LEARNING COMPUTATIONAL LOGIC THROUGH GEOMETRIC REASONING

Gopal Tadepalli

Computer Science Dept., College of Engineering, Anna University Chennai, India

gopal@annauniv.edu, gopal.tadepalli@gmail.com

ИЗУЧАВАНЕ НА ИЗЧИСЛИТЕЛНА ЛОГИКА ЧРЕЗ ГЕОМЕТРИЧНИ РАЗСЪЖДЕНИЯ

Abstract: *Computers control everyday things ranging from the heart pacemakers to voice controlled devices that form an integral part of many appliances. Failures related to computers regularly cause disruption, damage and occasionally death. Computational logic establishes the facts in a logical formalism. It attempts to understand the nature of mathematical reasoning with a wide variety of formalisms, techniques and technologies. Formal verification uses mathematical and logical formalisms to prove the correctness of designs. Formal methods provide the maturity and agility to assimilate the future concepts, languages, techniques and tools for computational methods and models. The quest for simplification of formal verification is never ending. This summary report advocates the use of geometry to construct quick conclusions by the human mind that can be formally verified if necessary.*

Keywords: *Computational Logic; Formal Verification; Simplification; Geometry; Design.*

Introduction

Logic and mathematics are two sister-disciplines. Logic provides for a generic inference and reasoning. Logical principles and Logical inferences are vital in constructing chains of reasoning for formal verification and proofs. The associated notations are important in constructing higher abstractions to face the challenges of rapidly increasing complexity. Arguably, the everyday things controlled by computers need a simplification of this method at least for a typical human to arrive at a “Proof of Concept”.

Rapid advances in computer graphics provided the necessary thrust for visual techniques as a means of communication between people and machines. "Graphical User Interfaces [GUI]" is a just the beginning. Geometric modelling refers to the construction and manipulation of structures that represent the grounding of claims towards formal verification. Humans are endowed with unique and special ability to create, manipulate, rationalize and process highly elaborate structures of language, mathematics and music. This capability to abstract the structure is readily evident in a much simpler domain - the visual perception of regular geometric shapes such as squares, rectangles, and parallelograms. Geometric reasoning uses critical thinking, logical argument and spatial reasoning to find new relationships and solutions to the problems. Geometry enables the understanding and reasoning about the form of physical objects and spatial relations in terms of shapes, angles, dimensions, and sizes. This is crucial to many applications in artificial intelligence and advanced reasoning systems. The topics such as Tensors, Triangulation and "Graphical Processing Units" with interactive interfaces to manipulate a reasonably complex physical system enables the human to conclude on the outcome with a very high degree of assurance. Connected Devices and Connected People are no more the causes of concern. They are a part of quick solutions with Geometric Reasoning.

This report highlights the pros and cons of the proposed method.

Exhibition

Rooted in Logic with Algebra and Set Theory forming the trunk, the tree of mathematics produced many branches such as Probability, Geometry, Calculus, Trigonometry, Topology and Applied Mathematics. Mathematical thinking explains things in terms of equations. The formulation of the equations is based on the understanding of the problem context. Organizing problem information is important for specifying the context in terms of the properties that characterize a given set of equations. The equations can often be solved "analytically" i.e. the properties are derived using only equations. A mathematical model thus specified is considered cleaner and more "pure" when it is possible to derive properties analytically.

Computational thinking is about taking a mathematical model and asking **"How should this be implemented?"**. The first step is abstraction or modeling that is closely related to the concepts of theory and design. This step helps in focusing attention on details of greater importance. Abstraction is the basis for simulation and automation through computer programming.

"The essence of abstraction is preserving information that is relevant in a given context, and forgetting information that is irrelevant in that context." - John V. Guttag, A Former Professor, Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, USA

Both Mathematical and Computational thinking are iterative. The following working definitions are useful in constructing the respective models.

1. ***Reason***: the mental ability to form conclusions, judgments, or inferences. **Synonyms:** understanding, intellect, mind, intelligence.
2. ***Logic***: reasoning in adherence to strict principles of validity. **Synonyms:** reasoning, line of reasoning, rationale, argument, argumentation.

Logical Reasoning is the method of making logical statements or conclusions based on given conditions. Statements are justified by definitions, postulates, theorems or "conjectures". In 1988 E W Dijkstra wrote an article titled "On the Cruelty of Really Teaching Computing Science" [2]. In this article, Dijkstra argues that computer programming should be understood as a branch of mathematics with formal provability of a program as a major criterion for correctness. Formal Verification and Validation of Computer Programs has been one of the tough challenges ever since the first computer programs were written. The limitations of understanding mathematical logic for computation are one of the primary reasons for the elusive formal verification. This is illustrated using Digital Image Processing. Digital Image Processing is used for remote sensing, data transmission, medicine, robotics, computer vision, film industry and so on. Matrix is the abstract representation of grayscale images. Please see Figure 1 below.

