Bibliography

[Chen P., 1976] Chen P. The entity-relationship model – towards a unified view of data. ACM TODS. March 1976. v. 1, № 1.

[Batini Carlo, Ceri S., Navathe S.B. and Batini Carol, 1991]. Batini Carlo, Ceri S., Navathe S.B. and Batini Carol. Conceptual Database Design: an Entity/Relationship Approach. Addison-Wesley, Reading, MA, 1991.

[Thelheim B., 2000]. Thelheim B. Fundamentals of Entity-Relationship Modeling. Berlin, Springer-Verlag, 2000.

[Гарсиа-Молина Г., Ульман Дж., Уидом Дж., 2004]. Гарсиа-Молина Г., Ульман Дж., Уидом Дж. Системы баз данных. Полный курс.: пер. с англ. Москва„Вильямс", 2004.

[Дейт Дж., 1998]. Дейт Дж. Введение в системы баз данных, 8-е издание.: пер. с англ. Москва, „Вильямс", 2005.

[Коннолли Т., Бегг К., Страчан А., 2000]. Коннолли Т., Бегг К., Страчан А. Базы данных: проектирование, реализация и сопровождение. Теория и практика, 2-е изд.: пер. с англ. Москва, „Вильямс", 2000.

[Крёнке Д., 2003]. Крёнке Д. Теория и практика построения баз данных. 8-е изд. Санкт-Петербург, „Питер", 2003.

[Буй Д. Б., Кахута Н. Д., 2005]. Буй Д. Б., Кахута Н. Д. Властивості теоретико-множинних конструкцій повного образу та обмеження. Вісник Київського університету. Серія: фіз.-мат. науки. 2005. Вип. 2.

[Биркгоф Г., 1984]. Биркгоф Г. Теория решеток. Москва, Наука, 1984.

[Редько В. Н., Борона Ю. Й., Буй Д. Б., Поляков С. А., 2001]. Редько В. Н., Борона Ю. Й., Буй Д. Б., Поляков С. А. Реляційні бази даних: табличні алгебри та SQL-подібні мови. Київ, Видавничий дім „Академперіодика", 2001.

[Скорняков Л. А., 1982]. Скорняков Л. А. Элементы теории структур. Москва, Наука, 1982.

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ALGORITHM OF CONSTRUCTION OF ORDERING OF THE OBJECTS NEAREST TO THE ANY RELATION ON SET OF OBJECTS

Grigory Gnatienko, Oleksiy Gnatienko

Abstract: The problem of a finding of ranging of the objects nearest to the cyclic relation set by the expert between objects is considered. Formalization of the problem arising at it is resulted. The algorithm based on a method of the consecutive analysis of variants and the analysis of conditions of acyclicity is offered.

Keywords: ranking, the binary relation, acyclicity, basic variant, consecutive analysis of variants

Introduction

Various expert estimations are used at decision-making on all an extent of a history of mankind. Many practical problems cannot be solved without application of expert estimations. One of the most widespread approaches at an expert estimation of objects is their ordering.

The problem of ordering of set of objects in degrees of display of some properties is one of the primary goals of expert reception of estimations [Литвак, 1983]. The essence of a problem will consist in definition of the full order on set of compared objects under the set partial order.

Among problems of decision-making the problem of linear ordering of objects is allocated with a plenty of concrete applications and a unconditional urgency of a theme. This problem traditionally is in the center of attention of researchers and the quantity of the works devoted to questions of construction optimum in this or that sense of linear orders on set of compared objects is very great [Миркин, 1976].

Practical application of problems of ranging is very various [Левин, 1987]. Such problems arise, for example, at the decision of

- a problem of definition of sequence of loading and unloading of a transport spacecraft;
- a finding of sequence of elimination of malfunctions of various systems;
- the complex analysis of quality of production;
- the analysis of characteristics of production and allocation of the main parameters of quality;
- a finding of bottlenecks in some complex systems possessing such properties as stability, controllability, self-organizing;
- designing of liaison channels between units in information networks;
- expert reception of estimations of projects of development of branches of a national economy or scientific researches;
- planning of building of residential areas, etc.

