Using this method we have found out the adaptive search algorithm to be optimal for the class of functions with the lowest changing speed.

The using a number of metrical equivalence classes solved by an algorithm as an effectiveness measure was also suggested.

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Authors' Information

Victor Mikhailovich Nedel'ko – Institute of Mathematics SB RAS, Laboratory of Data Analysis, 660090, pr. Koptyuga, 4, Novosibirsk, Russia, e-mail: nedelko@math.nsc.ru

Svetlana Valeryevna Nedel'ko – Institute of Mathematics SB RAS, Laboratory of Data Analysis, 630090, pr. Koptyuga, 4, Novosibirsk, Russia, e-mail: nedelko@math.nsc.ru

FORMALIZATION OF STRUCTURAL CONSTRAINTS OF RELATIONSHIPS IN MODEL „ENTITY-RELATIONSHIP"

Dmytro Buy, Lyudmila Silveystruk

Abstract: The basic conceptions of the model „entity-relationship" as entities, relationships, structural constraints of the relationships (index cardinality, participation degree, and structural constraints of kind (min, max)) are considered and formalized in terms of relations theory. For the binary relations two operators (min and max) are introduced; structural constraints are determined in terms of the operators; the main theorem about compatibility of these operators' values on the source relation and inversion to it is given here.

Keywords: entity, relationship, index cardinality, participation degree, structural constraints of kind (min, max).

ACM Classification Keywords: E.4 Coding and information theory – Formal models of communication

Introduction

Data design is the creation process of logical presentation of database structure. There are different approaches to the data design, which have its own admirers. One of such models is ER-model (Entity-Relationship model, the model „entity-relationship"). This model became traditional and most popular.

This model was introduced by P. Chen in 1976. It is necessary to point out, that this model is constantly being developed and modified. More over the model „entity-relationship" has come into the set of many CASE-instruments, which have made their own contribution into its evolution. Recent studies in the ER-design are represented in C. Batini and S. Cari works, in the work of B. Thelheim [Batini Carlo, Ceri S., Navathe S.B. and Batini Carol, 1991; Thelheim B., 2000].

Today there isn't any unique generally accepted standard for ER-model, but there is the set of general constructions, which lie in the basis of most variants of model [Гарсиа-Молина Г., Ульман Дж., Уидом Дж., 2004, chapter 2; Дейт Дж., 1998, part III, chapter 14; Коннолли Т., Бегг К., Страчан А., 2000, part II, chapter 5; Крёнке Д., 2003, part II, chapter 3].

It is necessary to mark that ER-model isn't a formal model, or to be more precise it is not such a model in the first place. Actually it consists of mainly informal conceptions, but formal aspects present in it too. In this work we formalize the basic conceptions of model as entities, relationships and structural constraint of relationships.

Entities and relationships

The basic conceptions of the model „entity-relationship" are *entities*, *attributes* and *relationships*.

It is necessary to mention, that the model's concept doesn't have any generally accepted precise interpretation; moreover, there is essential distinction in terminology; therefore all variants we find in the literature are specified in the table.

Table 1 – Basic conceptions of ER-model and their names

Notion	1 variant	2 variant	3 variant	4 variant
Entity (consists of own occurrences; contains, generates the own occurrences)	Entity type	Entity set	Entity type	Entity class
Entity occurrence (belongs to the own entity, generated by the own entity)	Entity occurrence	Entity occurrence	Entity	Entity occurrence
Attribute	Property	Attribute	Attribute	Attribute
Relationship (consists of own occurrences; contains, generates the own occurrences)	Relationship type	Relationship	Relationship type	Relationship class
Relationship occurrence (belongs to the own entity, generated by the own entity)	Relationship occurrence	Relationship occurrence	Relationship	Relationship occurrence
Source	[Дейт Дж., 1998, part III, chapter 14]	[Гарсиа-Молина Г., Ульман Дж., Уидом Дж., 2004, chapter 2]	[Коннолли Т., Бегг К., Страчан А., 2000, part II, chapter 5]	[Крёнке Д., 2003, part II, chapter 3]

Further we shell follow the third variant.

