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## MATHEMATICAL MODELS OF DOMAIN ONTOLOGIES<sup>1</sup>

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**Abstract:** *In this article the notion of a mathematical model of domain ontology is introduced. The mathematical apparatus (unenriched logical relationship systems) is essentially used. The representation of various elements of domain ontology in its model is considered. These elements are terms for situation description and situations themselves, knowledge and terms for knowledge description, mathematical terms and constructions, auxiliary terms and ontological agreements. The notion of a domain model is discussed. The notions of a precise ontology and precise conceptualization are introduced. The structures of situations and knowledge and also their properties are considered. Merits and demerits of various classes of the domain ontology models are discussed.*

**Keywords:** *Domain ontology, domain ontology model, ontology language specification, kernel of extendable language of applied logic, unenriched logical relationship systems, enriched logical relationship systems, enrichment of logical relationship system.*

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### Introduction

A few different definitions for the notion of domain ontology have been suggested by now. But every definition has certain flaws. Because different interpretations of the notion of a domain ontology are used when different problems related to domain ontologies are solved, it may be deduced that now there is no universally accepted definition of the notion. This article suggests another definition of the notion of domain ontology. As this takes place, the mathematical apparatus (unenriched logical relationship systems) introduced in [1-3] is essentially used.

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### A Mathematical Model of a Domain Ontology

An unenriched logical relationship system [3] can be considered as a domain ontology model, if each of its logical relationship has a meaningful interpretation that a community of the domain agrees with, and the whole system is an explicit representation of a conceptualization of the domain understood both as a set of intended situations and as a set of intended knowledge systems of the domain. Some examples of unenriched logical relationship systems and their meaningful interpretations as models of simplified domain ontologies were given in [1-2]. Models of ontologies for medicine close to real notions of the domain were described in [5]. Models of ontologies for physical and organic chemistry and also for roentgen fluorescent analysis were described in [6-9]. Model of ontology for classical optimizing transformations is described in [10-12].

Information concerning a finite (real or imaginary) fragment of a real or imaginary reality (the fragment may be related to a finite part of the space and to a finite time lapse) will be called a situation (a state of affairs in terms of [4]), if this fragment contains a finite set of objects and a finite set of relations among them.

Objects and relations (including unary ones) among them depending on situations are designated by special domain terms which will be called terms for situation description. Objects in situation models can be represented: by elementary mathematical objects (numbers and so on); by names having neither sort nor value [1] (such a name is a designation of an object); by structural mathematical objects (sets, n-tuples, and so on) constructed of elementary or structural mathematical objects or names having neither sort nor value by composition rules defined in the language of applied logic.

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The set of names having neither sort nor value and used as designations of objects (and their components) in situation models can be determined explicitly or implicitly in a domain ontology model. In the former case, all these names appear in sort descriptions for unknowns. In the latter case, all these names are constituents of parameter values. In the domain ontology model the names of these parameters are used for describing sorts of unknowns. If a domain ontology model determines some names having neither sort nor value then these names have the same meaning in every situation of the domain. A domain ontology model can determine only some of the names having neither sort nor value and used in situations for designating objects (and their components). In this case these names are determined by a model of situation and may have different meaning in different situations.

Unknowns represent relations among objects depending on situations. In different situations the relations corresponding to the same unknown can be different. Every objective unknown designates a role that in each situation an (unique) object of the situation plays, and also in every situation there is its own object playing the role. Every functional unknown designates a set of functional relations. For each situation this functional relation is the one among objects of the situation. For different situations these relations corresponding to the same unknown can be different. Analogously, every predicative unknown designates a set of nonfunctional relations. For each situation this nonfunctional relation (it may be empty) is the one among objects of the situation. For different situations these relations corresponding to the same unknown can be different.

Thus, every unknown can be considered as a designation of a one-to-one correspondence between situations and the values of the unknown in these situations.

The sort description for an unknown determines the set of value models for the unknown. In any (real or imaginary) situation only an element of this set can be a value of the unknown. Thereby, the sort description for an unknown determines a model of the capacity for the concept designated by the unknown. A model of the capacity for a concept can be both a finite and infinite set.