Problems of qualitative and quantitative ranging are considered. At the decision of such problems wide application was received with a method of pair comparisons. The set of works [Гнатиенко, 1993] is devoted to the analysis of the specified problems.

The problem of definition of ranging of objects is the widespread problem of the theory of decision-making. This problem is solved various methods. At application of expert estimations for ranging objects the information both from one expert and from expert group can be used.

As the person frequently supposes infringement of a condition of transitivity of relations at estimations of objects even at the decision of a problem of ranging one expert in relations between objects can appear cycles. In such cases there is a problem of definition of the ranking nearest to the relation set by the expert.

Problem Definition

Let it is necessary to find ranking (the linear order) n objects of set A, the nearest to the cyclic relation set by a matrix of pair comparisons

$$P = (p_{ij}), \quad i, j \in I = \{1,...,n\}. \tag{1}$$

Elements of a matrix P express result of comparison of objects $a_i \in A, \quad i \in I,$ with indexes $i, j \in I,$ and are defined thus:

$$p_{ij} = \begin{cases} 1, & if \ a_i \succ a_j, \\ 0, & if \ i = j, \\ -1, & if \ a_i \prec a_j, \end{cases}$$

$$p_{ij} + p_{ji} = 0, \ \forall i, j \in I, \ i \neq j,$$

where “$\succ$“ - a symbol of the relation of preference between objects.

Construction of the linear order generally demands entering enough the big changes in initial structure of preferences of a kind (1) on set of objects A. The problem of a finding of the order is a complex combinatory problem, NP - difficult in strong sense [Миркин, 1976]. Therefore algorithms of local optimization, heuristic algorithms or the algorithms basing a method of branches and borders are applied to construction of ranking R*. Methods of the consecutive analysis of variants in the offered interpretation for this class of problems were not applied, though their use in this area of researches is perspective.

The linear order nearest to the set relation of a kind (1), we shall search as

$$R^* = Arg \min_{R \in \Re} d(P, R),$$

where $\Re$ - set of matrixes which correspond to linear orders n objects, d(R,P) – distance between ranking $R \in \Re$, which is under construction, and set cyclic relation P of a kind (1).

For measurement of distance between set relation P and ranking R, we shall use the most widespread in this class of problems metrics Hamming

$$d(P,R) = 0{,}5 \sum_{i \in I} \sum_{j \in I} \left| p_{ij} - r_{ij} \right|,$$

where p_{ij}, r_{ij} – accordingly elements of matrixes P and R.

Formalization of a Problem

As matrixes of relation P and R are slanting symmetric they can be written down as vectors C and X with elements

$$c_t = p_{ij}, \quad x_t = r_{ij}, \\ t = (i-1)n + j - (i+1)i/2, \ 1 \le i < j \le n. \tag{2}$$

Then the distance between relations P and R will be written down as

$$d(P,R) = \sum_{j \in J} \left| c_j - x_j \right|, \quad j = 1,\dots, n(n-1)/2 = N, \quad J = \{1,\dots,N\}.$$

The problem of a finding of the linear order nearest to set on set of objects to the relation (1), is formalized as

$$\sum_{j \in J} \left| c_{ij} - x_{ij} \right| \ \to \ \min, \tag{3}$$

$$x_j \in X_j^0 = \{-1,1\}, \quad j \in J, \tag{4}$$

$$x \in D^A \subset X^0, \quad X^0 = \prod_{j \in J} X_j^0, \tag{5}$$

where D^A – set of vectors of a kind (2) which correspond{meet} to acyclic relations between objects.

Specificity of a problem (3) - (5) will be, that its decision $x \in X$ should satisfy to a condition of acyclicity as the relation which is set by matrix R, should belong to a class of linear orders.

We shall consider a chain of objects a_{i1}, a_{i2}, a_{i3} with the set relations of preference which we shall designate symbols $a_{i1}\ \pi\ a_{i2}\ \pi\ a_{i3}$, $i_1,i_2,i_3 \in L$, $\pi \in \{\succ,\prec\}$.

