STRUCTURAL ANALYSIS OF CONTOURS AS THE SEQUENCES OF THE DIGITAL STRAIGHT SEGMENTS AND OF THE DIGITAL CURVE ARCS

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Abstract: Recognition of the object contours in the image as sequences of digital straight segments and/or digital curve arcs is considered in this article. The definitions of digital straight segments and of digital curve arcs are proposed. The methods and programs to recognize the object contours are represented. The algorithm to recognize the digital straight segments is formulated in terms of the growing pyramidal networks taking into account the conceptual model of memory and identification (Rabinovich [4]).

Keywords: contour, segments of digital lines arc of digital curves

Introduction

The structural analysis of the object contours in the image as the sequences of straight segments and of curve arcs is one of the tasks of image processing in the artificial intelligence systems.

In most cases the IMAGE can be CONSIDERED as the PART of a PLANE, where the objects are located. The objects optical parameters – optical density, color, texture – are different from the similar parameters of rest of the IMAGE – background. A BORDER, i.e. contour, IS inalienable PROPERTY of every object and is a simply connected sequence consisting of segments of lines and arcs of curves. The IMAGE, as a rule, is discrete. Accordingly, the straight segments and curve arcs, the contours consist of, ARE the *digital straight segments* and the *digital curve arcs*.

Automatic segmentation of arbitrary contour on the digital straight segments and/or the digital curve arcs is the purpose of the work.

There are algorithms to select the *digital straight segments* in a contour [1,2]. The curvilinear image elements, represented as splines, Bezie curves, etc., are used in a great number of applications. At the same time the arcs of arbitrary curves, as elements of the description of contours, practically is not used for recognition of image contours mainly by reason of the absence of common determination of the arc of arbitrary digital curve. The use of curve arcs as the structural elements of image contours description would approach its description to intuitional, natural representation of images by a man, substantially would shorten the expenses of memory for storage of image and image processing time. As an example we will consider the description of contours of binary images, which are obtained, with the use of tools of widespread graphics editor Corel Draw.

On fig.1 the contours of three identical objects are represented, which are not to be distorted by noises. Each of the objects contains the arc of ellipse and differs from the other objects by spatial position and rotation angle.

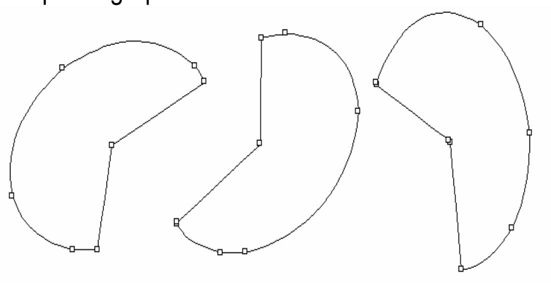

Fig.1 Selection of arcs of ellipses in three identical, rotated in relation to each other objects by means of Corel Draw

The boundary points which divide contours into the curve arcs and the straight segments are marked by the squares. Identical arcs, which belong to different objects, are represented by sequences containing the different amount of different arcs of curves. Each of the identical objects in the image is represented with the different elements. Such description of objects can not be directly used in intelligence systems for interpretation of images supposes hard image processing enough.

The represented example shows existence of the problem with the images not distorted by noises and the actuality to solve it.

Contour as the sequence of L-elements

When the image contours to be digitized the straight segments and the curve arcs transform to digital straight segments and digital curve arcs. We will consider the discrete image, as two-dimensional cell complex [1]. The pixels are two-dimensional square elements for the given case. Besides pixels *there* are *cracs* and points. Cracs are the sides of pixels, being one-dimensional elements. Points are the end points of *cracs* and angular points of pixels. Then the image object contour is the connected *crac* sequence*,* to be a *boundary* between the object and background pixels. The characteristic features of straight segments and curve arcs to a great extent are lost as a result of digitization.

In fig. 2 the example of the objects initial contour, formed by a curve arc and a straight segment, and also its digital equivalent, as the *crac* sequence, are represented. Some connected parts of the *crac* sequence can be united in *L-elements*.