A model of a (real or imaginary) situation is a set of values of the unknowns for the unenriched logical relationship system representing a domain ontology model. A model of a situation can be represented by a set of value descriptions for the unknowns.

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### Knowledge Models and Terms for Knowledge Description

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If an unenriched logical relationship system is a model of a domain ontology then any of its enrichments is a model of a knowledge system for the domain. If a model of a domain ontology is an unenriched logical relationship system  $O$  without parameters, then the ontology model introduces all the terms for description of the domain. In this case any enrichment  $k$  of the system  $O$  is a set of logical relationships – restrictions on the interpretation of names representing empirical or other laws of the domain. Since this enrichment does not introduce any new names it cannot contain any sort descriptions for names [3].

If a model of a domain ontology is an unenriched logical relationship system with parameters, then the parameters of the system are the domain terms which are used for knowledge description.

If a model of domain ontology is a pure unenriched logical relationship system  $O$  with parameters then any enrichment  $k$  of the system  $O$  is a set  $\alpha_P$  of the parameter values for the system  $O$  [3]. A value of an objective parameter determines a feature of the domain, a set of names for situation description, or a set of parameter names. Every enrichment (a knowledge base) can introduce new names as compared with the ontology – terms for situation and knowledge description. Functional and predicative parameters represent empirical or other laws of the domain. The value of every functional or predicative parameter is some relation among terms and/or domain constants. In this case domain knowledge is described at a higher level of abstraction than in the case when a domain ontology model is an unenriched logical relationship system without parameters. The values of parameters can be represented by a set of propositions – value descriptions for names.

If a model of a domain ontology is a mixed unenriched logical relationship system  $O$  with parameters, then any enrichment  $k$  of the system  $O$  is a pair  $\langle \Phi', \alpha_P \rangle$ , where  $\Phi'$  is a set of logical relationships (restrictions on the interpretation of names) representing a part of empirical or other domain laws, and  $\alpha_P$  is a set of parameter values for the system  $O$  representing the other domain laws [3]. In this case domain knowledge is represented at

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two levels of abstraction: as logical relationships among unknowns of the system  $O$  and as relations among terms of the domain (as parameter values of the system  $O$ ).

The sort description for a parameter determines the set of value models for the parameter. In any knowledge model only an element of this set can be a value of the parameter. Thereby, the sort description for a parameter determines a model of the capacity for the concept designated by the parameter. A model of the capacity for a concept can be both a finite and infinite set.

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### Mathematical Terms and Constructions. Auxiliary Terms

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The language of applied logic [1] determines mathematical terms and constructions used for domain description in that the unenriched logical relationship system which is an ontology model for the domain is represented. The kernel of the applied logic language [1] determines a minimal set of logical means for domain description. The standard extension of the language [1] apart from additional logical means introduces arithmetic and set-theoretic constants, operations and relations. Every specialized extension [2] of the language gives us a possibility to define both additional logical means and constants, operations and relations of other divisions of mathematics. The specialized extensions Intervals and Mathematical quantors of language [2] introduce integer-valued and real-valued intervals, and also mathematical quantifiers. Other examples of mathematical terms which can be introduced by specialized extensions are operations of differentiation and integration, predicates of optimization, and the like.

Mathematical objects (names, numbers, sets,  $n$ -tuples, and the like) serve to represent models of elementary and combined domain objects. Mathematical functions and relations represent the properties of domain objects which are kept with mathematical models in place of domain objects. In every domain a specific mathematical apparatus is used, as a rule. This property of domains is represented by specialized extensions of the language in domain ontology models. At the same time, the practice shows that the same mathematical apparatus can be used for description of different domains. In this case, for description of ontology models of these domains the same specialized extensions of the applied logic language given by the names of these extensions can be used.