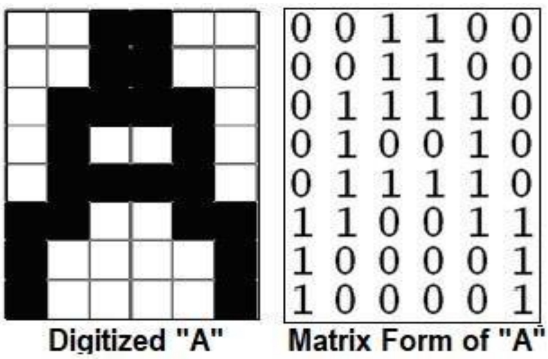

Figure 1. Matrix Representation of a Digitized Grayscale Image

Color images are represented by three matrices with each matrix specifying the quantity of Red, Green and Blue that make the image. Please see Figure 2 below.

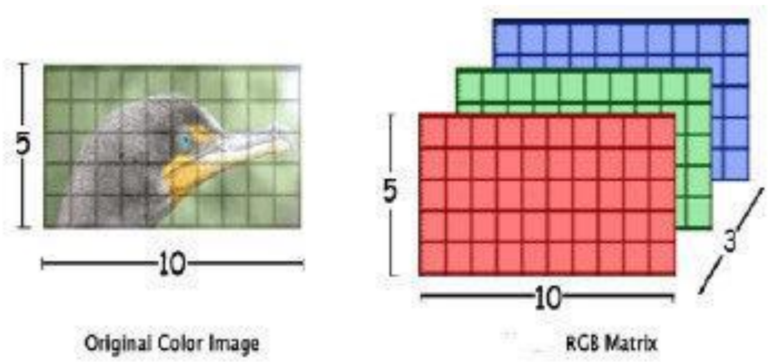

Figure 2. The Three Matrices or the RGB Matrix of the Color Image

The mathematics typically used for processing of the images is mentioned below.

- Linear Algebra Basics
- Matrix Calculus
- Singular Value Decomposition (SVD)
- Eigenvalue Decomposition
- Low-rank Matrix Inversion

There are good software packages like MATLAB that help in bridging the representations in mathematics and abstraction for computation. Human mind is capable of processing geometric information very rapidly. The Delaunay triangulation method and the graphical processing unit [GPU] result in the faster automation of this processing. Computational models are now capable of working with very complex image sets and their analytics. Please see Figure 3 below.

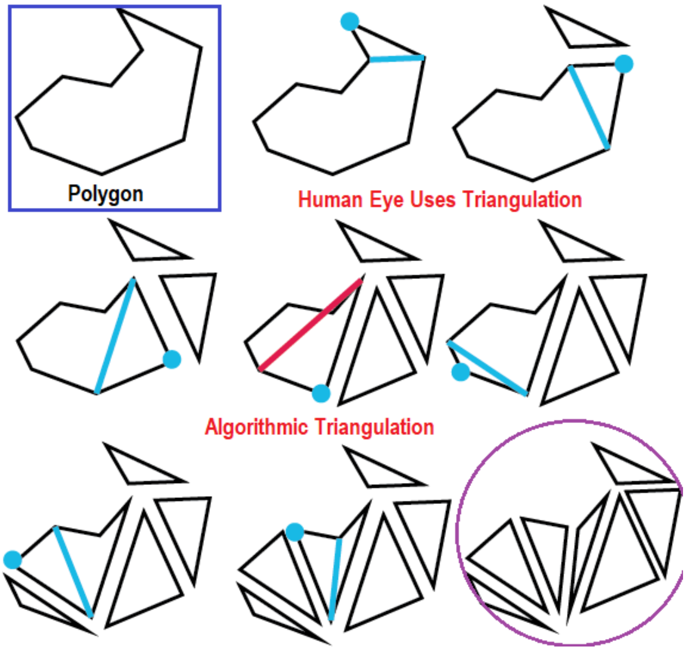

Figure 3. Illustration of "Triangulation".

