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SERIES IN LE ROY TYPE FUNCTIONS: THEOREMS IN THE COMPLEX PLANE

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Abstract

Series defined by means of the Le Roy type functions are considered in this paper. These series are studied in the complex plane and their behaviour is investigated 'near' the boundaries of their domains of convergence.

Key words: Le Roy functions and generalizations, series in Le Roy type functions, Abel theorem, Tauber theorem

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1. Introduction. In two recent papers Gerhold [1] and, independently, Garra and Polito [2] introduced the new special function (see also [3])

$$F_{\alpha,\beta}^{(\gamma)}(z) = \sum_{k=0}^{\infty} \frac{z^k}{[\Gamma(\alpha k + \beta)]^{\gamma}}, \quad z \in \mathbb{C},\ \alpha, \beta \in \mathbb{C},\ \gamma > 0, \tag{1}$$

which turns out to be an entire function of the complex variable z for all values of the parameters such that

$$\Re(\alpha) > 0, \beta \in \mathbb{C}, \gamma > 0. \tag{2}$$

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As a matter of fact, the entire function (1) is introduced for the values of parameters $\alpha > 0$, $\beta > 0$, $\gamma > 0$, and on a later stage its definition is extended to the range (2) by Garrappa, Rogosin and Mainardi [3].

The function $F^{(\gamma)}_{\alpha,\beta}(z)$ is closely related to the classical modified Bessel function of the first kind $I_0(2\sqrt{z})$, as well as to the 2-parametric Mittag-Leffler function $E_{\alpha,\beta}(z)$ (see the recent monographs [4] and [5]). It is also closely related to the multi-index generalizations of $E_{\alpha,\beta}(z)$ (with $2m$ and $3m$ parameters, $m = 1, 2, \dots$, i.e. the so-called multi-index Mittag-Leffler functions (for their definitions and properties see e. g. [5–9]). Actually, if the parameter γ is a positive integer, the function $F^{(\gamma)}_{\alpha,\beta}$ can be presented by the multi-index Mittag-Leffler functions. The function (1) is also related to the so-called Le Roy function $F^{(\gamma)}$, named after the great French mathematician Édouard Louis Emmanuel Julien Le Roy (1870–1954), namely $F^{(\gamma)}(z) = F^{(\gamma)}_{1,1}(z)$. Le Roy himself used it in [10] to study the asymptotics of the analytic continuation of the sum of power series. This reason for the origin of Le Roy function sounds somehow close to Mittag-Leffler's idea to introduce the function $E_\alpha(z)$ for the aims of analytic continuation (we have to note that Mittag-Leffler and Le Roy were working on this idea in competition). The Le Roy function is involved in the solution of problems of various types; in particular it has been recently used in the construction of a Conway–Maxwell–Poisson distribution [11] which is important due to its ability to model count data with different degrees of over- and under-dispersion [12, 13].

For the sake of brevity, we often use in this paper the name Le Roy type function for the function $F^{(\gamma)}_{\alpha,\beta}$ defined by (1).

In the series of papers [14–16, 18], as well as in the recent book [5], we studied series in different systems of special functions of Fractional calculus, and we proved various results connected with their convergence in the complex plane. In this paper, we discuss the results for the Le Roy type functions (1), previously obtained, which are needful here. We consider series in Le Roy type functions and investigate their behaviour 'near' the boundaries of their domains of convergence.

2. Inequalities and asymptotic formula. For our purpose we need the family of functions

$$F^{(\gamma)}_{\alpha,n}(z) = \sum_{k=0}^{\infty} \frac{z^k}{[\Gamma(\alpha k + n)]^\gamma}, \quad z \in \mathbb{C};\ n \in \mathbb{N},\ \alpha > 0,\ \gamma > 0, \tag{3}$$

where $\mathbb{N}$ means the set of positive integers. Below is given an inequality, connected to the functions (3) and an asymptotic formula for them. Namely, the following results, proved by the author in [18], are formulated.

