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LOCAL ACTIVITY IN MEMRISTOR-BASED CHAOTIC SYSTEM MODEL*

Angela Slavova, Ronald Tetzlaff, Maya Markova

In this paper a memristor-based chaotic system model will be studied. Dynamics of such model will be investigated. Local activity theory will be applied in order to determine the edge of chaos domain of the parameter set in which the model under consideration can exhibit complexity. Simulations and applications will be provided.

1. Introduction

In 1963 an American meteorologist Lorenz tried to simulate the weather changes by bringing forward system equations. The Lorenz system became the first chaotic model which revealed the complex and fundamental behavior of the nonlinear dynamical systems [6]. Chaos can be described as a kind of random change or an irregular movement occurring in a deterministic system, and chaotic state can be considered as the fourth state besides equilibrium state, periodic state and quasi periodic state. Recent years increasing attention have been given to the theory and application research of chaos [4,7,11]. Numerous new chaotic systems are found and constructed constantly. This makes people to have a deeper understanding of the chaotic phenomenon, to enrich and improve the research content of chaos theory. Moreover, chaos and related theories will have a broad application in signal processing, electronic engineering, biology, ecology, etc.

On the other hand, an adjustable nonlinear element, memristor [3] can be used easily as nonlinear part in chaotic generator. Because of its characteristics

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of small volume and low power consumption, memristor can be an ideal choice for nonlinear elements in the chaotic circuit. Various kinds of chaotic systems based on memristors have received great attention from the researchers. In [5] Itoh and Chua adopted a flux-controlled piecewise linear memristor model to replace the Chua's diode in Chua's chaotic circuit. In this way first memristor-based chaotic system was constructed, which expanded people's understanding of its characteristics. Then some other new memristor chaotic circuits have been realized [9] in which under certain conditions of circuit parameters different shapes of chaotic attractor can be generated.

But the current researchers mostly focus on the nonlinear characteristics of the memristor, and ignore its resistance variability. That is why this paper aims to build a bridge between memristor parameters and chaotic systems by applying local activity theory.

In Section 2 we shall introduce the memristor-based chaotic system. Section 3 will deal with application of local activity theory in order to determine edge of chaos region for the CNN memristor-based chaotic model which will be constructed. In Section 4 we shall present bifurcation diagram of the CNN model under consideration. We shall provide simulations of the obtained theoretical results throughout the paper.

2. Memristor-based chaotic system

Let us consider the following chaotic system model:

$$\begin{aligned} \frac{dx}{dt} &= -ay \\ \frac{dy}{dt} &= bx + cy - yz + gf(-|x|) \\ \frac{dz}{dt} &= -dxy - ez + y^2, \end{aligned} \tag{1}$$

where the variables $x, y, z \in \mathbf{R}$ are the state variables, a, b, c, d, e are real constants, and $f(-|x|)$ refers to the charge of the memristor given by the formula below:

$$f(x) = \begin{cases} \dfrac{x - n_3}{R_{OFF}} & x < n_1 \\ \dfrac{\sqrt{2kx + M^2(0)} - M(0)}{k} & n_1 < x < n_2 \\ \dfrac{x - n_4}{R_{ON}} & x \geq n_2 \end{cases} \tag{2}$$

$n_1 = \dfrac{R_{OFF}^2 - M^2(0)}{2k}$, $n_2 = \dfrac{R_{ON}^2 - M^2(0)}{2k}$, $n_3 = \dfrac{[R_{OFF} - M(0)]^2}{2k}$, $n_4 = \dfrac{[R_{ON} - M(0)]^2}{2k}$, x is the magnetic flux, which is input to the memristor, $k = \dfrac{(R_{ON} - R_{OFF})\mu_v R_{ON}}{D_2}$. If we set the parameters for the memristor model (1), (2) as: $R_{ON} = 100\Omega$, $R_{OFF} = 20k\Omega$, $M(0) = 16k\Omega$, $D = 10nm$, $\mu_v = 10^{-14}m^2s^{-1}v^{-1}$, $a = 12$, $b = 2.2$, $c = 0.1$, $d = 22$, $e = 0.5$, $g = 10^4$, the system generates a typical chaotic attractor, different from other obtained results [4] (see Fig. 1):

