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LEONTIEF MODEL ANALYSIS WITH FUZZY PARAMETERS BY BASIC MATRIXES METHOD

Vladimir Kudin, Grigoriy Kudin, Alexey Voloshin

Abstract: The basic matrixes method is suggested for the Leontief model analysis (LM) with some of its components indistinctly given. LM can be construed as a forecast task of product's expenses-output on the basis of the known statistic information at indistinctly given several elements' meanings of technological matrix, restriction vector and variables' limits. Elements of technological matrix, right parts of restriction vector LM can occur as functions of some arguments. In this case the task's dynamic analog occurs. LM essential complication lies in inclusion of variables restriction and criterion function in it.

Keywords: Leontief model, quantitative and qualitative analysis, fuzzy set, basic matrix, membership function.

Introduction

Mathematical apparatus of fuzzy sets is the way of indefinite parameters assigning, the values of which are unknown until the moment of decision-making. One of the mechanisms of vagueness removal in parameters assigning at model construction is the presence in the outline the decision-making person (DMP). DMP is aimed in workmanlike manner to determine the model's structure, to indicate the mechanism of vagueness removal at its formation [Orlovskij, 1981]. LM essential complication (LM) [Leontief, 1972], [Hass, 1961] is the inclusion of restrictions on variables' meanings (values) [Orlovskij, 1981]. One of the LM peculiarities is the inclusion of mathematical problems analysis series of linear systems as systems of linear algebraic equation (SLAE) with the quadratic nondegenerate matrix of restrictions, linear algebraic inequalities (SLAI), with the corresponding matrix of restrictions and also the tasks of linear programming (TLP) [Voloshin, 1993], [Vojnalovich, 1987], [Vojnalovich, 1988], [Kudin, 2002]. Realization of model's qualitative analysis [Orlovskij, 1981] predetermines as well the inclusion of quantitative analysis of its structural elements' consistency [Voloshin, 1993], [Vojnalovich, 1987], [Vojnalovich, 1988], [Kudin, 2002]. We can differentiate the following stages of analysis:

- testing of mathematical and computer-assisted non-degeneracy of restrictions matrix, determination of its rank's value;
- directing correction of restrictions matrix's rank's value with the means of changing its single elements (in case of necessity);
- revelation of LM common features itself and the restrictions on variables – solubility (insolubilities);

- determination of LM restriction's peculiarities for polyhedral set of relative constraints on variables, realization, in case of necessity, directed changes;
- solution finding at solvability by compatibility;
- solutions properties establishment.

Problem formulation

Let's examine LM variants, which one can achieve as the result of canonical model equivalent transformations:

1. (SLAE) of mode

$$Au = C \ , \tag{1}$$

2. (SLAI) of mode

$$Au \le C \ , \tag{2}$$

3. SLAI models (2) can be investigated in presence of aspect's criterion function

$$\max_{u \in R^m} \mathrm{Bu} \ , \tag{3}$$

as the task of linear programming model's analysis (2)-(3), in which $A = \{a_{ij}\}_{\substack{i=1,\bar{m} \\ j=1,\bar{m}}}$ is the quadratic undegenerate matrix with dimension $(m \times m)$, $a_j = (a_{j1}, a_{j2}, ..., a_{jm}),\ j \in J = I = \{1, 2, .., m\}$ – matrix's lines, A , $u = (u_1, u_2, ..., u_m)^T$ – variables' vector,, $B = (b_1, b_2, ..., b_m),\ C = (c_1, c_2, .., c_n)^T$ – vector of gradient criterion function and model's constraints, $a_j u \le c_j,\ j \in J$ half-space which is determined by hyperplane $a_j u = c_j,\ j \in J$.

The elements of constraints are considered as $a_l u \le c_l,\ l \in J$, normals of which occupies k th can undergo changes in accordance with the correlations $a_l(t)u \le c_l(t),\ l \in J$, where $a_l(t) = (a_{l1}(t),\ a_{l2}(t),\ ...\ , a_{lm}(t)),\ l \in J$ (element of model a_{lr} becomes equal $a_{lr}(t),\ \ r \in I$, and c_l will be $c_l(t)$). All certain functions that depend from the argument $t \in (-\infty, +\infty)$ of the class C^2 . Such changes in model's elements can be construed as the price changes impact in the range $t \in (t_H, t_B)$ on the value of model's technological elements (2) by k th resource.