Lemma 1. *Let $z \in \mathbb{C}$, $\alpha > 0$, $\gamma > 0$, $n \in \mathbb{N}$ and let $K \subset \mathbb{C}$ be a nonempty* *compact set. Then there exists an entire function* $\vartheta_{\alpha,n}^{(\gamma)}$ *such that*

$$F_{\alpha,n}^{(\gamma)}(z) = \frac{1}{[\Gamma(n)]^{\gamma}} \, (1 + \vartheta_{\alpha,n}^{(\gamma)}(z)), \quad z \in \mathbb{C}. \tag{4}$$

Moreover, there exists a constant $C = C(K)$*,* $0 < C < \infty$*, such that*

$$\max_{z \in K} \left| \vartheta_{\alpha,n}^{(\gamma)}(z) \right| \leq C \, \frac{[\Gamma(n)]^{\gamma}}{[\Gamma(\alpha + n)]^{\gamma}}, \tag{5}$$

for all the natural numbers n*.*

Theorem 1. *Let* $z \in \mathbb{C}$*;* $n \in \mathbb{N}$*,* $\alpha > 0$*,* $\gamma > 0$*. Then*

$$F_{\alpha,n}^{(\gamma)}(z) = \frac{1}{[\Gamma(n)]^{\gamma}} \, (1 + \vartheta_{\alpha,n}^{(\gamma)}(z)), \quad \vartheta_{\alpha,n}^{(\gamma)}(z) \to 0 \ \text{as } n \to \infty, \tag{6}$$

uniformly in the max-norm on the compact sets in $\mathbb{C}$*.*

Remark 1. According to the asymptotic formula (6), it follows that there exists a natural number M such that the functions $[\Gamma(n)]^{\gamma} \, F_{\alpha,n}^{(\gamma)}(z)$ do not vanish for any n great enough (say $n > M$).

3. Series in Le Roy type functions. For simplicity of studying, we introduce auxiliary functions, related to the Le Roy functions, adding $\widetilde{F}_{\alpha,0}^{(\gamma)}(z)$ just for completeness, namely:

$$\widetilde{F}_{\alpha,0}^{(\gamma)}(z) = 1, \ \widetilde{F}_{\alpha,n}^{(\gamma)}(z) = z^n \ [\Gamma(n)]^{\gamma} \ F_{\alpha,n}^{(\gamma)}(z), \ n \in \mathbb{N}; \ \alpha > 0, \ \gamma > 0, \tag{7}$$

and we stydy the series in these functions:

$$\sum_{n=0}^{\infty} a_n \, \widetilde{F}_{\alpha,n}^{(\gamma)}(z), \tag{8}$$

for $z \in \mathbb{C}$ and with complex coefficients a_n $(n = 0, 1, 2, \dots)$.

Our main objective is to study the convergence of the series (8) in the complex plane. Such kind of results may be useful for studying the solutions of some fractional order differential and integral equations, expressed in terms of series (or series of integrals) in special functions of the type (7) (as for example in Kiryakova [19] in a more general case).

4. Cauchy–Hadamard type theorem and corollaries. Let us denote by $D(0; R)$ the open disk with the radius R and centred at the origin, and let the circle $C(0; R)$ be its boundary, i.e. $D(0; R) = \{z : \ |z| < R\}$ and $C(0; R) = \{z : \ |z| = R\}$ $(z \in \mathbb{C})$. In this section we discuss results, mainly obtained in [18], corresponding to the classical Cauchy–Hadamard theorem and Abel lemma for the power series. The theorem of the Cauchy–Hadamard type for the series (8) is formulated below.

Theorem 2 (of Cauchy–Hadamard type)**.** *Let $z \in \mathbb{C}$, $n \in \mathbb{N}$, $\alpha > 0$, $\gamma > 0$. Then the domain of convergence of the series* (8) *with complex coefficients a_n is the disk $D(0; R) = \{z : \ |z| < R\}$ with a radius of convergence*

$$R = 1/\limsup_{n\to\infty} (\ |a_n|\ )^{1/n}. \tag{9}$$

The cases $R = 0$ and $R = \infty$ are included in the general case.