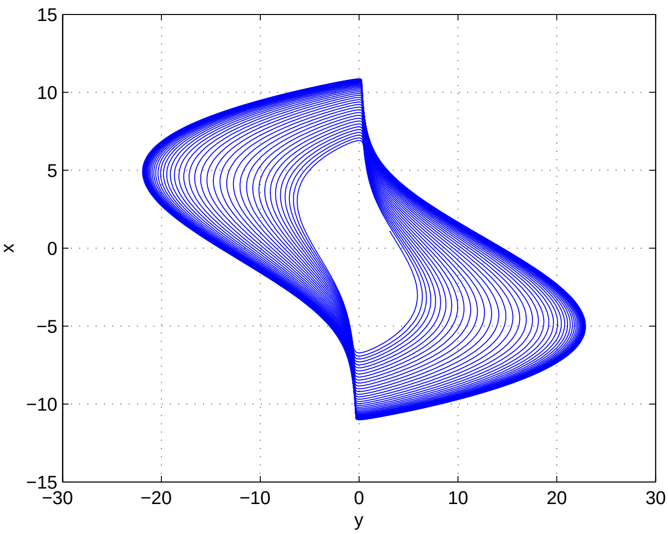


Figure 1: Chaotic attractor described by memristor-based system (1), (2)

Similar to the above chaotic system is studied in [4], but without memristor incorporated in the system. In this paper we are interested to study the dynamical behaviour of (1), (2) by rigorous mathematical analysis based on local activity theory. Intuitively, a cell is locally active if it is endowed with some excitable innate potential, such that under certain conditions, it can become mathematically alive, capable of exhibiting oscillation and chaos.

Definition 1. (Complexity)[2] *A spatially continuous or discrete medium made of identical cells which interact with all cells located within a neighbourhood* *(called the sphere of influence), with identical interaction laws is said to manifest complexity if the homogeneous medium can exhibit a nonhomogeneous static or spatio-temporal pattern, under homogeneous initial and boundary conditions.*

All complexity related examples, problems such as pattern generation, wave propagation and oscillations can be analysed and explained via local activity. We shall proposed below a constructive algorithm for the exact determination of so called phenomena edge of chaos [2,8].

3. Local activity in memristor-based chaotic system

In this section we shall apply local activity theory in order to determine the edge of chaos region in our memristor based chaotic system (1), (2). We shall first discretize the model into Cellular Nonlinear Networks (CNN) architecture [1]. According to [2,8] a rigorous analytical test can be performed at the cell level in order to determine whether a cell is locally active or not. In the local activity domain, it is possible to determine cell parameters for which both unstable and stable dynamics of the cell can spawn emergent behaviors. We shall call edge of chaos (EC) domain a region in the parameter space of a dynamical system in which chaotic behaviour appears. Numerical simulations of the CNN dynamics corresponding to a large variety of cell parameters chosen on, or nearby, the edge of chaos confirmed the existence of a wide spectrum of complex behaviors, many of them with computational potentials in image processing and other applications [8].

We shall try to define more precisely this phenomena below.

To begin with we shall map the memristor-based chaotic system (1), (2) into CNN architecture:

$$
(3) \qquad \left| \begin{array}{l} \dfrac{dx_{ij}}{dt} = -ay_{ij} \\[2ex] \dfrac{dy_{ij}}{dt} = bx_{ij} + cz_{ij} - y_{ij}z_{ij} + gf(-|x_{ij}|) \\[2ex] \dfrac{dz_{ij}}{dt} = -dx_{ij}y_{ij} - ez_{ij} + y_{ij}^2 \end{array} \right.
$$

We can consider the above discretized system as the state equations of CNN [1].

We look for the equilibrium points of (3) which satisfy the following system [12]:

$$(4)\qquad \begin{vmatrix} -ay_{ij} = 0 \\ bx_{ij} + cz_{ij} - y_{ij}z_{ij} + gf(-|x_{ij}|) = 0 \\ -dx_{ij}y_{ij} - ez_{ij} + y_{ij}^2 = 0 \end{vmatrix}$$

The only equilibrium point of the system (3) is $E_0 = (0, 0, 0)$. In general, the number of equilibrium points is not equal to the order of the system [12].

Mathematically, the signal must be infinitesimally small in order to model the cell by only the linear terms in its Taylor series expansion. This in turn allows us to apply well-known linear mathematics and derive explicit analytical criteria for the cell to be locally active at the equilibrium point where the Taylor series expansion is computed. This also proves that complexity originates from infinitesimally small perturbations, notwithstanding the fact that the complete system is typically highly nonlinear. According to the local activity theory [2,8] we should linearize the system (3) at the equilibrium point E_0 in order to find the cell coefficients.