It is suggested that the system contains $P = \{1, 2,\ ...\ , p\}$ experts. Each expert forms its own membership function $\mu_p(t)$, $p \in P$. These functions are piecewise linear, for which the expert determines levels of value $\lambda^{(p)},\ p \in P$. This means that at $1 \ge \mu_p(t) \ge \lambda^{(P)}$ is defined the range of value change $T_p,\ p \in P$, where $T_p = [t^{(-)}_{p(H)}, t^{(+)}_{p(B)}] \subseteq (-\infty, +\infty),\ p \in P$ [Orlovskij, 1981]. The resultant range variable's modification $T = (t_H,\ t_B)$ coordinated by P experts can be determined, e.g., as $T = \bigcap_{p=1}^{P} T_p$ (or $T = \bigcup_{p=1}^{P} T_p$). Model (1) is investigated in the area of E^m . At the presence in the outline the experts' decision making (DMP) phase of qualitative analysis of models (1), (2)-(3) defines a consequent task, as the task of quantitative analysis – investigations at given levels $\lambda^{(p)},\ \ p = \{1, 2, ...p\}$ indicated by experts of elements changes constraints' influence according to the correlation $a_l(t)u \le c_l(t),\ l \in J$ at $t \in T$ on an earlier chosen optimal decision.

In the work is suggested the development of sequential analysis' methodology [Voloshin, 1987] and the basic matrixes method (BMM) [Kudin, 2002] for realization of quantitative analysis of functional changes' influence in LM on its properties – like nonsingularity of constraints matrix, optimal decisions of the original problem (2)-(3) at model's elements changes (k th constraint) in the form of $a_l(t)u \le c_l(t),\ l \in J$, $t \in T$.

Main principles of Basic Matrixes Method (BMM)

In the suggested BMM are introduced horizontal basic matrixes [Vojnalovich, 1987], [Vojnalovich, 1988], [Kudin, 2002]. During interpretation of task's solution the basic matrixes are consecutively changing by leading in and out of her lines-perpendiculars of constraints. In the common case in the model under consideration the number of restrictions exceeds the number of variables mode (2), and in this case in LM $m = n$:

Definition 1. The matrix, which is made up from m linearly independent perpendiculars of constraints (2), we will consider to be basic, and the solution of corresponding to her system of equations $A_б u_0^T = C^0$ as well basic. Two basic matrixes that differ in one line are called adjacent.

Let: β_{ij}, $i, j \in I = \{1, 2, ..., m\}$ – elements of basic sub-matrix $A_б$, e_{ri} – elements of matrix $A_б^{-1}$, which is inverse to к $A_б$; $e_k = (A_б^{-1})_k$.– column of inverse matrix. Solution $u_0 = (u_{01}, u_{02}, ..., u_{0m})$ of equations set $A_б u^T = c^0$, where, in general case, c^0 – subvector C, the components of which consist of right parts of constraints (2), forming normals basic matrix $A_б$; $\alpha_r = (\alpha_{r1}, \alpha_{r2}, ..., \alpha_{rm})$ - expansion vector of normals of constraints $a_r u_1 \le c_r$ by the lines of basic matrix $A_б$, $\alpha_0 = (\alpha_{01}, \alpha_{02}, ..., \alpha_{0m})$ - expansion vector of criterion function's gradient (3) by the lines of basic matrix $A_б$, $\Delta_r = a_r u_0^T - c_r$ – excess of r[th] constraint (2) in the top u_0; $J_б$, J_H, $J = J_б \cup J_H$ - an indexes' set of basic and nonbasic constraints (2). In the work [Vojnalovich, 1987] are given the connections formulas of basic solution, expansion coefficient of constraints normals and criterion function (3), inverse matrix's coefficients, discrepancies of constraints and criterion function's values at transfer to basic matrix $\overline{A}_б$, which is formed from matrix $\overline{A}_б$ with the change of its line a_k to a_l, that is not included into the basic matrix $A_б$. The inserted quantities in the new basic matrix $\overline{A}_б$ we will call – elements of basic matrixes' method and will be marked with a sign-line above, i.e. $\overline{\beta}_{ij}$, $\overline{\alpha}_r$, $\overline{\Delta}_k$, $\overline{e}_{ri}$, $\overline{\alpha}_0$. Let $a_{i1}, a_{i2}, ..., a_{im}$ - are constraints normals, $a_j u^T \le c_j$, $j \in J_б$, where $J_б = \{i_1, i_2, ..., i_m\}$ - are constraints indices, the normals of which form the lines of basic matrix $A_{б,}$ a_l - is a normal of constraints $a_l u \le c_l$, $\alpha_l = (\alpha_{l1}, \alpha_{l2}, ..., \alpha_{lm})$ - – are the coefficients of vector's a_l expansion into lines of basic matrix $A_{б,}$.