Thus, the considered series (8) absolutely converges in the disk $D(0; R)$ with the radius R, given by (9), like in the classical theory of the power series. Additionally, it turns out that the convergence of the discussed series is uniform inside the disk, i.e., the following corollary, similar to the classical Abel lemma, holds.

Corollary 2.1. *Let $z \in \mathbb{C}$, $n \in \mathbb{N}$, $\alpha > 0$, $\gamma > 0$, and let the series* (8) *converge at the point $z_0 \neq 0$. Then it converges absolutely in the disk $D(0; |z_0|)$ and on the compact subsets of the disk $D(0; R)$ (R defined by* (9)*), as well.*

Theorem 2 and Corollary 2.1 have been obtained in [[18]]. Here we give one more corollary.

Corollary 2.2. *Let $z \in \mathbb{C}$, $n \in \mathbb{N}$, $\alpha > 0$, $\gamma > 0$, and let the series* (8) *diverge at the point $z_0 \neq 0$. Then it is divergent for each z with $|z| > |z_0|$.*

Proof. Assuming the opposite, the trueness of the corollary follows by getting a contradiction. □

Note that the very disk of convergence is not obligatorily a region of uniform convergence and on its boundary the series may even be divergent. More precise results, connected with the behaviour of the series (8) 'near' the boundary $C(0; R)$ are obtained and discussed in the next sections.

5. Abel type theorem. Let $z_0 \in \mathbb{C}$, $0 < R < \infty$, $|z_0| = R$ and g_φ be an arbitrary angular region with size $2\varphi < \pi$ and with a vertex at the point $z = z_0$, which is symmetric with respect to the straight line passing through the points 0 and z_0, and let d_φ be the part of the angular region g_φ, closed between the angle's arms and the arc of the circle centred at the point 0 and touching the arms of the angle. The following inequality is verified for $z \in d_\varphi$ ([[5]], p. 21):

$$|z - z_0| \cos\varphi < 2(|z_0| - |z|). \tag{10}$$

The next theorem refers to the uniform convergence of the series (8) in the set d_φ and the limit of its sum at the point z_0, provided $z \in D(0; R) \cap g_\varphi$.

Theorem 3 (of Abel type)**.** *Let $\{a_n\}_{n=0}^{\infty}$ be a sequence of complex numbers, R be the real number defined by* (9) *and $0 < R < \infty$. If $f(z; \alpha, \gamma)$ is the sum of the series* (8) *in the region $D(0; R)$, i.e.*

$$f(z; \alpha, \gamma) = \sum_{n=0}^{\infty} a_n \widetilde{F}_{\alpha,n}^{(\gamma)}(z), \quad z \in D(0; R),$$

and this series converges at the point z_0 of the boundary $C(0; R)$, then:

(i) *The following relation holds*

$$\lim_{z\to z_0} f(z;\,\alpha,\,\gamma) = \sum_{n=0}^{\infty} a_n \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0), \tag{11}$$

provided $z \in D(0;R) \cap g_\varphi$.

(ii) *The series* (8) *converges uniformly in the max-norm on compact subsets of the region* d_φ.

Idea of proof. Beginning with (i), we consider the difference

$$\Delta(z) = \sum_{n=0}^{\infty} a_n \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - f(z;\,\alpha,\,\gamma) = \sum_{n=0}^{\infty} a_n (\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - \widetilde{F}^{(\gamma)}_{\alpha,n}(z)) \tag{12}$$

and represent it in the form

$$\Delta(z) = \sum_{n=0}^{k} a_n (\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - \widetilde{F}^{(\gamma)}_{\alpha,n}(z)) + \sum_{n=k+1}^{\infty} a_n (\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - \widetilde{F}^{(\gamma)}_{\alpha,n}(z)).$$