The corresponding Jacobian matrix which consists of these cell coefficients is:

$$(5)\qquad J_0 = \begin{bmatrix} 0 & -a & 0 \\ \tilde{b} & c & 0 \\ 0 & 0 & -e \end{bmatrix},$$

where $\tilde{b} = \begin{cases} b + \dfrac{10^4}{R_{OFF}} & x < n_1 \\ b + \dfrac{10^4}{M(0)} & n_1 < x < n_2 \\ b + \dfrac{10^4}{R_{ON}} & x \geq n_2 \end{cases}.$

Then we can find the trace and determinant of the Jacobian matrix (5) $Tr(J_0)$ and $\Delta(J_0)$ In our case, $Tr(J_0)_{E_0} = c - e < 0$, $\Delta(J_0)_{E_0} = \tilde{b}ea > 0$. It can be shown [2], that $Tr(J_0) < 0$ and $\Delta(J_0) > 0$ are the only region which corresponds to locally asymptotically stable [12] equilibrium points.

Now we are ready to define the stability and locally active region at the equilibrium point E_0, $SLAR(E_0)$. According to [2] we have the following definitions:

Definition 2. *Stable and Locally Active Region $SLAR(E_0)$ at the equilibrium point E_0 the CNN model (3) is such that $Tr(E_k) < 0$ and $\Delta(E_k) > 0$.*

In our case for CNN model (3) $SLAR(E_0)$ can be defined as the following parameter set:

$$
(6) \qquad \left| \begin{array}{llll} c < e & \text{and} & ea\left(b + \dfrac{10^4}{R_{OFF}}\right) > 0 & \text{for} \quad x < n_1, \\ c < e & \text{and} & ea\left(b + \dfrac{10^4}{M(0)} > 0\right) & \text{for} \quad n_1 < x < n_2, \\ c < e & \text{and} & ea\left(b + \dfrac{10^4}{R_{ON}}\right) > 0 & \text{for} \quad x \geq n_2. \end{array} \right.
$$

The simulation of this region is given on Fig. 2:

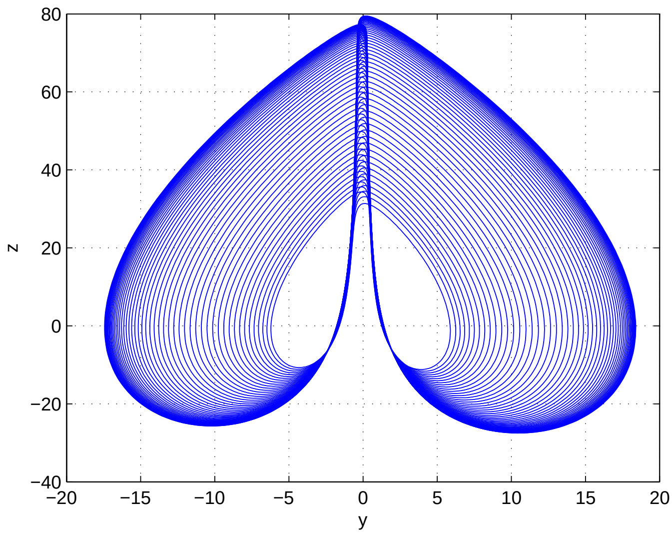


Figure 2: $SLAR(E_0)$ region for the CNN model (3)

Now we shall define EC domain following the standard definition below [2]:

Definition 3. *CNN is operating in the edge of chaos regime if and only if there is at least one equilibrium point which is both locally active and stable.*

In this way the following theorem hold:

Theorem 1. *CNN model* (3) *of the memristor-based chaotic system* (1), (2) *operates in edge of chaos if and only if the conditions* (6) *are satisfied. This means that there is at least one equilibrium point which is both locally active and stable.*

We can show that an uncoupled cell on the edge of chaos may cause CNN to oscillate under the appropriate choice of the parameters. However, it does not imply that it is always possible to find some such parameter set to destabilize an otherwise homogeneous solution. Actually, it is possible to prove that, in general, such a set of destabilizing parameters exist only for a proper subset of the edge of chaos parameter domain, which is called the sharp edge of chaos domain. By definition, a cell on the edge of chaos, but not on the sharp edge of chaos cannot be destabilized by any locally-passive coupling networks. The sharp edge of chaos parameter domain is a proper subset of the edge of chaos parameter domain, which in turn is a proper subset of the local activity parameter domain: Sharp Edge of Chaos $\subset$ Edge of Chaos $\subset$ Local Activity.