Basic sub-variant b, which is generated by the three of objects (a_{i1}, a_{i2}, a_{i3}), we shall name elements of a vector of a kind (2) with components

$$b = (c_{j1}, c_{j2}, c_{j3}), \ 1 \le j_1 < j_2 < j_3 \le N, \ \ c_{j1}, c_{j2}, c_{j3} \in c, \ \ c_{j1}, c_{j2}, c_{j3} \in \{-1,1\}, \tag{6}$$

which values answer relations of a kind

$$\left(a_{i1}\ \pi\ a_{i2},\ \ a_{i2}\ \pi\ a_{i3},\ \ a_{i3}\ \pi\ a_{i1}\right),\ \ a_{i1}, a_{i2}, a_{i3} \in A,\ \ \pi \in \{\succ,\prec\}.$$

The basic sub-variant is a minimal subset of objects from set A, on which it is possible to reveal a cycle.

Allowable basic sub-variant we shall name a basic sub-variant which is generated by the three of objects, relations between which satisfy to a condition of acyclicity.

Full variant (variant of length N) we shall name a vector which answers the full binary relation on set of objects.

Allowable variant x^D we shall name a full variant which answers the acyclic relation on set of all n objects, that is $x^D \in D^A$.

At check of an admissibility of basic variants of a kind (6) which are formed by objects a_{i1}, a_{i2}, a_{i3}, $1 \le i_1 < i_2 < i_3 \le n$, it is necessary to consider relations as ($a_{i1}\pi\ a_{i2}$, $a_{i2}\pi\ a_{i3}$, $a_{i1}\underline{\pi}\ a_{i3}$), where $\underline{\pi}$ - inversion of the relation π: $a_{i1}\underline{\pi}\ a_{i3} \Leftrightarrow a_{i3}\pi\ a_{i1}$, $\pi = \{\succ,\prec\}$.

Set $X^s = \{\cup X_j^s$, j=1,...,N}, $X^s \subseteq X^0$, s=1,2,..., we shall name reduced (concerning initial set X^0).

For the decision of a problem (3)-(5) procedure of reduction of set of allowable decisions Z on a condition $X^s=(X^s_1 \times X^s_2 \times \ldots \times X^s_N)$ of acyclicity of the relation which corresponds to the decision of a problem of a finding of strict resulting ranging of objects of set A is used. The analysis of variants in view of a condition of acyclicity of the decision is carried out with use of the procedure described in [Гнатиенко, 2005].

Let's designate set of all possible values which can get elements of a basic variant, through B^0. Sets of a kind B^0 are formed of set X^s by association of three various columns of a matrix X^s: $X^0_{j1} \cup X^0_{j2} \cup X^0_{j3}$, $1 \le j_1 < j_2 < j_3 \le N$, $X^0_{j1}, X^0_{j2}, X^0_{j3} \in X^s$. Capacity of this set is equal $|B^0| = 6$.

Basic set $B^0 = B^0_1 \times B^0_2 \times B^0_3$, $B^0_i = (-1,1)^T$, i=1,…,3, we shall name set of elements of a matrix of which values of basic vectors get out.

Columns of basic set $B^0_i = (-1,1)^T$, i=1,2,3, we shall name subsets of basic set.

The reduced basic set B^s, $B^s \subset B^0$, s=1,2,..., we shall name a matrix which is formed of a matrix B^0 by removal from it separate elements.

It is known [Макаров, 1982], that for matrixes of pair comparisons with elements of a kind (2) requirement of absence of cycles is equivalent to the requirement of absence of cycles of length three (T=3).

As the top triangular matrix of a matrix P^i, $i \in I$, contains the full information on all matrix indexes of objects need to be considered only on increase, that is $1 \le i_1 < i_2 < i_3 \le n$. Indexes of elements of a vector of relations between objects also satisfy to conditions $1 \le j_1 < j_2 < j_3 \le N$.