Thus, mathematical terms and constructions have more or less universally accepted designations, syntax and semantics. They are separated from a domain ontology by their definition in the applied logic language (in its kernel and extensions) rather than in the unenriched logical relationship system representing the ontology model. They are associated with the domain ontology by the fact that the name of the logical theory representing the set of logical relationships contains the names of all the extensions used for description of this theory. Using mathematical terms and constructions with this interpretation does not constrain the possibility of unenriched logical relationship systems application for representation ontologies of different domains, and mathematics is among them. In the latter case mathematical terms and constructions play the role of elements of the metalanguage with completely defined syntax and semantics, and the other terms play the role of terms of (the domain) mathematics, their semantics being defined by an ontology.

Auxiliary terms are introduced to make a domain ontology description more compact. A value of an auxiliary term is defined by the values of other domain terms: of mathematical terms, terms for situation descriptions, terms for knowledge descriptions and other auxiliary terms. The definitions of auxiliary terms are represented by a set of value descriptions for names in a domain ontology model.

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### Ontological Agreements

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Ontological agreements about a domain are represented by a set of restrictions on the interpretation of names of the unenriched logical relationship system which is an ontology model of the domain. Ontological agreements are explicitly formulated agreements about restrictions on the meanings of the terms in which the domain is described (additional restrictions on capacity of the concepts designated by these terms).

If a domain ontology model is an unenriched logical relationship system without parameters, then all the ontological agreements are only constraints of situation models. The set of ontological agreements, in this case, can be empty, too. If a domain ontology model is an unenriched logical relationship system with parameters then the set of ontological agreements can be divided into three nonintersecting groups: constraints of situation models, i.e. the agreements restricting the meanings of terms for situation description; constraints of knowledge

models, i.e. the agreements restricting the meanings of terms for knowledge description; agreements setting up a correspondence between models of knowledge and situations, i.e. the agreements setting up a correspondence between the meanings of terms for situation and knowledge description. Every proposition of the first group must contain at least one unknown or a variable whose values are unknowns and cannot contain any parameters; every proposition of the second group must contain at least one parameter or a variable whose values are parameters and cannot contain any unknowns; every proposition of the third group must contain at least one parameter or a variable whose values are parameters and at least one unknown or a variable whose values are unknowns. In doing so, the definitions of auxiliary terms should be taken into account.

Now let us define the informal notion of domain ontology using the formal notion of a domain ontology model. The part of information about a domain, which is represented by an ontology model of the domain, will be called an ontology of the domain. It immediately follows that a domain ontology contains a set of capacity concept definitions for situations description (it cannot be empty), a set of capacity concept definitions for knowledge description (it can be empty), characteristics of mathematical apparatus for domain description, a set of auxiliary term definitions (it can be empty), a set of restrictions on the meaning of terms for situation description (it can be empty), a set of restrictions on the meaning of terms for knowledge description (it can be empty), and a set of agreements setting up a correspondence between meanings of terms for situation description and for knowledge description (it can be empty).

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### A Domain Model

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If an unenriched logical relationship system  $O$  is a domain ontology model and  $k \in \text{En}(O)$  is a knowledge model of the domain, then the enriched logical relationship system  $\langle O, k \rangle$  [3] is a model of the domain. In this case the set of solutions  $A(\langle O, k \rangle)$  is a model of the domain reality. Thus, the domain ontology model  $O$  determines a class of the domain models  $\{\langle O, k \rangle | k \in \text{En}(O)\}$ . Every domain model consists of two parts: an ontology model  $O$  that is the same for the whole class and a knowledge model  $k$ , which is specific for a particular domain model  $\langle O, k \rangle$ .

The set of all possible situations in a domain which have ever taken place in the past, are taking place now and will take place in the future will be called the reality of the domain. Thus, the reality has the property that the persons studying the domain, the developers of its conceptualization and its models do not know the reality completely. Only a finite subset of situations forming the reality and having taken place in the past is known (although the information forming these situations also can be not completely known). We will suggest that relative to any conceptualization of a domain the hypothesis on its adequacy is true: the reality is a subset of the set of intended situations. In view of the reality definition it is evident that this hypothesis cannot be verified. Hence, every adequate conceptualization imposes certain limitations on the notion of the reality.