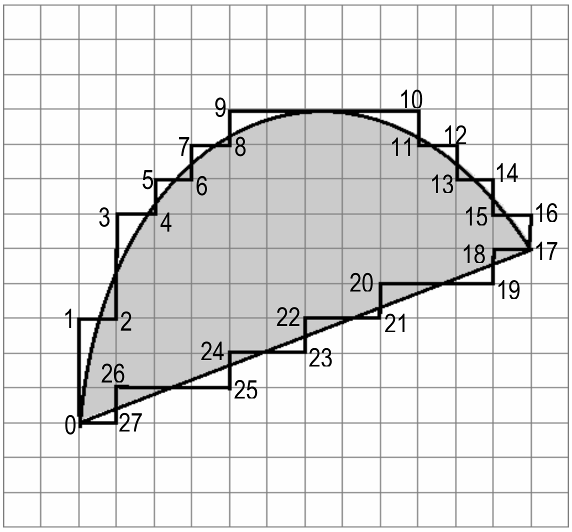

Fig.2 The example of the contour digitization

An L-element is the connected crac sequence of vertical (horizontal) orientation, which contains no more than one crac of horizontal (vertical) orientation.

As in the work [2], we will use such L-element configuration in which the *crac* (if it is present) different in the orientation relatively to the other cracs of the L-element is contained at the end of the L-element. Each L-element is defined with such parameters: *g* – direction in relation to initial its point *of* (using *g* = 0 – for direction upward, 1 – to the right, 2 – downward, 3 – to the left); *l* – amount of cracs of the same direction (*l* = 1,2,.); *q* – the last crac direction in relation to previous cracs value *g* (*q* = -1, if the last crac is directed to the left in relation to direction *of g*, +1 - to the right, 0 - coincides with direction *of g*). For the L-element (0-2) *g*=0, *l*=3, *q*=+1. For the L-element (27-0) *g*=3, *l*=1, *q*=0.

Determination of segments of digital lines in the sequence of L-elements of contour

Lets (x_1,y_1), (x_2,y_2) are integer-valued co-ordinates of straight segment first and last points. The segment slope ratio is defined by the relation of differences of its co-ordinates $n=\Delta x=|x_1 - x_2|$ and $m=\Delta y=|y_1 - y_2|$, which, in the general case, is not an integer. We will put for definiteness, that $n > m$. The digital straight segment of arbitrary slope ratio can be set by means of two types of L-elements, having identical directions, their lengths are equal accordingly $l, l+1$, thus $l \le n/m \le l+1$. The order of L-elements alternation determines the digital straight segment structure and is defined by the values of members of continued fraction $[l;\, k_1, k_2,., k_t]$ or

$$\frac{n}{m} = l + \frac{r}{m} = l + \frac{1}{k_1} + \frac{r_1}{r} = \ldots = l + \frac{1}{k_1} + \frac{1}{k_2} + \frac{r_2}{r_1} = \ldots = l + \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \ldots + \frac{1}{k_t}. \qquad (1)$$

As follows from the formula (1), *l* is the integer part from the division *of n* by *m* - corresponds *l* cracs of the same direction in succession in the digital straight segment. Together with the joining perpendicular crac they form the L-element of *l* length.

K1-element is the sequence in succession of k_1 L-elements having identical with each other parameters, but the last L-element having the length equal l+1 or l, according to n,m values; k_1 defines the "length" of the K1-element.

Likely, K2-element is the sequence in succession of k_2 L-elements having identical parameters with each other, but the last K1-element having the length equal k_1+1 or k_1 or according to n,m values; k_2 defines the "length" of the K2-element; et cetera to the exhausting of members of continued fraction. The numerator *r* determines the amount of L-elements of length l+1 in this digital straight segment, and also the K1-element amount. In turn numerator r_1 determines the amount K2- element lengths k_1±1 in this digital straight segment. In general numerator r_{t-1} determines the amount K_{t-1}- element lengths k_{t-1}±1 in this digital straight segment.

