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VIBRATION OF DRY ICE PLACED ON A METALLIC STRUCTURE

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Vibration of a dry ice block placed on a metallic beam structure is analyzed under two steps, i.e. a sublimation step under high heat flux using one-dimensional linearized heat conduction equation neglecting gaseous flow and the following jump stage of solid dry ice due to high gas pressure. Eigenfrequency of the metallic structure can be detected experimentally due to crashing by the dry ice block.

1. Introduction

Sounds produced by a metallic object placed on a dry ice block are well-known phenomena and frequently reported as singing spoon [1], coin vibration [2], which is due to rapid increase of pressure over the metallic surface contacted through sublimation of dry ice according to very high heat flux from metal with high thermal conductivity. Under these phenomena, inverse of the average period of metal (spoon, coin) motion (up and down) to dry ice is sufficiently smaller than the lowest natural (eigen) frequency of the metallic object (usually in audible frequency). Conversely, in case of of placing a dry ice block on a metallic structure, sublimation of dry ice produces its vibration (up and down movement), e.g. [3], where for moderate size of metallic structure except commercial thin honeycomb one its lowest natural frequency falls in the audible range, and up and down cycle of the dry ice block is much smaller than the natural frequency of the metallic structure.

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In this report, the phenomena produced in case of placing a dry ice block on nearly two-dimensional metallic structure are analyzed theoretically and compared with measured data.

2. Analysis

2.1. Characteristics of metallic structure

For a thin shell or a beam of uniform thickness under a plane stress condition

$$\Omega = \frac{\omega L}{\frac{h}{L}\sqrt{E/\rho}},$$

Ω: eigenvalue depending on geometrical configuration, dimensionless constant(s) if any, type of boundary conditions, and a kind of wave form
ω: natural frequency (rad/s) of the metallic structure = emitting sound frequency
L: characteristic length
h: characteristic thickness
E: Young's modulus
ρ: density of the metal

For a rectangular section beam of clamped edges (L = beam length),

$$\sqrt{12}\,\Omega = \sigma^2, \ \cos\sigma \ \cosh\sigma = 1, \ (\sigma > 0).$$

The minimum σ is $\sigma = 4.730(\approx \frac{3}{2}\pi)$. For an annular plate of clamped edges in case of the ratio of the inner radius to the outer radius = 0.3 for axisymmetric wave $\sqrt{12(1-\nu^2)}\,\Omega \approx 45.2$ for the lowest, [4], L = outer radius, ν: Poisson's ratio, and for most metals the value ν lies between 0.25 and 0.44. This situation is roughly analogous to a metallic curved cup with a small flat bottom and a tightly bent upper edge, where L is the circumference length measured along a curved path.

A steel cup example shown in Fig. 1 is given by

$$\Omega = 14.8, \ \omega = 1185 \text{ Hz}, \ L = 53 \text{ mm}, \ \sqrt{E/\rho} = 5.0 \times 10^3 \text{ m} \cdot \text{s}^{-1},$$

$$h = 0.28 \text{ mm}, \ b \text{ (radius of the bottom)} = 10.5 \text{ mm}, \ b/(L+b) = 0.16,$$

$$a \text{ (top radius)} = 35 \text{ mm}, \ \text{depth} = 37 \text{ mm}.$$

The sound wave form and spectrum is produced through an isolated blow by a cylinder covered with rubber (lower own eigenfrequency).

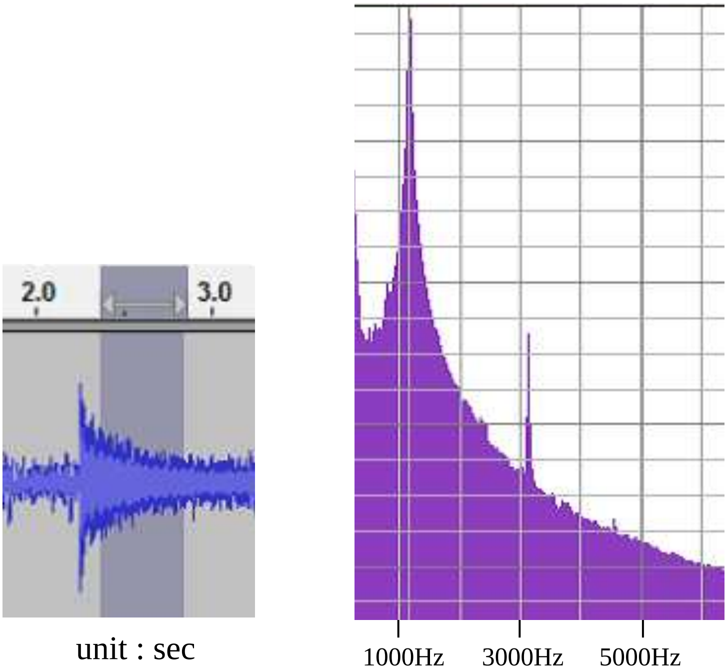


Figure 1: Wave form (left) and spectrum (right)