$A(\langle O, k \rangle)$  is an approximation of the unknown set of models of all situations which are members of the domain reality. It is apparent that the better  $A(\langle O, k \rangle)$  approximates the reality the more adequate the domain model  $\langle O, k \rangle$  is. A model of a domain is adequate to the domain, if the set of models of all the situations forming the domain reality is equal to the solution set of the enriched logical relationship system which is a model of the domain, i.e. the reality approximation is precise.

We will consider only such domain ontology models  $O$  that there is the adequate model  $\langle O, k \rangle$  of the domain in the class of models of the domain determined by the ontology model  $O$  (the hypothesis on existence of the adequate domain model). The hypothesis on existence of the adequate domain model is stronger than the hypothesis on conceptualization adequacy. The first hypothesis states that there is such a knowledge base (an element of the set  $\text{En}(O)$ ) that  $A(\langle O, k \rangle)$  is the same as the set of models of all the situations of the domain reality. The second one states only that the latter set is a subset of the set of models of all the intended situations. Inasmuch as the reality is not completely known (not all the situations which took place in the past and take place at present are known, and no future situation is known either), it is unknown for any domain model how well the reality model approximates the reality. Thus, it is impermissible to hold about any domain model that it is an adequate model of the domain. At the same time, a criterion of inadequacy can be formulated: a model of a domain represented by an enriched logical relationship system is an inadequate model of the domain, if such a situation is known which took place in the reality that its model is not a solution of the logical relationship system. If a domain ontology model  $O$  is given, and inadequacy of a domain model  $\langle O, k \rangle$  is revealed, then experts

usually look for some other model of the domain knowledge  $k' \in \text{En}(O)$ , so that the domain model  $\langle O, k' \rangle$  won't be inadequate with respect to the available data (known situations). If in the process of storing empirical data (extending the set of known situations) it becomes clear that inadequacy of the current domain model is sufficiently often found, and that the model has to be permanently modified, and that this process leads to constant increasing of the number of empirical laws and/or to constant growth of complexity of the knowledge model, then an aspiration may arise for finding another conceptualization of the domain and an ontology representing it (changing the paradigm) and for finding an adequate model of the domain within the restrictions of the new conceptualization.

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### A Precise Ontology and Conceptualization

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A domain ontology will be called precise, if the set of situation models forming the conceptualization represented by the ontology is equal to the set  $\bigcup_{k \in \text{En}(O)} A(\langle O, k \rangle)$ , where  $O$  is an unenriched logical relationship system

that is a model of the ontology, i.e. the approximation of the conceptualization determined by the ontology is precise.

A conceptualization will be called precise, if it is the same as the domain reality. It is apparent that precise conceptualizations are impossible for the domains related to the real world. But conceptualizations are possible for theoretical (imaginary) domains (mathematics, theoretical mechanics, theoretical physics and so on) for which their precision is postulated.

If an ontology and conceptualization are precise, then the unenriched logical relationship system  $O$  being a model of this ontology must have the following property: if  $\langle O, k \rangle$  is the adequate model of the domain where  $k \in \text{En}(O)$ , then  $A(\langle O, k' \rangle) \subseteq A(\langle O, k \rangle)$  for any  $k' \in \text{En}(O)$ . If  $O$  is an unenriched logical relationship system without parameters, then the empty set of propositions is this  $k$ .

The question arises of whether in the case of precise conceptualization the empty knowledge base is always consistent with the adequate domain model. Let us discuss this question using the example of an ontology of mathematics. An ontology of mathematics (or any one of its branches) consists of definitions and axioms. Any conceptualization of mathematics is assumed to be precise. At the same time, mathematical knowledge consists of theorems (lemmas, corollaries and so on) and their proofs. Since in mathematics any theorem is a logical consequence of the ontology, the theorems impose no additional restrictions on the reality model. Thus, both the empty knowledge base and a knowledge base containing any set of theorems determine adequate (and equivalent [1]) models of mathematics. The role of theorems is to make explicit the properties implicitly given by the ontology, and the role of proofs is to make evident the truth of theorems. Some theorems can have inflexible form (identities, inequalities and so on). So a mixed unenriched logical relationship system with parameters can be a natural ontology model for mathematics where terms identities, inequalities and others describe knowledge.