Under the digital straight segment with the co-ordinates of first and end points (x_1,y_1), (x_2,y_2) we will understand the sequence of L-elements having identical directions g,q, where integer-valued lengths are equal according to **l**, **l***+1, and* $l \leq n/m \leq l+1$, *where* $n = | x_1 - x_2 |$ *and* $m = | y_1 - y_2 |$, *thus, the order of L-elements alternation is defined by the values of members of continued fraction n/m.*

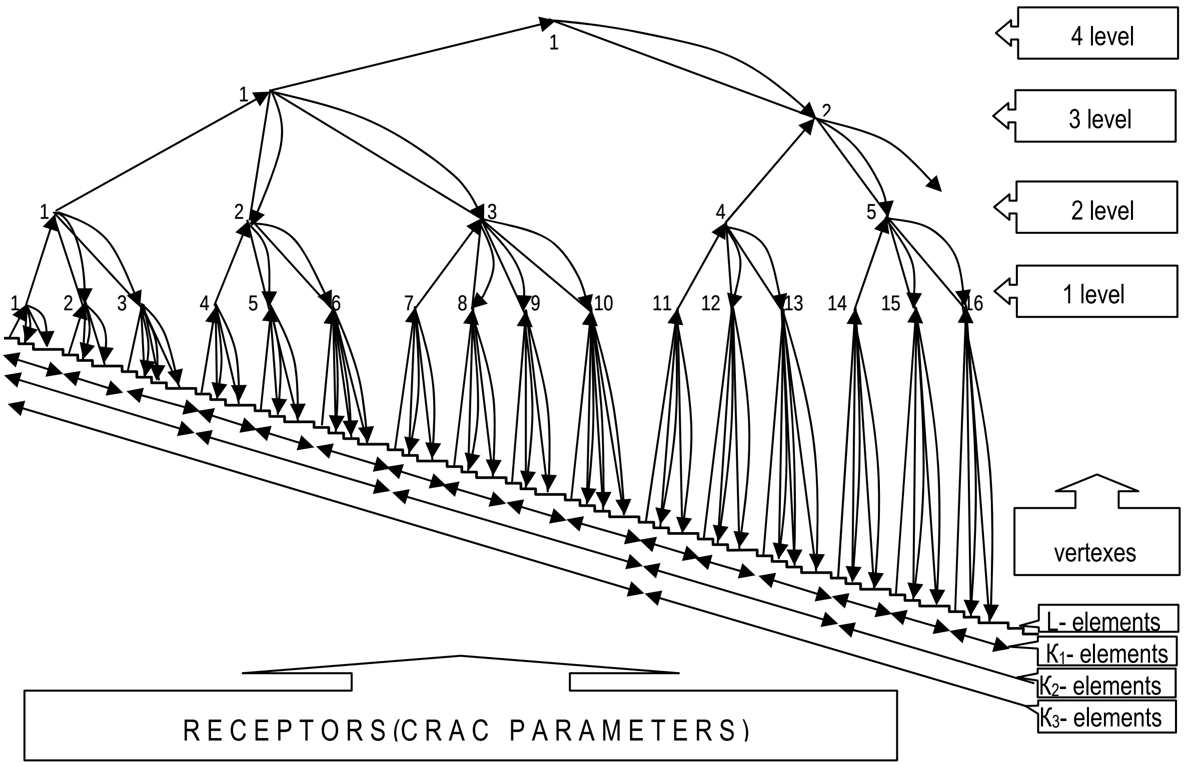

Fig.3 The process of digital straight segment image forming in terms of growing pyramidal networks

The algorithm to select digital straight segments in crac sequence was formulated in the work [3]. It is possible to represent the process of digital straight segment image forming by the mentioned algorithm in terms of growing pyramidal networks [5] (fig.3) taking into account the work by Z. L. Rabinovich [4]. The special case of the growing pyramidal network, when the vertexes subsets have not intersections, is examined. The crac parameters, which form L-elements, are used as receptors. The vertexes of the network are tied-up with conceptors and feed-backs. Conceptors are represented by ascending direct pointers and lines without pointers. Feed-backs are represented by descending pointers. Successive procedure to form the digital straight segment image is examined - from left to right. The algorithm to form digital straight segment image in the arbitrary crac sequence consists in the following.

Starting conditions. The L-elements counter s_0=1; the common amount of L-elements in the sequence is equal N; the vertexes counters of t level: s_1= 1, s_2= 1,., s_t= 1; t = 1; the work constant p = 0.