2.2. Analysis

It is assumed that during time $[0, t_0]$ (t_0: residence time) a dry ice block of mass M is fixed stationarily on a horizontal metallic structure of a beam type, and that it will jump vertically upward with initial speed v_0 and will fall down to the original position after time passage t_f. During residence time, small displacement of the dry ice block due to sublimation is ignored, and flow of gaseous CO_2 over the contact surface is assumed to be small, so that the viscosity of the gaseous atmosphere can be neglected. Energy equation of the metallic structure (beam) under one-dimensional assumption

$$\rho\, c\, \frac{\partial \theta}{\partial t} = \lambda\, \frac{\partial^2 \theta}{\partial x^2},$$

θ: (absolute temperature) – T_a
t: time
x: coordinate measured vertically upward ($x \leq 0$, $x = 0$: surface)
T_a: absolute atmospheric temperature

ρ: metal density
c: heat capacity of the metal
λ: thermal conductivity of the metal
θ_0: $\equiv T_a -$ (sublimation temperature)

Initial condition: $\theta(x, +0) = 0 (x < 0)$
Although there exists some thermal contact resistance between a dry ice block and a metallic structure [5, 6, 7], as a thermal boundary condition at $x = 0$ the following applies:

$$\theta(0, t) = -\theta_0 (t > 0).$$

The far away boundary condition is

$$\theta(-\infty, t) = 0 (t > 0) .$$

The solution is

$$\theta(x, t) = -\theta_0 \left\{ 1 + \mathrm{erf}\left(\frac{\alpha x}{\sqrt{t}} \right) \right\} ; \; \alpha = \frac{1}{2}\sqrt{\frac{\rho c}{\lambda}}.$$

Width of the beam (different from aluminum honeycomb of order 0.001 in width) is assumed to be sufficiently greater than the mean clearance y_0 (defined later), so that the pressure variation with time at the contact surface is considered uniform. The constitutive equation of gaseous CO_2 in the clearance at $0 < t < t_0$

$$\frac{dp}{dt} = \frac{\dot{m}}{S_0 \, y_0} \, R \, T_a,$$

$\dot{m}$: sublimation rate
S_0: effective clearance surface area
y_0: mean clearance at the contact surface $\approx$ surface roughness (usually 0.0006 mm - 0.1 mm) of the beam $\gg$ precision of the so-called gauge block (max. 0.2×10^{-6} m in case of 200 mm gauge)
R: gas constant of $CO_2 = \Re/(0.044 \mathrm{kg} \cdot \mathrm{mol}^{-1})$, $\Re$: universal gas constant
p: pressure

Continuity of heat flux at $x = 0$:

$$S_1 q_0 = \dot{m} H,$$

H: latent heat of evaporation
S_1: effective sublimation surface area
q_0: heat flux at the surface of the beam

$$q_0 = -\lambda \frac{\partial}{\partial x} \theta(0, t) = \theta_0 \, \frac{2\alpha\lambda}{\sqrt{\pi t}}.$$

Thus

$$p - p_a = \frac{4\alpha\lambda}{\sqrt{\pi}}\,\frac{RT_a}{Hy_0}\,\theta_0\,\sqrt{t} \times \frac{S_1}{S_0},$$

p_a: atmospheric pressure

The extra force product F acting to the dry ice due to $p - p_a, (0 < t < t_0)$ is given by

$$F = (S_0 + S_1)\int_0^{t_0} (p - p_0)\, dt.$$

Initial speed v_0 of the dry ice block leaving the surface at $t = t_0$ is given by $v_0 = F/M$ (M: mass of dry ice block). The ratio of time average total pressure F/t_0 acting on the dry ice to the magnitude of gravity (Mg) would be sufficiently greater than unity, which is necessary for the dry ice block to jump and is satisfied if $|x_0|/y_0 > 1$. x_0: heat penetrating depth location, $t_0 \equiv (\alpha x_0)^2$. Falling down time t_f (without rotation of dry ice block) is

$$t_f = \frac{2v_0}{g}.$$

All the more it is expected that

$$\frac{1}{t_0 + t_f} \geq \text{(frequency of dry ice block jump)}.$$

3. Result

In case of steel beam with $L = 415$ mm, $h = 45$ mm, (width $= 5$ mm), $\sigma = 4.73$, configuration is shown in Fig. 2, and isolated natural (theoretical) frequency $=$ 1350 Hz as in Fig. 3 through an isolated blow by a hammer with higher eigenfrequency, whereas the lowest peak sound frequency produced in dry ice vibration