Entity. We shall interpret the entity type as a set, and entity as an element of this set.

Relationship. Richly in content, relationship is an association between n entities, which are named its participants. The number of relationship participants is named the relationship degree or relationship arity. Separate relationships form the relationship type, arity of all relationships of one relationship type is identical; thus, arity is the characteristic of relationship type. Here is complete analogy with arity of logic-mathematical relation and quantity of the corteges' components, of which (corteges) the relation consists of.

Let us consider binary relationships. In general case binary relationship is able to join any entity of some entity type with the entity of any other entity type, in particular, with any entity of the same entity type (in particular binary relationship can join entity with itself; here is the complete analogy with the reflexive of binary relations; let us mention, that for such relationships an unsuccessful term „recursive relationship" is used sometimes).

We shall specify the relationship types as logic-mathematical relations; in particular, binary relationship types as binary relations.

Index cardinality. There are standard constraints in this model, which are imposed on the relationship types. Index cardinality is one of such constraints. Index cardinality assigns the quantity of possible relationships for every entity-participant of relationship; speaking more precisely, this index assigns the quantity of the entity which are associated with the fixed entity.

Relationship types with the indexes cardinality *„one-to-one" (1:1)*, *„many-to-one" (M:1)*, *„one-to-many" (1:M)* and *„many-to-many" (M:N)* are selected among the binary relationship types.

Let us suppose that R is relationship type, which connects the entities types E and F. For adequate formalization of index cardinality, we shall interpret the entities types E, F as the sets E, F accordingly, and relationship type R as the binary relation R, were $R \subseteq E \times F$ (the order of sets in the Cartesian product is substantial). As usual $R^{-1} \subseteq F \times E$ is the inverse relation to R.

Intercommunication between the indexes cardinality of relationship type and functionality properties of binary relations R, R^{-1} is shown in table 2.

Table 2 – Intercommunication between the index cardinality and functionality of binary relations and values of operator max

		Relation R	
		R is functional	R isn't functional
Relation R^{-1}	R^{-1} is functional	R – „one-to-one" (1:1)	R – „many-to-one", (M:1), which is directed from F to E R – „one-to-many", (1:M), which is directed from E to F
		$\max(R) \le 1 \wedge \max(R^{-1}) \le 1$	$2 \le \max(R) \le \infty \wedge \max(R^{-1}) \le 1$
	R^{-1} isn't functional	R – „many-to-one", (M:1), which is directed from E to F R – „one-to-many", (1:M), which is directed from F to E	R – „many-to-many" (M:N)
		$\max(R) \le 1 \wedge 2 \le \max(R^{-1}) \le \infty$	$2 \le \max(R) \le \infty \wedge 2 \le \max(R^{-1}) \le \infty$

As we can see, relationship type is „one-to-one" when both relations R, R^{-1} are functional; relationship type is „one-to-many" (equivalently „many-to-one") when exactly one of these relations is functional; finally, relationship type is „many-to-many" when both these relations aren't functional.

So, it is possible to do a derivation: restriction „ one-to-many" („many-to-one") is related to functionality (equivalently: to injectivity); „one-to-one" – to simultaneous functionality and injectivity; finally, „many-to-many" – to the binary relations of general kind. It is necessary to take into account here the obvious logical communication between functionality and injectivity: binary relation is functional if and only if, when inverse relation is injective (see, for example [Буй Д. Б., Кахута Н. Д., 2005, assertion 1]).

Participation degree of entity in relationship. There is another constraint for the relationships types – *participation degree of entity in relationship*. One of the possible interpretations: the participation degree determines dependence of existence of some entity type on participation in the relationship type of other entity type (at least, for total participation in relationship; see farther).

There are two kinds of entity type participations in the relationship type: total and partial. Lets R be the relationship type, and the entity type E be the participant of relationship type R (mark that notion of participant naturally carried from relationships to the relationship types). The characteristic property is: if every entity of type E is at least in one relationship in accordance to the relationship type R, this participation of entity type E in the relationship type R is named *total*, in other case (i.e. there is the entity of type E, which isn't in relationship with every entity of other entity type) – *partial*.