According to Remark 1, there exists a positive integer M such that $\widetilde{F}^{(\gamma)}_{\alpha,n}(z_0) \neq 0$ when $n > M$. Let $k > M$ and $p > 0$. By using the notations

$$\beta_k = 0,\ \beta_m = \sum_{n=k+1}^{m} a_n \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0),\ m > k, \quad \gamma_n(z) = 1 - \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z)/\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0),$$

and the Abel transformation (see in ([20], vol.1, Ch.1, p.32, 3.4:7)) for the expression $\sum_{n=k+1}^{k+p} a_n (\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z)) = \sum_{n=k+1}^{k+p} (\beta_n - \beta_{n-1})\gamma_n(z)$, we obtain:

$$\sum_{n=k+1}^{k+p} a_n (\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z)) = \beta_{k+p}\gamma_{k+p}(z) - \sum_{n=k+1}^{k+p-1} \beta_n(\gamma_{n+1}(z) - \gamma_n(z)).$$

In order to estimate the module of the difference (12) we consider the last relation and we firstly find the estimations of $|\gamma_{n+1}(z) - \gamma_n(z)|$ and $|\gamma_{k+p}(z)|$. For this purpose, letting the natural number $n > k$, we apply the Schwarz lemma (see in ([20], vol.1, Ch.1, p.317)) for them. Let now ε be a positive number. After additional estimations, bearing in mind (10), we establish that there exists $\delta = \delta(\varepsilon)$ such that inequality $|\Delta(z)| = |\sum_{n=0}^{\infty} a_n (\widetilde{F}^{(\gamma)}_{\alpha,\,n}(z_0) - \widetilde{F}^{(\gamma)}_{\alpha,\,n}(z))| < \varepsilon$ holds, provided $z \in d_\varphi$ and $|z - z_0| < \delta(\varepsilon)$, which proves equality (11).

In order to prove (ii), we also use inequality (10) which is the crucial point of the proof. So, let $z \in d_\varphi$. Setting

(13) $$S_k(z) = \sum_{n=0}^{k} a_n \widetilde{F}^{(\gamma)}_{\alpha,n}(z), \quad S_k(z_0) = \sum_{n=0}^{k} a_n \widetilde{F}^{(\gamma)}_{\alpha,n}(z_0), \quad \lim_{k\to\infty} S_k(z_0) = s,$$

we obtain $S_{k+p}(z) - S_k(z) = \sum_{n=0}^{k+p} a_n \widetilde{F}^{(\gamma)}_{\alpha,n}(z) - \sum_{n=0}^{k} a_n \widetilde{F}^{(\gamma)}_{\alpha,n}(z) = \sum_{n=k+1}^{k+p} a_n \widetilde{F}^{(\gamma)}_{\alpha,n}(z)$. According to Remark 1, there exists a natural number N_0 such that $\widetilde{F}^{(\gamma)}_{\alpha,n}(z_0) \neq 0$ when $n > N_0$. Let $k > N_0$ and $p > 0$. Then, using the denotation $\gamma_n(z;z_0) = \widetilde{F}^{(\gamma)}_{\alpha,n}(z)/\widetilde{F}^{(\gamma)}_{\alpha,n}(z_0)$, we can write the difference $S_{k+p}(z) - S_k(z)$ as follows:

$$S_{k+p}(z) - S_k(z) = \sum_{n=k+1}^{k+p} (\beta_n - \beta_{n-1})\gamma_n(z;z_0),$$

with $\beta_n = S_n(z_0) - s$. Applying after that the Abel transformation (see in ([20], vol.1, ch.1, p.32, 3.4:7)), we estimate the module $|S_{k+p}(z) - S_k(z)|$ in a similar way to (i), and so we succeed to prove the theorem. □

6. Tauber type theorem. In this section we provide two theorems, inverse of Theorem 3. We consider the series $\sum_{n=0}^{\infty} a_n$, $a_n \in \mathbb{C}$. Let $z_0 \in \mathbb{C}$, $|z_0| = R$, $0 < R < \infty$, $\widetilde{F}^{(\gamma)}_{\alpha,n}(z_0) \neq 0$ for $n = 0, 1, 2, \ldots$. For the sake of brevity, denote

$$F^*_{n,\alpha,\gamma}(z;z_0) = \frac{\widetilde{F}^{(\gamma)}_{\alpha,n}(z)}{\widetilde{F}^{(\gamma)}_{\alpha,n}(z_0)}.$$