The list XY of the co-ordinates of digital straight segment ends is empty.

1. The L-element number s_0 parameters is defined and is passed to the vertex number s_1. To go to the following L-element s_0:= s_0 +1; Jump to point 2. If s_0>N, to store the value s_1 and to set s_1 at the starting state: p = s_1; s_1=1; Jump to point 3.

2. Using the feed-back arc, the L-element parameters of vertex number s_1 is compared to the parameters of the following in order L-element number s_0. If the result of comparison is positive (the examined L-elements form K1-

element) Jump to point 1. Otherwise the K1- element construction is completed. If the built K1- element is completed, go to the next vertex of the first level s_1:= s_1 +1; Jump to point 1. Otherwise, the digital straight segment is completed, the vertexes counters are set to the starting state; the co-ordinates of the digital straight segment ends are stored; Jump to point 1.

3. Parameters of the Kt-element are passed from the vertex of *t* level number s_t to the vertex of *t*+1 level number s_{t+1} using a conceptor, go to the new vertex of *t* level s_t:= s_t +1; Jump to point 4. If s_t > p, the value of s_t is stored and s_t is set to the starting state: p:= s_t; s_t:=1; go to the following level t:=t+1. If amount of vertexes of the next level s_t>1, Jump to point 3, otherwise, go to End of algorithm.

4. Using the feed-back arc the Kt-element parameters of vertex number s_{t+1} are compared to the Kt-element parameters of the vertex number s_t. If the result of comparison is positive (the examined Kt-elements form Kt+1-element) Jump to point 3. Otherwise, the K1- element construction is completed. If the built Kt+1- element is completed, go to the next vertex of the *t* level: s_t+1:= s_{t+1} +1; Jump to point 3. Otherwise, the digital straight segment is completed, the vertexes counters are set to the starting state; the co-ordinates of the digital straight segment ends are stored the list *XY*; Jump to point 1.

5. The End of algorithm: forming the list of the co-ordinates of the digital straight segment ends.

The offered algorithm allows restoring of the objects contours of digitized image from the crac sequence to the digital straight segment sequence. However the contours of the initial image consisted not only of straight segments but also of curve arcs. In the next sections of the article methods and algorithms to restore the digital curve arcs of the digital straight segment will be developed. So, the object contours of digitized image will be represented not only as the *sequence of digital straight segments* but the *sequence of digital straight segments and/or of digital curve arcs*.

Determination of arc of digital curve

The determination of digital curve arc will be formulated in this section. This determination allows setting or rejecting a fact: some part of contour sequence, which consists of digital straight segments, is formed as a result of digitizing of some arbitrary curve arc.

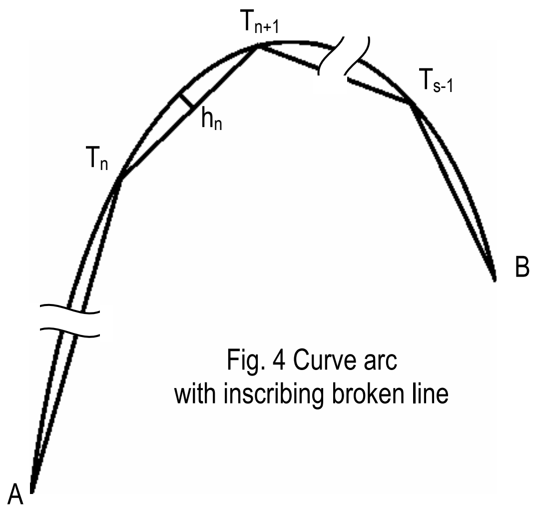

Fig. 4 Curve arc
with inscribing broken line

We will suppose that arcs of curves using in the graphic images, display the segments of continuous functions with continuous derivative ones.