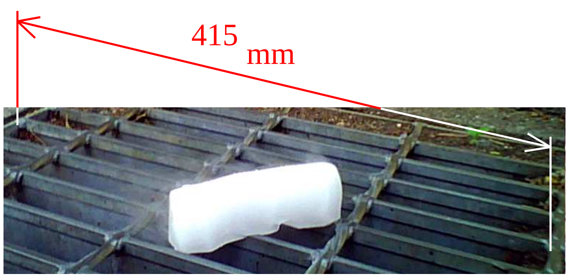


Figure 2: Configuration

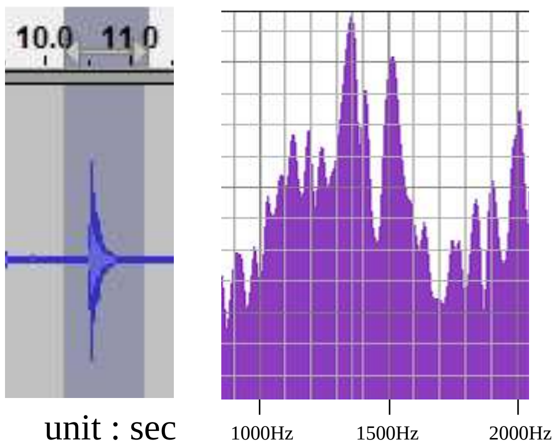


Figure 3: Wave and spectrum (isolated)

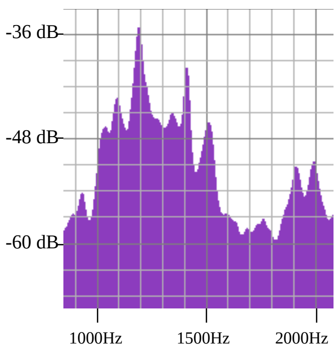


Figure 4: Spectrum

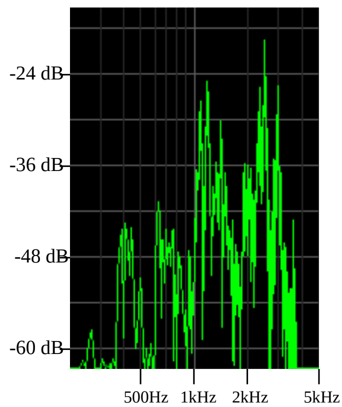
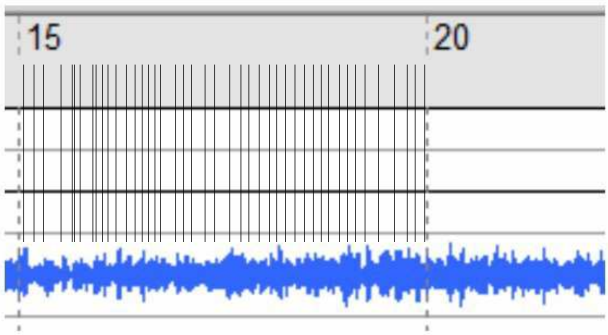


Figure 5: Instantaneous spectrum

is 1200 Hz as in Fig. 4 with jumping frequency 8 Hz as in Fig. 6. In Fig. 4, times corresponding to vertical bars indicate the time of fallen down of a dry ice block. If $y_0 = 10^{-4}$ m, $|x_0| = 10^{-4}$ m, $\alpha = 130\ \mathrm{s}^{1/2} \cdot \mathrm{m}^{-1}$ (steel), then $t_0 = 1.7 \times 10^{-4}$ s. For dry ice, Poisson's ratio = 0.25 [8], Young's modulus = 4.5 GPa after applying shear modulus [9], and density $\approx 1500\ \mathrm{kg} \cdot \mathrm{m}^{-3}$. Mean force acting to the dry ice between the residence time t_0 is given by F/t_0 and

$$F/t_0 = \frac{|x_0|}{y_0} \frac{8}{3\sqrt{\pi}} \frac{RT_a}{H} \lambda \alpha^2 \theta_0 \frac{S_1}{S_0} \times (S_0 + S_1)\,.$$



unit: sec

Figure 6: Wave with time

In case of $H = 3.7 \times 10^5$ J $\cdot$ kg^{-1} (dry ice), $R = 188.6$ J $\cdot$ kg$^{-1} \cdot$ K^{-1} (CO_2), $\lambda = 53.5$ J $\cdot$ m$^{-1} \cdot$ s$^{-1} \cdot$ k^{-1} (steel), assuming $T_a = 293$ K , $\theta_0 = 99$ K $= (79 + 20)$ K, $S_0 = S_1 = 2.5 \times 10^{-5}$ m^2 with $|x_0|/y_0 = 1$,

$$F/t_0 = 1000 \text{ N}.$$

For $M = 50 \times 10^{-3}$ kg, $t_f = 0.69$ s, and $v_0 = F/M = 3.4$ m/s.

4. Conclusions

Vibration of a dry ice block placed on a metallic structure of moderate size excluding honeycomb of order 0.001 in thickness is analyzed quantitatively, using one-dimensional model through sublimation. The vibration form at the current condition is found to be quite different from an isolated blow vibration.

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