Let the series $\sum_{n=0}^{\infty} a_n F^*_{n,\alpha,\gamma}(z;z_0)$ be convergent for $|z| < R$ and

(14) $$F(z) = \sum_{n=0}^{\infty} a_n F^*_{n,\alpha,\gamma}(z;z_0), \quad |z| < R.$$

Theorem 4 (of Tauber type). *If $\{a_n\}_{n=0}^{\infty}$ is a sequence of complex numbers with*

(15) $$\lim\{na_n\} = 0,$$

and there exists

(16) $$\lim_{z\to z_0} F(z) = S \quad (|z| < R, z \to z_0 \text{ radially}),$$

then the series $\sum_{n=0}^{\infty} a_n$ is convergent and $\sum_{n=0}^{\infty} a_n = S$.

Idea of proof. Let z belong to the segment $[0, z_0]$. By using the asymptotic formula (6) for the Le Roy type functions, we obtain:

$$a_n F^*_{n,\alpha,\gamma}(z; z_0) = a_n \left(\frac{z}{z_0}\right)^n \frac{1 + \vartheta^{(\gamma)}_{\alpha,n}(z)}{1 + \vartheta^{(\gamma)}_{\alpha,n}(z_0)} = a_n \left(\frac{z}{z_0}\right)^n \left(1 + \widetilde{\vartheta}^{(\gamma)}_{\alpha,n}(z; z_0)\right). \tag{17}$$

Then $\widetilde{\vartheta}^{(\gamma)}_{\alpha,n}(z; z_0) = O\left(\frac{1}{n^{\alpha\gamma}}\right)$, due to (5) and the Γ-functions quotient property.

Let us write $\sum_{n=0}^{\infty} a_n F^*_{n,\alpha,\gamma}(z; z_0)$ in the form

$$\sum_{n=0}^{\infty} a_n F^*_{n,\alpha,\gamma}(z; z_0) = \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^n \left(1 + \widetilde{\vartheta}^{(\gamma)}_{\alpha,n}(z; z_0)\right), \tag{18}$$

Denoting $w_n(z) = a_n \left(\frac{z}{z_0}\right)^n \widetilde{\vartheta}^{(\gamma)}_{\alpha,n}(z; z_0)$, we consider the series $\sum_{n=0}^{\infty} w_n(z)$ and we prove that

$$\lim_{z\to z_0} \sum_{n=0}^{\infty} w_n(z) = \sum_{n=0}^{\infty} \lim_{z\to z_0} w_n(z) = 0. \tag{19}$$

Then, bearing in mind (14) and (18), we can obtain that

$$\lim_{z\to z_0} F(z) = \lim_{z\to z_0} \sum_{n=0}^{\infty} a_n F^*_{n,\alpha,\gamma}(z; z_0) = S = \lim_{z\to z_0} \sum_{n=0}^{\infty} a_n \left(\frac{z}{z_0}\right)^n. \tag{20}$$

Now, from (20) and by the classical Tauber theorem for the power series, it follows that the series $\sum_{n=0}^{\infty} a_n$ converges and $\sum_{n=0}^{\infty} a_n = S$. □

Theorem 5 (of Littlewood type). *If $\{a_n\}_{n=0}^{\infty}$ is a sequence of complex numbers with $a_n = O(1/n)$, and there exists $\lim_{z\to z_0} F(z) = S \quad (|z| < R, z \to z_0$ radially), then the series $\sum_{n=0}^{\infty} a_n$ is convergent and $\sum_{n=0}^{\infty} a_n = S$.*

Ending we note that the proof of Theorem 5, as well as the specific details of the proofs, following the ideas given here, will be provided elsewhere.

REFERENCES

[1] Gerhold S. (2012) Asymptotics for a variant of the Mittag-Leffler function, Integr. Trans. Spec. Func., **23**(6), 397–403.