According to our investigations above equilibrium point E_0 satisfies the conditions (6), therefore we have sharp edge of chaos (see Fig. 3):

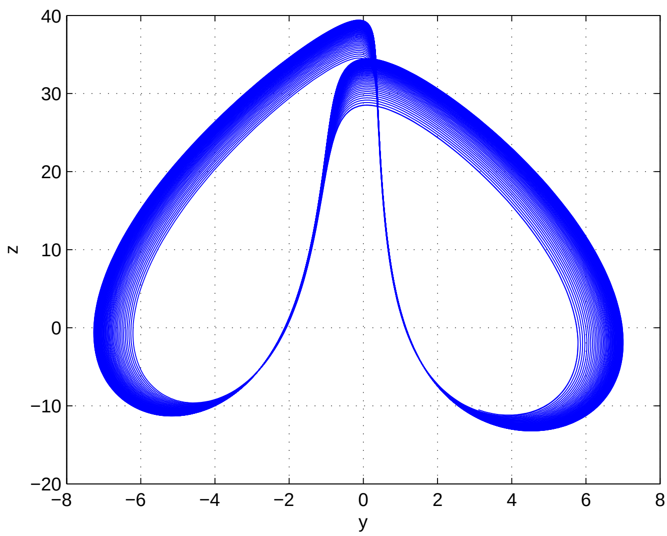
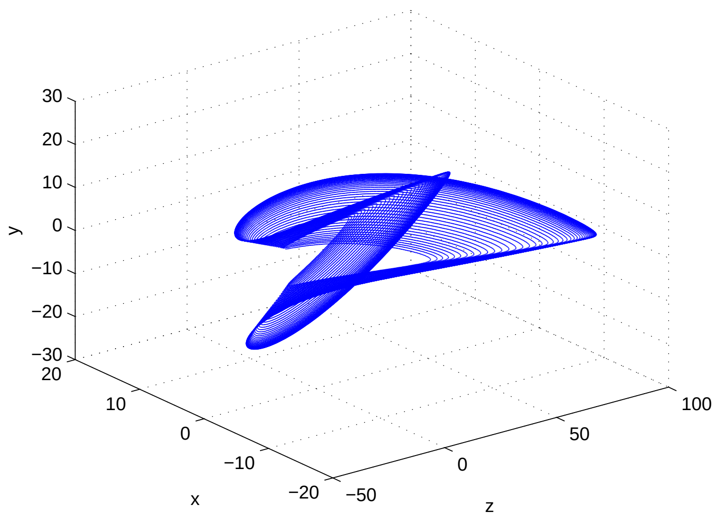
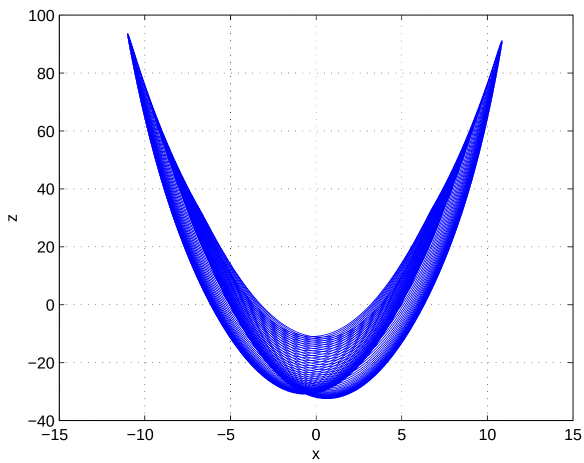


Figure 3: Sharp edge of chaos domain for CNN model (3)



(a) Mirror symmetric attractor.



(b) Non chaotic attractor.

Figure 4: Influence of the memristor parameter g on the dynamic behavior of CNN model (3)

Remark. In general, by properly selecting the value of memristor parameter g, the corresponding mirror symmetric attractor can be generated when the polarity of the g is changed under the identical initial condition (see Fig. 4 (a)). But then by changing the value of parameter g , system (3) will no longer produce chaotic motion under the original parameters (see Fig. 4 (b)). Therefore the flexibility of the selection of g value expand the application range of system (3). System (3) can not only be applied to the practical applications which need to produce chaotic motion such as secure communication and so on, but it can also be used in the process of suppressing the chaos because of the function of suppressing chaos.

Some interesting conclusions can be done about the effect of memristors on chaotic systems: (a) when the polarity of the memristor is changed, a mirror image of the chaotic attractors will appeared in the system; (b) along with the proper choose of the memristor parameters, the chaotic motion of system will be suppressed and enhanced, and therefore the system can be applied to the practice on either generating chaos signal or suppressing chaotic interference.