Lemma 1. (linear independence criterion of vectors system). An essential and sufficient condition of matrix row's linear independence models $a_{i_1}, a_{i_2}, ..., a_{i_{k-1}}, a_l, a_{i_{k+1}}, ..., a_{i_m}$, which are formed by substitution of the row a_{i_k}, that occupies k[th] row in basic matrix $A_б$, for row a $_l$, is the fulfillment of the condition $\alpha_{lk} \ne 0$.

Theorem 1. (About the connection between adjacent basic matrixes) Between the expansion coefficient of constraints normals (2) and criterion function (3) for the rows of basic matrix, elements of inverse matrix, basic solutions, discrepancy of constraints (2) and meanings of criterion function for two adjacent basic matrixes exist the following correlations

$$\overline{\alpha}_{rk} = \frac{\alpha_{rk}}{\alpha_{lk}}, \quad \overline{\alpha}_{ri} = \alpha_{ri} - \frac{\alpha_{rk}}{\alpha_{lk}} \alpha_{li,} \quad r = \overline{0, n}; \; i = \overline{1, m}; \; i \ne k; \tag{4}$$

$$\overline{e}_{rk} = \frac{e_{rk}}{\alpha_{lk}}, \quad \overline{e}_{ri} = e_{ri} - \frac{e_{rk}}{\alpha_{lk}} \alpha_{li,} \quad r = \overline{1, m}; \; i = \overline{1, m}; \; i \ne k; \tag{5}$$

$$\overline{u}_{0j} = u_{0j} - \frac{e_{jk}}{\alpha_{1k}} \Delta_l, \quad j = \overline{1, m}, \tag{6}$$

$$\overline{\Delta}_k = -\frac{\Delta_l}{\alpha_{lk}}, \quad \overline{\Delta}_r = \Delta_r - \frac{\alpha_{rk}}{\alpha_{lk}} \Delta_l, \quad r = \overline{1, n}; \qquad r \ne k; \tag{7}$$

$$B\overline{u}_0 = Bu_0 - \frac{\alpha_{0k}}{\alpha_{lk}} \Delta_l, \tag{8}$$

moreover, the condition that the matrix remains basic at substitution by vector a_l k[th] row of basic matrix $A_б$, is the fulfillment of condition $\alpha_{lk} \ne 0$, by the term of supporting basic solution permissibility is $\alpha_{lk} < 0$, the growth of criterion function's value $\alpha_{0k} < 0$.

The proof of lemma 1 and theorem 1 is based on theoretical statements that are stated at [Vojnalovich, 1987], [Vojnalovich, 1988], [Kudin, 2002].

Correlation (4)-(8) will be basic for the creation of search algorithm not only optimal solution but carrying out the analysis of LM properties with the means of basic matrixes method.

Definition 2. Feasible basic decision u_0 is optimal if $Bu_0 \geq Bu$ for all u that meet (2).

Theorem 2. For the basic decision is optimal u_0 is needed and not negativity of expansion coefficients of criterion function vector normal is sufficient (3) on the rows of basic matrix $A_{б}$, i.e. $\alpha_{ok} \geq 0$ for all $k = \overline{1,m}$, moreover the task (2),(3) with the square nonsingular matrix of constraints has a unique solution only when $\alpha_{0i} > 0,\ i = \overline{1,m}$, while is needed and is sufficient the condition of not uniqueness of task's solution is $\exists\, i \in I$ of such, that $\alpha_{0i} = 0$, in which the solution set is not limited.

Optimal criterion validity arises from formula (8) theorem 1.

Consequence 1. At the variable's values, the function where $\alpha_{lk}(t) = a_l(t)(A_b^{-1})_k \neq 0,\ t \in [t_0, t_k]$ is preserved unchangeable the rank quantity while changing the constraint k LM into constraint $a_l(t)u \leq c_l(t),\ l \in J$ where $t \in T$. The values t under which $\alpha_{lk}(t) = a_l(t)(A_b^{-1})_k = 0,\ t \in [t_0, t_k]$, $l \notin J_б$ diminish the system's rank (constraints matrix of model (2) becomes degenerated).