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### The Structure of Situations and Knowledge

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The set of the unknowns whose values form a model of a situation will be called the structure of the situation model. We will say that models of two situations have the same structures, if the sets of the unknowns forming the structures of these situations are the same. From this point of view, the models of all the situations belonging to the reality model of any domain model have the same structures, if this domain model is an enriched logical relationship system. As for the structures of intended situation models determined by a domain ontology model that is an unenriched logical relationship system, three cases are possible.

1. A domain ontology model is an unenriched logical relationship system without parameters. In this case all intended situation models have the same structures.
2. A domain ontology model is an unenriched logical relationship system with parameters, none of parameter values being able to contain unknowns. In this case all intended situation models also have the same structures.
3. A domain ontology model is an unenriched logical relationship system with parameters, values of some parameters being able to contain unknowns. In this case the models of the situations belonging to the reality

models of different models of the domain (consistent with different knowledge models) can have different structures depending on knowledge models.

The structures of all the situations determined by the ontology model of example 1 of article [3] are the same. They are formed by the unknowns diagnosis, partition for a sign, moments of examination, blood pressure, strain of abdomen muscles, and daily diuresis. The structures of all the situations determined by the ontology model of example 6 of article [3] also are the same. They are formed by the unknowns cubes, balls, rectangular parallelepipeds, length of an edge, volume, substance, and mass.

The parameter signs in example 2 of article [2] contains unknowns (see propositions 2.2.1 and 2.2.13 in [2]). Thus, situation models determined by this ontology model can have different structures. In [2] an example of a knowledge model for this ontology model was given (see example 3, propositions from 3.1.1 to 3.1.9). The structure of situation models corresponding to that knowledge model is formed by the unknowns diagnosis, partition for a sign, moments of examination, strain of abdomen muscles, blood pressure and daily diuresis. If in another knowledge model of the same ontology model the parameter signs has the different value

signs = {pain, temperature, discharge},

and the other parameters have some proper values, then the structure of situation models corresponding to this knowledge model is formed by the unknowns diagnosis, partition for a sign, moments of examination, pain, temperature and discharge, i.e. these structures differ from one another.

Using parameters whose values contain unknowns makes it possible "to hide" some terms used for situation description in domain ontology model description. At the same time, the meanings of these unknowns are completely determined by the propositions describing the sorts of these unknowns (see proposition 2.2.13 of example 2 of [2]): models of concepts designated by these unknowns are determined, for any unknown its meaning in a situation is determined (either the unknown is a name of a role, a functional relation or an unfunctional one), for every name of relation the number of its arguments, the sorts of its arguments and the sort of its result are determined.

The set of parameters of the unenriched logical relationship system being a domain ontology model will be called the structure of domain knowledge model. It follows from this definition that if an unenriched logical relationship system without parameters is a domain ontology model, then any knowledge model of this domain has no structure. If a mixed unenriched logical relationship system with parameters is a domain ontology model, then a part of any knowledge model has a structure but the other its part has no structure. If a pure unenriched logical relationship system with parameters is a domain ontology model, then all parts of any knowledge model of the domain have a structure. Let a domain ontology model be an unenriched logical relationship system with parameters. If no parameter value in its turn contains a parameter, then all domain knowledge models for this conceptualization have the same structures. If values of some parameters in their turn contain parameters, then different knowledge models of the domain can have different structures.

Using parameters whose values contain parameters makes it possible "to hide" some terms used for knowledge description in domain ontology model description. At the same time, the meanings of these terms are completely determined by the propositions describing the sorts of these terms.

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## A Comparison between Different Ontology Model Classes

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Now let us discuss the question about capabilities of domain models and domain ontology models, which are enriched and unenriched logical relationship systems of different classes.

Let us consider several aspects of the term "domain ontology".

1. If a conceptualization contains intended situations of different structures, then any ontology representing this conceptualization cannot have a model in the class of unenriched logical relationship systems without parameters but can have a model in the class of the systems with parameters.
2. If a conceptualization contains concepts designated by terms for knowledge description, then no ontology representing this conceptualization can have a model in the class of unenriched logical relationship systems without parameters, but it can have a model in the class of the systems with parameters.