To formalize the concept of total and partial participation is possible by means of the relation projection (table 3). In previous denotations we have a next table, where $\pi_1^2(\mathrm{R})$, $\pi_2^2(\mathrm{R})$ are projections of binary relation with respect to the first and second components accordingly.

Table 3 – Intercommunication between participation degree and relation projections and values of operator min

Relation Projections	Participation degree of entity type in relationship	Values of operator min
$\pi_1^2(\mathrm{R}) = \mathrm{E}$	participation of entity type E in the relationship type R is total	$0 < \min(\mathrm{R})$
$\pi_1^2(\mathrm{R}) \subset \mathrm{E}$	participation of entity type E in the relationship type R is partial	$\min(\mathrm{R}) = 0$
$\pi_2^2(\mathrm{R}) = \mathrm{F}$	participation of entity type F in the relationship type R is total	$0 < \min(\mathrm{R}^{-1})$
$\pi_2^2(\mathrm{R}) \subset \mathrm{F}$	participation of entity type F in the relationship type R is partial	$\min(\mathrm{R}^{-1}) = 0$

Structural constraints of kind (min, max). There is the alternative variant of considering constraints on relationships type, so called structural constraints, which demand maximal and minimum values specification. As it will be shown farther, these constraints allow representing more information about relationship. Formalizing such structural constraints is possible by means of the notion of the whole image. As previously mentioned we interpret entities types E, F as nonempty sets E, F; the elements of such sets we denote as $x, y, \ldots$.

Let $x \in \mathrm{E}$, we denote by $\mathrm{R}[x]$ the whole image of singleton set $\{x\}$ relatively to the relation R; by definition $\mathrm{R}[x] \overset{def}{=} \{ y \mid y \in \mathrm{F} \wedge < x, y > \in \mathrm{R} \}$ – the set of all elements of set F, which are in the relationship R with the element x.

It is assumed that the sets E, F are no more then countable and all whole images of singletons are finite; such restriction is natural taking into account the finiteness of all objects.

We denote by $\mathrm{Im}(\mathrm{R}) \overset{def}{=} \{ |\mathrm{R}[x]| \,|\, x \in \mathrm{E} \}$ the set of cardinalities of whole images of all elements of set E. It is clearly, that $\mathrm{Im}(\mathrm{R})$ is the nonempty subset of natural numbers, finite (bounded above) or infinite (unbounded above). This set always has the least element, which we denote by $\min(\mathrm{R})$. In general case this set doesn't have the greatest element, so we introduce the following notation, where ∞ is some element that doesn't belong to the set of natural numbers:

$$\max(\mathrm{R}) = \begin{cases} \text{greatest element of set } \mathrm{Im}(\mathrm{R}), \text{ if } \mathrm{Im}(\mathrm{R}) \text{ is the finite set;} \\ \infty, \text{ in another way.} \end{cases}$$

In fact we added to the set of natural numbers N with standard order $\leq$ the greatest element ∞, converting it into a complete lattice $< N', \leq >$, where $N' \overset{def}{=} N \cup \{\infty\}$, being $n < \infty$ for all $n \in N$ (on the occasion of the conditional completeness and filling of the conditional complete lattices see, for example, [Биркгоф Г., 1984, chapter V, p. 153-154]).

Directly from the definition it follows

$$\min(\mathrm{R}) = \prod \mathrm{Im}(\mathrm{R}), \ \max(\mathrm{R}) = \coprod \mathrm{Im}(\mathrm{R}), \qquad (*)$$

where $\prod, \coprod$ are used for denotation of infimums and supremums accordingly (in the complete lattice N').

A main task consists in research of logical communication between the introduced operators values on the source relation (R) and on the relation inverse to source (R^{-1}). This task will be solved in the given below theorem, proof of which uses next lemmas about properties of $\min$, $\max$ operators.