Euclid's elements is not only a mathematical step forward but was a step forward in Mathematical Logic. Rene Descartes, Isaac Newton, Spinoza and Abraham Lincoln thrived on the geometrical structure for reasoning to arrive at conclusions in real life. Clifford Geometric Algebra is by far the best unified mathematics formulation for science and engineering. In mathematics, tensors are geometric representations of objects to describe linear relations among the geometric vectors, scalars, and other tensors. The tensor is a physical quantity, which is described by a number of components, whose values depend on the coordinate system. In this way, the vector is also tensor. Please see Figure 4 below.

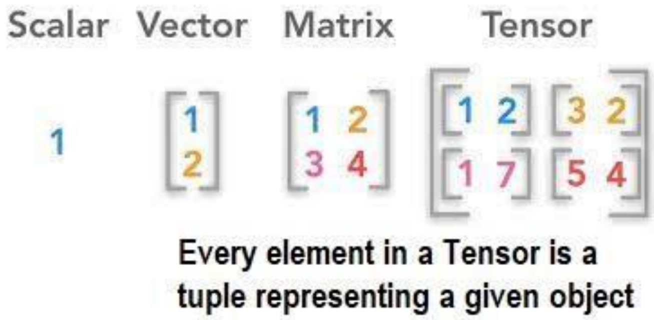

Figure 4. An example of a Tensor.

Rapid advances in Computational methods enable very fast processing of the algebra, geometry and the calculus of Tensors. The result is an amazing set of transformations that come close to the way human eye processes the real-world objects. Geometric Proof can be constructed from the tensor analysis as a sequence of statements starting from a given set of premises leading to a valid conclusion. Each statement is derived logically from previous statements. Each statement is supported by a reason such as definition, postulate, or "conjecture". The simple mathematics and geometry result in human like processing of real world at very high speeds. There are many software packages that make complex processing readily simulated and studied [1], [3], [4], [5], [6] and [7]. The hardware for computation is also becoming faster to work with complex tensors using the triangulation method.

Conslusion

Geometry enables deductive reasoning. It uses facts, definitions, accepted properties and laws of logic to form a logical argument. A geometric proof is a good way to learn logical reasoning for computation. It can be a proof without words. Geometric reasoning helps one learn the computational logic much faster is the submission in this report.

References

[1] Almstrum, V.L. (1994). "Limitations In The Understanding of Mathematical Logic by Novice Computer Science Students", Dissertation, Doctor of Philosophy, The University Of Texas at Austin, May, 1994.

[2] Dijkstra, E. W. (1989). "On the cruelty of really teaching computing science", Communications of the ACM, 32(12), pp. 1398–1404, 1989.

[3] Hunt, C.; Ropella, G.; Park, S. et al. (2008). "Dichotomies between computational and mathematical models", Nature Biotechnology 26, 737–738, 2008. DOI: https://doi.org/10.1038/nbt0708-737

[4] Paulson, Lawrence C. (2018). "Computational logic: its origins and applications", Proceedings of the Royal Society A, Volume 474, Issue 2210, pp. 1-14, February 2018. DOI: https://doi.org/10.1098/rspa.2017.0872

[5] Sablé-Meyer, M.; Fagot, J.; Caparos, S.; van Kerkoerle, T.; Amalric, M.; and Dehaene, S. (2021). "Sensitivity to geometric shape regularity in humans and baboons: A putative signature of human singularity", Proceedings of the National Academy of Sciences, Vol. 118, No.16, 2021. DOI: https://doi.org/10.1073/pnas.2023123118

[6] Shoeb, A.; Edwards, H.; Connolly, J.; Bourgeois, B.; Treves, T.; Guttag, J. (2004). "Patient-specific seizure onset detection", The 26th Annual International Conference of the IEEE Engineering in Medicine and Biology Society. Vol. 1., pp. 419–422, 2004. DOI: https://doi.org/10.1016/j.yebeh.2004.05.005

[7] Syed, Z.; Leeds, D.; Curtis, D.; Guttag, J.; Nesta, F.; Levine, R. A. (2006). "Audio-Visual Tools for Computer-Assisted Diagnosis of Cardiac Disorders", 19th IEEE Symposium on Computer-Based Medical Systems (CBMS'06), pp. 207–212. 2006. DOI: https://doi.org/10.1109/CBMS.2006.50

Received: 13-05-2023 Accepted: 29-06-2023 Published: 24-07-2023
Cite as:
Gopal, T. (2023). "Learning Computational Logic through Geometric Reasoning", Science Series "Innovative STEM Education", volume 05, ISSN: 2683-1333, pp. 7-12, 2023. DOI: https://doi.org/10.55630/STEM.2023.0501