3. If a conceptualization contains concept classes and determines properties of the concepts belonging to these classes, and concepts themselves are introduced by domain knowledge, then no ontology representing this conceptualization can have a model in the class of unenriched logical relationship systems without parameters but can have a model in the class of the systems with parameters.
4. If in a conceptualization some restrictions on meanings of terms for situation description depend on the meaning of terms for knowledge description, then any ontology representing this conceptualization cannot have a model in the class of unenriched logical relationship systems without parameters, but it can have a model in the class of the systems with parameters.
5. The more compactly and clearly domain ontology models of a class describe agreements about domains, the better the class is. In this regard unenriched logical relationship systems without parameters require for every term for situation description to appear explicitly in these agreements. For real domains (such as medicine) the models of their ontologies turn out immense because of large number of these terms. At the same time, the systems with parameters describe agreements about domains for groups of terms, rather than only for isolated terms through using terms for knowledge description. In doing so the majority of the terms for situation description and some terms for knowledge description do not appear explicitly in agreement descriptions (they are replaced by the variables whose values are terms from appropriate groups). As a result, a model of agreements becomes compact and agreements themselves become more general.
6. The more understandable knowledge bases represented in terms of an ontology are for domain specialists, the better the class of domain ontology models is. In this respect unenriched logical relationship systems without parameters represent knowledge bases as sets of arbitrary logical formulas. The more complex these formulas are, the more difficult it is to understand them. At the same time, the systems with parameters introduce special terms for knowledge description. The meanings of these terms are determined by ontological agreements, and their connection with terms for situation description among them. In real domains these terms are commonly used to ease mutual understanding and to make communication among domain specialists economical. The meanings of these terms are, as a rule, understood equally by all domain specialists. The role of these terms is to represent domain knowledge as relation tables (sets of atomic formulas, of simple facts). It is considerably easier for domain specialists to understand the meanings of these simple facts than the meanings of arbitrary formulas.
7. The more precise approximation of a conceptualization model a class of domain ontology models assumes, the better it is.

First, let us remark that it follows from the theorem about eliminating parameters of enriched logical relationship systems [3] that if there is a domain model represented by an enriched logical relationship system with parameters which determines an approximation of the domain reality, then there is a model of the domain represented by an enriched logical relationship system without parameters which determines the same approximation of the domain reality. In this regard domain models represented by enriched logical relationship systems with parameters offer no advantages over domain models represented by enriched systems without parameters.

As for domain ontology models, every one represented by an unenriched logical relationship system determines some approximations for both the set of intended domain situation models and for the set of intended domain knowledge models. If a model  $O_P$  of a domain ontology represented by an unenriched logical relationship system with parameters determines an approximation  $\bigcup_{k \in \text{En}(O_P)} A(< O_P, k >)$  of the set of intended domain situation

models, then the unenriched logical relationship system  $O_X$  without parameters quasiequivalent to  $O_P$  determines the approximation  $\bigcup_{k \in \text{En}(O_X)} A(< O_X, k >)$  of the same set of intended situation models [1]. Let  $h : \text{En}(O_P) \rightarrow$

$\text{En}(O_X)$  be the map defined by the theorem about eliminating parameters of unenriched logical relationship systems and  $H = \{h(k) \mid k \in \text{En}(O_P)\}$ . Then  $\bigcup_{k \in \text{En}(O_X)} A(< O_X, k >) = \bigcup_{k \in \text{En}(O_P)} A(< O_X, h(k) >) \cup$

$\bigcup_{k \in \text{En}(O_X) \setminus H} A(< O_X, k >)$ ; but by the theorem about eliminating parameters of enriched logical relationship systems

$$\bigcup_{k \in \text{En}(O_P)} A(\langle O_X, h(k) \rangle) = \bigcup_{k \in \text{En}(O_P)} A(\langle O_P, k \rangle), \text{ i.e. } \bigcup_{k \in \text{En}(O_X)} A(\langle O_X, k \rangle) = \bigcup_{k \in \text{En}(O_P)} A(\langle O_P, k \rangle) \cup \bigcup_{k \in \text{En}(O_X) \setminus H} A(\langle O_X, k \rangle).$$

Thus, the approximation of the set of intended situation models determined by the system  $O_X$ , is less precise than the approximation represented by the system  $O_P$ .