Lemma 1. For the any binary relation R the following statements are valid:

1. $\min(R) \le \max(R)$, and what is more $\max(R) = \infty \Rightarrow \min(R) < \max(R)$;
2. $\min(R) = \max(R) \Leftrightarrow \forall x\, y\, \left(x, y \in E \Rightarrow |R[x]| = |R[y]|\right)$;
3. let k is such natural number, that $\min(R) = \max(R) = k$; then $\forall x\, \left(x \in E \Rightarrow |R[x]| = k\right)$;
4. let k is such natural number, that $\forall x\, \left(x \in E \Rightarrow |R[x]| = k\right)$; then $\min(R) = \max(R) = k$;
5. $R = \varnothing \Leftrightarrow \min(R) = \max(R) = 0$; and what is more
$R = \varnothing \Leftrightarrow \min(R) = \max(R) = \min(R^{-1}) = \max(R^{-1}) = 0$;
6. $\pi_1^2(R) \subset E \Leftrightarrow \min(R) = 0$, $\pi_1^2(R) = E \Leftrightarrow \min(R) > 0$;
7. $0 < \min(R) < \max(R) \Rightarrow \left|\pi_1^2(R)\right| \ge 2$;
8. $\max(R) = \infty \Rightarrow \left|\pi_1^2(R)\right| = \left|\pi_2^2(R)\right| = \omega$,, where ω is the cardinal of the countable sets;
9. R is functional $\Leftrightarrow$ $\max(R) \le 1$. □

The filling of tables 2, 3 follows from statements 6, 9 of above lemma with reference to operators $\min, \max$. Consequently, the index cardinality and participation degree are very simply expressed by means of the entered operators.

Lemma 2. (the value of operators $\min, \max$ on the finite universal relation) Let $|E| = l > 0$, $|F| = k > 0$ and $U(l,k) \overset{def}{=} E \times F$ is universal relation on the sets E, F, then $\min(R) = \max(R) = l$ and $\min(R^{-1}) = \max(R^{-1}) = k$. □

The values of operators $\min, \max$ depend not only on the argument-relation (in previous denotations R), but also on the parameter – such set, that the first components of relation pairs belong to it (set E); therefore more precisely would be to write, for example, $\min_E(R)$ instead of $\min(R)$. The next lemma specifies dependence on this set-parameter.

Lemma 3. Let the relation R and the sets E, F, E′, are such, that $R \subseteq E \times F$ and $E \subset E'$; then $\min_{E'}(R) = 0$ and $\max_E(R) = \max_{E'}(R)$. □

So, the own extension of set-parameter has an influence only on the operator value $\min$, which possibly not equal to zero becomes equal to zero.

The next lemma considers the case, when the relation is equal to the union of compatible in pairs relations (compatibility is understanding in terms of [Редько В. Н., Борона Ю. Й., Буй Д. Б., Поляков С. А., 2001], i.e. $U \approx V \overset{def}{\Leftrightarrow} U \mid X = V \mid X$,, where $X \overset{def}{=} \pi_1^2 U \cap \pi_1^2 V$ is intersection of projections of relations with respect to the first component, and $U \mid X$, $V \mid X$ is the restrictions of the binary relations to the set X), in the lemma the operators values $\min, \max$ on the source set are expressed in terms of the values of the same operators on the sets from the union.

Lemma 4. Let the relation R is such, that $R = \bigcup_{i \in I} R_i$, where all relations R_i, $i \in I$ are compatible in pairs. Then $\max_E(R) = \coprod_{i \in I} \max_{E_i}(R_i)$, where the sets E, E_i, are such, that $\pi_1^2(R) \subseteq E$, $\pi_1^2(R_i) \subseteq E_i$ for all $i \in I$. Besides, denoting by G and G_i the projections on the first component of relations R and R_i accordingly, the equality $\min_G(R) = \prod_{i \in I} \min_{G_i}(R_i)$ is valid. □

The proof follows from equality $\mathrm{Im}_G(R) = \bigcup_{i \in I} \mathrm{Im}_{G_i}(R_i)$, equality (*) and well known statement about supreme (infimum) of the union of sets (see, for example, [Скорняков Л. А., 1982, § 1, theorem 9]).

