a system of ordinary differential equations, with a scalar pay-off function. It is found that the result of the game is not changed, if the players use positional or program strategies. For the considered differential game, it is interesting that the ε– Slater saddle points are not equivalent and there exists two ε– Slater saddle points for which the values of all components of the vector pay-off function at one of them are greater than the respective components of the other ε– saddle point.

An Approximate Algorithm for Steiner Tree Problem in Graphs

Tatjana Stancheva [2]

Let $G = [V, E]$ be a finite connected undirected graph with set of vertices V and set of edges E with weights on its edges. If W is a fixed subset of V find a connected subgraph T among all connected subgraphs of G with minimum weight spanning a set of the specified vertices W.

The algorithm is based on the shortest paths in the graph generated by the original one such that between every two vertices there is only one shortest path. The aim of the algorithm is to find the possible Steiner's vertices and to include them in the solution.

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Some Properties of Graphs which are Cartesian Product of Two Directed Cycles

Raina Tarandova [1]

Here some results and algorithms have been presented.

The Cartesian product $C_p \times C_{p+2}$ of two directed cycles has the properties: it is hypo-hamiltonian if $p = 4k + 3$; if any vertical edge of it is reversed then a Hamilton directed cycle is obtained; if any two horizontal edges of it are reversed then Hamilton directed cycle is also obtained.

The Cartesian product $C_p \times C_{rp+1}$ of two directed cycles is hypo-hamiltonian when: $r = 1$, or r and p are even, or r and p are odd.

The written algorithms are as follows: the algorithm for finding the longest directed cycle in $C_p \times C_{p+2}$ when $p = 4k + 3$; the algorithm for determining the longest directed cycle in $C_p \times C_{p+1}$ (this algorithm can be used for defining directed Hamilton cycle in $C_{2a} \times C_{2b}$, when g..c.d. $(a, b) = 1$); the algorithm for obtaining a directed Hamilton cycle in $C_{3a} \times C_{3b}$, when g.c.d. $(a, b) = 1$.

Stable and Unstable Sets for the Wave Equation with Nonlinear Damping and Source Terms

G. Todorova [2]

We study the interaction between the nonlinear damping and the nonlinear source term for the initial boundary value problem

$$\Box u + a u_t |u_t|^{m-1} = b u |u|^{p-1} \quad (t, x) \in [0, T] \times \Omega \tag{1}$$

$$u(0, x) = \varphi(x), \; u_t(0, x) = \psi(x) \tag{2}$$

$$u(t, x) = 0 \qquad (t, x) \in [0, T] \times \partial\Omega \tag{3}$$

Here $a, b > 0$, $p, m > 1$ and Ω is a bounded domain in R^n $n \geq 1$, with smooth boundary $\partial\Omega$.

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We separate a suitable domain for the parameters p, m where the damping term $au_t|u_t|^{m-1}$ dominates over the source $bu|u|^{p-1}$ and the global solution exists for any initial data.

Second, we determine another domain where the influence of the source is stronger and the solution blows up for finite time for suitable large initial data.

We determine the so called stable set W^* and unstable set V^* for the initial data. The behaviour of the solution for the Cauchy problem for the equation (1) depends on the set to which the initial data belong. If the initial data belong to the stable set W^* the global solution exists. If the initial data belong to the unstable set V^* the solution blows up for finite time.

A Coincidence Theorem for Densifying Mappings and Applications

Ts. Y. Tsachev [1] and V. G. Angelov [1]

A fixed point theorem for densifying mappings in locally convex linear topological spaces is proved. As a corollary a coincidence theorem for the same class of mappings is obtained. An application to the existence of generalized solutions (introduced recently by V. Angelov) of ordinary differential equations in Banach spaces is made.

Minimax Problem on Special Ordered Sets

Vl. Tzokov [2] and N. Yanev [2]

We study the following problem: let $a_i \in \mathbb{R}^m$ for $i = 1, \ldots, n$ (we assume without loss of generality that all components of a_i are non-negative for $i = 1, \ldots, n$) and let the set $N = \{a_1, \ldots, a_n\}$ is partitioned into k disjoint subsets $N_1, \ldots, N_k$. The problem is to choose $a'_j \in N_j$ for $j = 1, \ldots, k$ such that the greatest component of the vector $a = \sum_{j=1}^{k} a'_j$ is minimal.

We prove that the problem is NP-hard in strong sense even in the case when the vectors in each subset N_j, $j = 1, \ldots, k$ are cyclic permutations of a given vector. Moreover, the complexity of the problem is the same if in each subset all cyclic permutations are included.

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The Crisis of the Bulgarian Economy and its Analysis on the Basis of the Nonequilibrium and Catastrophe States of the Leontief's Balance Model

Ilia Tzvetanov [1]

It is well known that the Leontief's balance model was the classical theoretical scheme according which the annual and five year plans of all East-European socialist economies of centralized management were built. Hence it is logically to link this model theoretically to the investigation of the balance nonequilibrium, economic crisis and catastrophe of the Bulgarian economy. The stability, nonstability and catastrophe of the Leontief's balance states are defined taking into consideration the influence of the three types of errors: the "planning errors", the "statistical errors" and the "calculation errors". The present study proposes original transforming approach which gives possibilities to investigate and construct the solutions of the four problems: balance "stability", "nonextremal instability", "extremal instability of first degree" and "catastrophe" of the Leontief's model.

The theoretical basis of this approach is the principle of "logical gradient properties" of the Produced and Consumed National Incomes near to the balance equilibrium state.

The application aspect of this investigation is revealed in the following directions:

a) to prove the hypothesis that the cross-sectional balance of the Bulgarian economy was instable earlier than 1980-ies and it was the original cause that brought about the crisis; this instability generated disbalances and indicated the large deficits in the end of the 1990-ies;

b) to construct the criteria of quantitative evaluation of the instability, of the crisis depth and to measure distances to the theoretical limit – the balance catastrophe;

c) to determine those relations among the future investments into production capacities and the relative growth of the gross input under which the balance will restore its productivity (stability) and these conditions will contribute to get out of the crisis and the catastrophe.

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Densely Multivalued Metric Projections

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Let $(X, \|\cdot\|)$ be a Banach space. For a non-empty set $M \subset X$ the set-valued mapping $P : X \longrightarrow M$ defined by $P(x, M) = \{y \in M : \|x - y\| = d(x, M)\}$ with distance function $d(x, M) = \inf\{\|x - z\| : z \in M\}$ is the metric projection generated by M.

It is known that $P(\cdot, M)$ is single-valued at most points, i.e. on a dense subset of G_δ type, provided that X is strictly convex and M is a compact (Stechkin). Zamfirescu showed that $P(\cdot, M)$ might be multi-valued on a dense set, as well, and that this is the typical case in the Euclidean space $\mathbb{R}^n$ when $n \geq 2$: In the Hausdorff metric space of compacta $\mathcal{K}(\mathbb{R}^n)$ most compacta (in the sense of Baire category) generate metric projections which are multi-valued on dense sets. This result of Zamfirescu has been extended by De Blasi and Myjak to the case of separable strictly convex Banach space X, $\dim X \geq 2$, for various Hausdorff complete metric spaces of non-empty sets.

Suppose X is a uniformly convex Banach space. The following result holds true:

Theorem There is a dense G_δ set of bounded and closed sets $\mathcal{U}$ in the Hausdorff set topology in X, $\dim X \geq 2$, such that a metric projection generated by $M \in \mathcal{U}$ is two-valued and u.s.c. on a dense subset of X.

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