Consequence's validity arises from lemma 1.

Consequence 2. Sufficient condition of optimality preservation by the means of changing k constraint of model in the form of $a_l(t)u \leq c_l(t),\ l \in J_б$ the normal of which occupies the k row in the basic matrix serves as fulfillment of correlations: $a_l(t)(A_\sigma^{-1})_k > 0$, $\frac{\alpha_{0k}}{\alpha_{lk}(t)} \geq 0$, $\frac{\alpha_{0i}}{\alpha_{0k}} \geq \frac{\alpha_{li}(t)}{\alpha_{lk}(t)}$. $t \in T,\ i \neq k,\ i \in I$. The condition of uniqueness of solution (2), (3) are $\frac{\alpha_{0k}}{\alpha_{lk}(t)} > 0,\ t \in T$, $\frac{\alpha_{0i}}{\alpha_{0k}} > \frac{\alpha_{li}(t)}{\alpha_{lk}(t)},\ t \in T,\ i \neq k,\ i \in I$.

Evidence. In accordance with lemma 1 and consequence 1 the condition of matrix's non-degeneracy model (2) is the implementation by all $t \in T$ the correlation $\alpha_{lk}(t) = a_l(t)(A_\sigma^{-1})_k \neq 0$. From formula (4) and theorem 2 follows that by substitution of k row of the basic matrix for the fulfillment of the optimality conditions is necessary to accomplish $\overline{\alpha}_{0k} = \frac{\alpha_{0k}}{\alpha_{lk}(t)} \geq 0$, $\overline{\alpha}_{0i} = \alpha_{0i} - \frac{\alpha_{0k}}{\alpha_{lk}(t)}\alpha_{li}(t) \geq 0,\ i \neq k,\ i \in I$, $t \in T$, as the optimality condition is $\alpha_{0i} \geq 0,\ i \in I$. From this it follows that the fulfillment of the condition $\alpha_{lk}(t) = a_l(t)(A_\sigma^{-1})_k \geq 0$ and $\frac{\alpha_{0i}}{\alpha_{0k}} \geq \frac{\alpha_{li}(t)}{\alpha_{lk}(t)}$, $t \in T$ for preservation of solution's optimality. From here is immediate, taking into account theorem 2, comes the validity of uniqueness (not uniqueness) conditions solutions (2)-(3).

Conclusions

The usage of simplex ideology on the basis of BMM at analysis of LM enables:

- to investigate the properties of solutions of SLAE and SLAI (1),(2) at changes in vectors of constraints;
- to carry out the analysis of LM properties by changing the values of separate elements and its components;
- to use the solution of basic LM at analysis of perturbed model;
- to control or directly to change the value system's rank;
- to discover a solution for the square system of equation by a fixed number of steps;
- to construct first task's solutions on the basis of trivial basic matrixes which exclude laborious pioneering calculations;
- to apply the diagram for tasks analysis which presuppose multistepness or recurrence of calculations on models by changes in model's components.

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APPROXIMATION OF EXPERIMENTAL DATA BY BEZIER CURVES

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Abstract: *Very often the experimental data are the realization of the process, fully determined by some unknown function, being distorted by hindrances. Treatment and experimental data analysis are substantially facilitated, if these data to represent as analytical expression. The experimental data processing algorithm and the example of using this algorithm for spectrographic analysis of oncologic preparations of blood is represented in this article.*

Keywords*: graphics, experimental data, Besie's curves*

ACM Classification Keywords*: I.4 Image processing and computer vision - Approximate methods*

Introduction

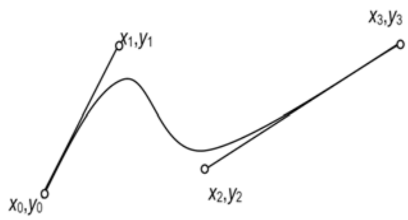

Fig. 1. Bezier curve

The experimental data, as a rule, represent the distorted by hindrances certain process fully determined by some unknown function *y= f(x)*, and distorted by hindrances. In most cases experimental data are represented as the graphical curves. The graphs, i.e. graphical curves is, apparently, the simplest and a long ago in-use means of cognitive presentation of experimental data in the most different scopes of human activity which allow to estimate evidently the qualitative property of the process, in spite of hindrances, measurement errors. Graphs displaying the same process, description of certain object can substantially differ