All variants of values $\min, \max$ for the relations are given in table 4, the lines of which correspond to the source relation, and columns – to the inverse relation. Next theorem answers the question about the compatibility (feasibility) of operators values $\min, \max$ on the relation and inverse relation (here compatibility is understood in general sense).

Theorem. There are the relations with the proper values $\min, \max$ for the cells of table 4, which designated +. There aren't relations with the proper values for the cells of table 4, which designated–.□

Table 4 – All variants of values min, max for the relations R, R^{-1} and their compatibility

		$l' \overset{def}{=} \min(R^{-1})$, $l \overset{def}{=} \max(R^{-1})$					
		$l'=l=0$	$l'=l>0$	$l'=0, l \geq 1$	$l'=0, l=\infty$	$l' \geq 1, l \geq l'$	$l' \geq 1, l=\infty$
$k' \overset{def}{=} \min(R)$, $k \overset{def}{=} \max(R)$	$k'=k=0$	+	–	–	–	–	–
	$k'=k>0$	–	+	+	+	+	+
	$k'=0, k \geq 1$	–	+	+	+	+	+
	$k'=0, k=\infty$	–	+	+	+	+	+
	$k' \geq 1, k \geq k'$	–	+	+	+	+	+
	$k' \geq 1, k=\infty$	–	+	+	+	+	+

The proof is based on the previous lemmas. So, filling of the first line and first column follows from statement 5 of lemma 1 (about characteristic property of empty relation). Note only that the proper relations are built by the unions of the finite universal relations (lemma 2) and the lemma 4 for the countable unions is used.

Table 4 is filled symmetrically (with regard to main diagonal), because the change of line on a column (or vice versa) corresponds to the situation when source relation and reverse to it only exchange its roles; thus the proof is needed only for cells, which are on a main diagonal and higher it. □

Therefore, except the special case of empty relation, for any distribution of min, max operators values there exists the relation, on which such values are achieved. In this sense there is no logical communication between the values operators of min, max on the source and inverse relations. It is for the reason that the whole images of singleton sets have local information about the relation (for example, functionality is expressed but injectivity isn't).

Conclusion

In the paper the basic concepts of ER model: entity, relationship, index cardinality, participation degree of entity in relationship, structural constraints of kind (min, max) were considered and specified in terms of relations theory.

After consideration of the constraints on the relationship types (tables 2-3) we can make the conclusion: structural constrains of kind (min, max) are more powerful than the index cardinality and participation degree.

The main task of future investigation is to formalize such ER-model concepts as attributes, multiway relationships, weak and strong entities types.

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Authors' Information

Dmytro B. Buy - Taras Shevchenko Kyiv National University, cybernetics department: Glushkova st. 2, b. 6, Kyiv, 03022, Ukraine; e-mail: buy@unicyb.kiev.ua

Lyudmila N. Silveystruk - Taras Shevchenko Kyiv National University, cybernetics department: Glushkova st. 2, b. 6, Kyiv, 03022, Ukraine; e-mail: slm-klm@rambler.ru

ALGORITHM OF CONSTRUCTION OF ORDERING OF THE OBJECTS NEAREST TO THE ANY RELATION ON SET OF OBJECTS

Grigory Gnatienko, Oleksiy Gnatienko

Abstract: The problem of a finding of ranging of the objects nearest to the cyclic relation set by the expert between objects is considered. Formalization of the problem arising at it is resulted. The algorithm based on a method of the consecutive analysis of variants and the analysis of conditions of acyclicity is offered.

Keywords: ranking, the binary relation, acyclicity, basic variant, consecutive analysis of variants

Introduction

Various expert estimations are used at decision-making on all an extent of a history of mankind. Many practical problems cannot be solved without application of expert estimations. One of the most widespread approaches at an expert estimation of objects is their ordering.

The problem of ordering of set of objects in degrees of display of some properties is one of the primary goals of expert reception of estimations [Литвак, 1983]. The essence of a problem will consist in definition of the full order on set of compared objects under the set partial order.

Among problems of decision-making the problem of linear ordering of objects is allocated with a plenty of concrete applications and a unconditional urgency of a theme. This problem traditionally is in the center of