Under the continuous curve lines [6], prescribed functions $x = \varphi(t)$, $y = \phi(t)$, we will understand the Zhordan curves without multiple points or simple arcs, in other words such, that for any two different values t' and t'' points proper to them on a plane M' [$\varphi(t')$, $\phi(t')$] and M'' [$\varphi(t'')$, $\phi(t'')$] are different. Lets $x = \varphi(t)$, $y = \phi(t)$, where $\varphi(t)$, $\phi(t)$ are the continuous functions of parameter *t*, defined on the segment [*a*, *b*]. If *t* is increasing from *a* to *b*, the point with the co-ordinates of *x*, *y* describes the arc *AB* (fig.4). We will consider breaking up of segment [*a*, *b*] with the points of division

$$a = t_0 < t_1 < \ldots < t_{s-1} < t_s = b, \qquad (2)$$

and let curve points *A*, T_1, …, T_{s-1}, *B* correspond to these points of division. Let us connect consistently by the straight segments the point *A* with the point T_1, the point T_1 with the point T_2, ., the point T_{s-1}, with the point *B*, so we will build the broken line, and will name this broken line as entered in the arc *AB*. The arc segment T_n, T_{n+1} (n = 0,1,…,*s*) is the figure, limited by the segment of broken line T_n, T_{n+1} and the proper arc link $\frown T_n, T_{n+1}$. The maximal length of line between a segment T_n, T_{n+1} and $\frown T_n, T_{n+1}$, which is perpendicular to the segment T_n, T_{n+1} is the height of arc segment h_n. Lets

$$\beta = \max_{n=0,1,\ldots,s-1} l(T_n, T_{n+1}) \qquad (3)$$

if β will tend to zero while *s* is increased, the length of any link of the entered broken line will tend to zero, as well as the height of every arc segment, for functions $\varphi(t)$, $\phi(t)$ are continuous.

While the curve arc and its entered broken line are mapped onto discrete space, the value of discontinuity is *d*, the entered broken line segments will be displayed by digital straight segments. The co-ordinate values of discrete space are integer-valued and multiple to *d*. So since a moment, when $h_n < d$, the segment heights will not be mapped in this space, because their lengths will become equal to zero. While $h_n < d$, discrete mapping of arc links will coincide with the proper links of entered broken line – the digital straight segments. We will name these digital straight segments, representing certain curve arc, as ***digital curve arc***. Thus, the contour, which initially consists of segments of lines and of arbitrary curve arcs, after digitizing is mapped as sequence of digital straight segments.

The task is to determine digital curve arcs in the contour sequence of the digital straight segments.

We will examine the pair of neighboring digital straight segments in a sequence which correspondents to curve arc. The pair of neighboring segments determines the finite difference of the second order. We will name two pairs which have a general segment as neighboring pair. Neighboring pair determines the finite difference of the third order.

If the finite differences of the second order are not equal to zero (for the integer-valued co-ordinates of points finite differences must be more than 1), maybe the pair of digital straight segments is part of a digital curve arc. In general, it is possible to conduct many curves through three points, defined by the pair of neighboring digital straight segments. At the same time, the arc segment heights values h_n does not exceed the space value of discontinuity *d* ($h_n < d$). Thus, for the pair of digital straight segments $T_{n-1}T_n$, T_nT_{n+1} to be the part of a digital curve arc, it is necessary to prove the existence of the curve which passes through points T_{n-1}, T_n, T_{n+1}, and the condition is executed for: $(h_{n-1}<d)\&(h_n<d)$.

Usually the curvature of a flat curve is equated with curvature of contiguous circumference arc [7]. By the tangent circumference to plane curve in the point *T*1 is named the limit position of the circumference passing through two points T_2 and T_3 neighboring to the point T_1, while T_2 and T_3 tending to T_1. Abandoning aside the decision of this task in general case, further we will examine the partial case of this task, when a circumference is the curve passing through three points. It is possible to formulate the next ***determination*** as result of the consideration.

Under the digital curve arc in two-dimensional discrete space of discontinuity d we will understand such sequence of digital straight segments, that through ending three points of every pair of neighboring segments it is possible to conduct such circumference, that its segment heights does not exceed d.

This determination is correct as far as it is correct to equate the segment of arbitrary curve arc, which corresponds to the pair of neighboring segments, with the arc of tangent circumference.