[2] Garra R., F. Polito (2013) On some operators involving Hadamard derivatives, Integr. Trans. Spec. Func., **24**(10), 773–782.

[3] Garrappa R., S. Rogosin, F. Mainardi (2017) On a generalized three-parameter Wright function of Le Roy type, Fract. Calc. Appl. Anal., **20**(5), 1196–1215, DOI: 10.1515/fca-2017-0063.

[4] Gorenflo R., A. A. Kilbas, F. Mainardi, S. V. Rogosin (2020) Mittag-Leffler Functions, Related Topics and Applications, Springer, DOI: 10.1007/978-3-662-61550-8.

[5] Paneva-Konovska J. (2016) From Bessel to Multi-Index Mittag Leffler Functions: Enumerable Families, Series in them and Convergence, London, World Scientific Publ., doi: 10.1142/q0026.

[6] Al-Bassam M. A., Y. F. Luchko (1995) On generalized fractional calculus and its application to the solution of integro-differential equations, J. Fract. Calc., **7**, 69–88.

[7] Kilbas A. A., A. A. Koroleva, S. V. Rogosin (2013) Multi-parametric Mittag-Leffler functions and their extension, Fract. Calc. Appl. Anal., **16**(2), 378–404, doi: 10.2478/s13540-013-0024-9.

[8] Kiryakova V. (1999) Multiindex Mittag-Leffler functions, related Gelfond-Leontiev operators and Laplace type integral transforms, Fract. Calc. Appl. Anal., **2**(4), 445–462.

[9] Kiryakova V. (2010) The multi-index Mittag-Leffler functions as important class of special functions of fractional calculus, Computers and Math. with Appl., **59**(5), 1885–1895, doi: 10.1016/j.camwa.2009.08.025.

[10] Le Roy É. (1900) Valéurs asymptotiques de certaines séries procédant suivant les puissances entères et positives d'une variable réelle, Darboux Bull. (2), **24**, 245–268 (in French).

[11] Conway R. W., W. L. Maxwell (1962) A queueing model with state dependent service rate, J. Industr. Engn., **12**(2), 132–136.

[12] Pogány T. (2016) Integral form of the COM-Poisson renormalization constant, Statistics and Probability Letters, **119**, 144–145.

[13] Santarelli M. F., D. Della Latta, M. Scipioni, V. Positano, L. Landini (2016) A Conway-Maxwell-Poisson (CMP) model to address data dispersion on positron emission tomography, Computers in Biology and Medicine, **77**, 90–101, DOI: 10.1016/j.compbiomed.2016.08.006.

[14] Paneva-Konovska J. (2008) Cauchy–Hadamard, Abel and Tauber type theorems for series in generalized Bessel–Maitland functions, C. R. Acad. Bulg. Sci., **61**(1), 9–14.

[15] Paneva-Konovska J. (2016) Periphery behaviour of series in Mittag-Leffler type functions, I, Intern. J. Appl. Math., **29**(1), 69–78, DOI: 10.12732/ijam.v29i1.6.

[16] Paneva-Konovska J. (2016) Periphery behaviour of series in Mittag-Leffler type functions, II, Intern. J. Appl. Math., **29**(2), 175–186, DOI: 10.12732/ijam.v29i2.2.

[17] Paneva-Konovska J. (2017) Overconvergence of series in generalized Mittag-Leffler functions, Fract. Calc. Appl. Anal., **20**(2), 506–520, DOI: 10.1515/fca-2017-0026.

[18] Paneva-Konovska J. (2020) Series in Le Roy type functions: Inequalities and convergence theorems, Intern. J. Appl. Math., **33**(6), 995–1007, DOI: 10.12732/ijam.v33i6.3.

[19] Kiryakova V. (2011) Fractional order differential and integral equations with Erdélyi–Kober operators: Explicit solutions by means of the transmutation method, AIP Conf. Proc., **1410**, 247–258, DOI: 10.1063/1.3664376.

[20] Markushevich A. (1967) A Theory of Analytic Functions., **1, 2**, Moscow, Nauka (in Russian).

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