4. Bifurcation diagram of our memristor-based chaotic CNN model

We shall present in this section structure of several bifurcation diagrams which we derive from the Definitions 2 and 3 for local activity and sharp edge of chaos.

In the simulations we set parameters $a = 12$, $b = 2.2$, $d = 22$, $e = 0.5$, $g = 10^4$, and make $c \subset (0, 2]$. Fig. 5 shows the bifurcation diagram of system (3) with respect to c. It indicates that when $0 < c \geq 2$, the system (3) is always in a state of chaotic, and parameter c has little influence on the system (3). From the bifurcation diagram in Fig. 5, it can be concluded that the amplitude of the output signal y of system (3) is decreased when increasing the parameter c.

By the above bifurcation diagrams, we can conclude that the changes of parameters a, b and c affect the state of the system. And the dynamic characteristics of thesystem are very rich, which verified that the model is a typical chaotic system.

Systems made of a large number of simple components interacting with each other in accordance with some coupling laws tend to operate in one of three possible regimes: 1). an ordered regime such as for example crystals; 2). disorder regime such as fluids, and 3). phase transition regime which separates them. The concept of local activity argues that only systems operating in the phase transition regime are capable of information processing and complexity, and the domain of parameters which gives rise to such regime is the edge of chaos domain.

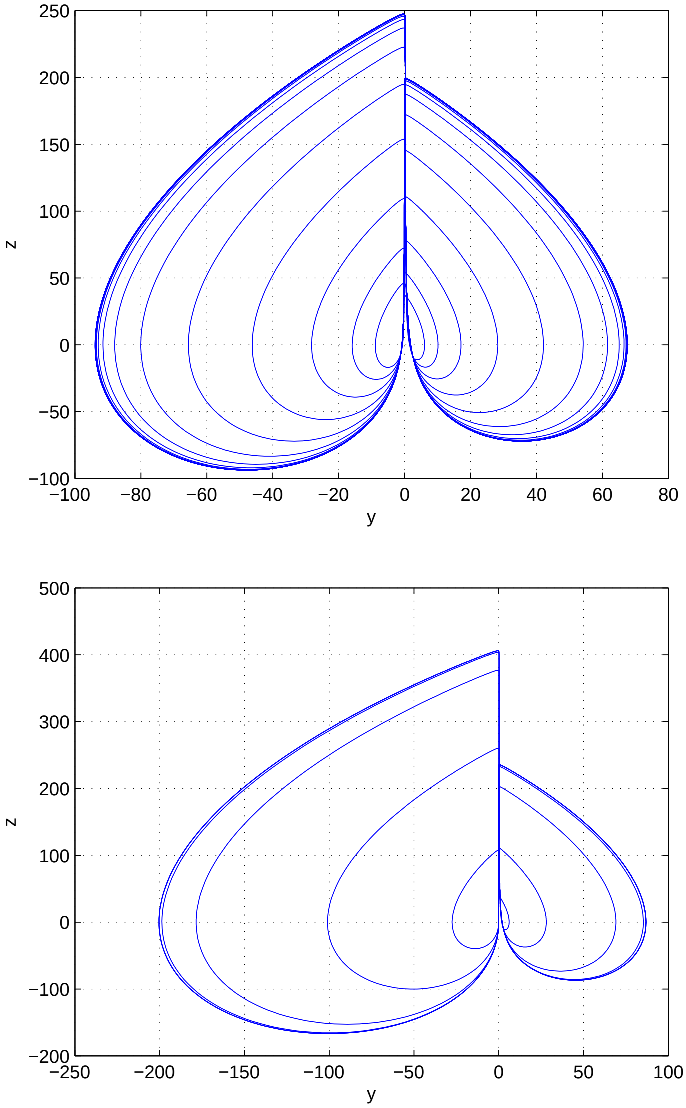


Figure 5: Bifurcation diagram of CNN model (3) for different values of parameter a, b, c

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Angela Slavova
Institute of Mathematics and Informatics
Bulgarian Academy of Sciences
Sofia, Bulgaria
email: slavova@math.bas.bg

Ronald Tetzlaff
Institute of Circuits and Systems
Faculty of Electrical and Computer Engineering
Technische Universität
Dresden, Dresden, Germany
e-mail: Ronald.Tetzlaff@tu-dresden.de

Maya Markova
Department of Applied Mathematics and Statistics
University of Russe
Russe 7017, Bulgaria
e-mail: maya.markova@gmail.com