The offered determination allows to define, whether a given pair of neighboring digital straight segments corresponds to some arc of the digital arbitrary curve. Lets build the circumference arc on points *Tn-1,Tn,Tn*+1 in accordance with the determination of digital curve arc, and estimate the deviation size of end of each segment from direction of line of preceding segment (fig.5). As the deviation size it is possible to choose segment length $T_{n+1}T^*$ when condition that $l(T_n'')\cong l(T_n,T_{n+1})$. Define the length T_nR_2 as a height of triangle $T_{n-1}T_n\ T_{n+1}$. Maximal distance is between the points of curve arc and proper segment of broken line SR_1 is equals *d*. At the same time

$$SR_1 = OT_{n-1} - OT_{n-1}\times \cos\alpha = r - r\times\cos\alpha = r(1-\cos\alpha).$$

The length of height $\Delta(T_{n-1}T_n\ T_{n+1})$

$$T_nR_2 = OT_n - OT_n\times \cos 2\alpha = r - r\times \cos\alpha = r\times(1-\cos 2a)= 2r\times (1-\cos2\alpha).$$

$$T_nR_2 / SR_1 = 2\times(1+\cos\alpha);\text{ or } T_nR_2 = 2\times(1+\cos\alpha)\times SR_1.$$

If $SR_1 \approx d$ and $\alpha \le 30°$, the height of triangle $(T_{n-1}T_n\ T_{n+1})$ $T_{n2} \le 3{,}85d$. It is not hard to see that the maximal deviation size is

$$Tn+1T' = 2´\ TnR2 \approx 7.7d. \qquad (4)$$

It means that the considered pair of segments could be corresponded to the digital curve arc, if the maximal deviation value T_nT^* is not more than 7.7d. Minimum deviation value must be more than *d*: $T_{n+1}T^* > d$, because if it is less one of the pair of segments grows into one digital straight segment.

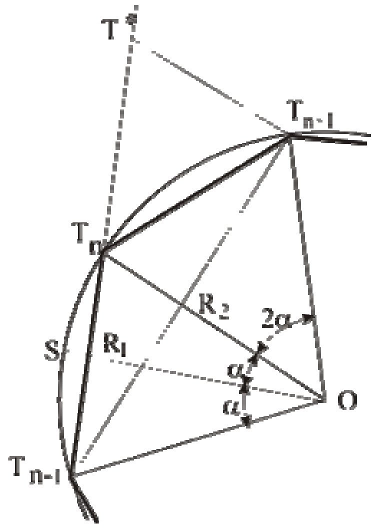

Fig. 5 Estimation of the deviation size of neighbouring segments ends

The example is demonstrated on a fig. 6 the program to recognize the object contours in the image as the sequences of digital straight segments and of digital curve arcs. The image is used from the fig. 1, but the offered algorithms are implemented in the program. Unlike the contours of fig. 1, got by means of the program of Corel Trace, the arcs of contours are represented without lying out of intermediate points on a few arcs regardless of different spatial positions and rotation angles for the each of objects.

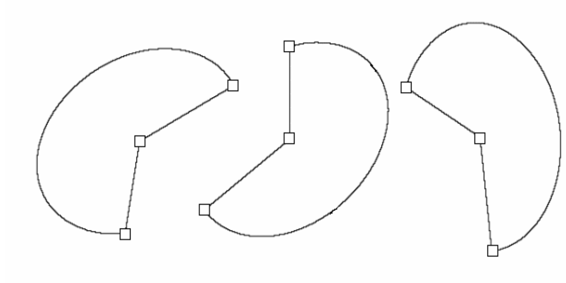

Fig. 6 Object segmentation using describing methods

Conclusion

The automatic construction of reflex of digital straight segment is considered, as the special case of growing pyramidal network taking into account the conceptual model of identification and memory.

Determination of digital curve arc is offered as the sequences of digital straight segments. The algorithms and programs developed on the basis of the attained results allow, unlike known, to execute segmentation of contours in a natural way on the digital straight segment and arcs of digital curves regardless of different spatial positions and rotation angles for the each of objects in the image.

The binary images contours processing had been developed without the loss of information. The questions of the half-tone image segmentation by the contours consisting of digital straight segment and of arcs of digital curves are the subject of the next works.

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