If a model  $O_P$  of a domain ontology represented by an unenriched logical relationship system with parameters determines an approximation  $\text{En}(O_P)$  of the set of intended domain knowledge models, then the unenriched logical relationship system  $O_X$  without parameters determines an approximation  $\text{En}(O_X)$  of the same set of intended knowledge models. In this case  $H$  is a subset of  $\text{En}(O_X)$ , i.e. the approximation of the set of intended knowledge models determined by the system  $O_X$  also is less precise than the approximation determined by the system  $O_P$ . In what follows we show some reasons of this fact.

Let us consider the case when a domain ontology model is a pure unenriched logical relationship system  $O_P$  with parameters. First, the constraints of knowledge models represented by  $O_P$  determine the set  $\text{En}(O_P)$  as a proper subset of the set of all possible interpretations of the system  $O_P$ 's parameters, whereas, if the system  $O_X$  without parameters is a domain ontology model, then this ontology model contains practically no restrictions on the set  $\text{En}(O_X)$ . Second, for the theorem about eliminating parameters of enriched logical relationship systems a set of formulas representing empirical and other domain laws can be deduced from every proposition setting up a correspondence between knowledge models and situation models and from parameter values. These formulas contain no parameters. It is obvious that the forms of these formulas are restricted and determined by the forms of propositions setting up a correspondence between knowledge models and situation models. At the same time, if a domain ontology is an unenriched logical relationship system  $O_X$  without parameters, then this system imposes no restrictions on the form of formulas entering its enrichments.

Let us consider the case when a domain ontology is a mixed unenriched logical relationship system  $O_P = \langle \Phi, P \rangle$  with parameters. In this case, if  $k \in \text{En}(O_P)$ , then  $h(k) = \Phi' \cup \Phi''$  where the propositions belonging to  $\Phi'$  are deduced from every proposition of  $\Phi$  setting up a correspondence between knowledge models and situation models and from parameter values (taking into account the parameter constraints) and  $\Phi''$  is such a set of propositions that  $\Phi_X \cup \Phi' \cup \Phi''$  is a semantically correct applied logical theory where  $\Phi_X$  is the set of all the propositions of  $\Phi$  which contain no parameters, i.e.  $H \subset \text{En}(O_X)$ .

Domain ontology models represented by unenriched logical relationship systems with parameters are thus seen to offer certain advantages over domain ontology models represented by unenriched logical relationship systems without parameters (see also [13]).

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## Conclusions

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In the article a notion "a mathematical model of a domain ontology" has been introduced, the representation of different elements of a domain ontology in this model – of terms for situation description and situations themselves; of knowledge and terms for knowledge description; of mathematical terms and constructions; of auxiliary terms, and ontological agreements has been considered. The structures of situations and knowledge and their properties have been considered. The notion "a domain model" has been discussed. Definitions of the notions "precise ontology" and "precise conceptualization" have been presented. Some merits and demerits of different domain ontology model classes have been discussed in details.

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## A METHOD OF ESTIMATING USABILITY OF A USER INTERFACE BASED ON ITS MODEL

Valeriya Gribova

**Abstract.** *The article presents a new method to estimating usability of a user interface based on its model. The principal features of the method are: creation of an expandable knowledge base of usability defects, detection defects based on the interface model, within the design phase, and information to the developer not only about existence of defects but also advice on their elimination.*

**Keywords:** *Ontology, defects, interface model, user interface development*

**ACM Classification Keywords:** *I.2.2 Artificial intelligence: automatic programming*

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### Introduction

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Quality and speed of software development are traditionally considered as a compromise where one of them is paid more attention than the other. However, to remain a competitive company developing software should not