We shall designate through ψ function of two arguments which values are calculated under the formula $j = \psi(i_1, i_2) = (i_1 - 1)n + i_2 - (i_1 + 1)i_1 / 2,\ 1 \le i_1 < i_2 < n$. For definition of indexes of objects which form relations with an index $j,\ j = 1,\ldots,N,$ it is applicable such formulas: $i_1 = \arg\max_{r=1,\ldots,n-1} n \cdot r - r/2 - r^2/2 < j;\ i_2 = j + i_1 \cdot (i_1 + 1)/2 - (i_1 - 1) \cdot n$. Through $\delta^1_j,\ j = 1,\ldots,N,$ we shall designate quantity of cycles of length three which form relations with an index $j,\ j = 1,\ldots,N$. Through $\delta^2_j,\ j = 1,\ldots,N,$ we shall designate quantity of cycles of length three which are formed by inversion of the relation with an index $j,\ j = 1,\ldots,N$. Quantity of all three n of objects, relations between which form cycles, is equal: $k3 = n \cdot (n-1) \cdot (n-2)/6$. Quantity of cycles on set n objects equally: $d = (n^3 - 4n)/24$ for even n and $d = (n^3 - n)/24$ for odd n.

Algorithm

In a problem of definition of the linear order nearest to the set cyclic relation, it is possible to present algorithm of the consecutive analysis and elimination of inadmissible elements in the following kind. The algorithm of the decision of a task (3) - (5) has the following kind.

Step 1. We shall put initial value of the decision of a task equal $x(j) = c(j),\quad j \in J$.

Step 2. We shall put also initial values $\delta_j = \delta^1_j = \delta^2_j = 0,\ \ j \in J$.

Step 3. The organization of a cycle: $j = 1,\ldots,N$. The variable of a cycle $j,\ j = 1,\ldots,N,$ is an index of the relation between objects with indexes i_1, i_2. In a body of this cycle steps 4-11 are carried out.

Step 4. Definition of indexes of objects i_1, i_2, which form relations with an index $j,\ j = 1,\ldots,N,$ $1 \le i_1 < i_2 \le n$.

Step 5. The organization of a cycle: $i = 1,\ldots,n$. The variable of a cycle $i,\ i = 1,\ldots,n,$ is an index of object. In a body of this cycle steps 6-10 are carried out.

Step 6. If $(i \ne i_1) \& (i \ne i_2)$, that we carry out steps 7-10 cycles on $i = 1,\ldots,n$.

Step 7. If $i < i_1$, that we believe values of indexes j_1, j_2, equal $j_1 = \psi(i, i_1)$, $j_2 = \psi(i, i_2)$. If $(x_j = x_{j_1}) \& (x_j = -x_{j_2})$, that $\delta_j^1 = \delta_j^1 + 1$. If $(-x_j = x_{j_1}) \& (x_j = x_{j_2})$, that $\delta_j^2 = \delta_j^2 + 1$.

Step 8. If $(i_1 < i) \& (i < i_2)$, that we believe j_1, j_2, equal $j_1 = \psi(i_1, i)$, $j_2 = \psi(i, i_2)$. If $(-x_j = x_{j_1}) \& (-x_j = x_{j_2})$, that $\delta_j^1 = \delta_j^1 + 1$. If $(x_j = x_{j_1}) \& (x_j = x_{j_2})$, that $\delta_j^2 = \delta_j^2 + 1$.

Step 9. If $i_2 < i$, that we believe j_1, j_2, equal $j_1 = \psi(i_1, i)$, $j_2 = \psi(i_2, i)$. If $(x_j = -x_{j_1}) \& (x_j = x_{j_2})$, that $\delta_j^1 = \delta_j^1 + 1$. If $(x_j = x_{j_1}) \& (-x_j = x_{j_2})$, that $\delta_j^2 = \delta_j^2 + 1$.

Step 10. Definition of values of differences $\delta_j = \delta_j^1 - \delta_j^2$, $j = 1,..., N$.

Step 11. End of the enclosed cycle on i, $i = 1,..., n$.

Step 12. End of a cycle on j, $j = 1,..., N$.

Step 13. Definition of the sums: $\Delta^1 = \sum_{j=1}^{N} \delta_j^1$, $\Delta^2 = \sum_{j=1}^{N} \delta_j^2$.

Step 14. Definition of the maximal value are nonviscous: $\delta^M = \max_{j=1,...,N} \delta_j^1$, and an index $j^M = \arg \max_{j=1,...,N} \delta_j^1$.

Step 15. Change of the relation with an index j^M, $j^M \in \{1,..., N\}$, on inverse to it: $x(j^M) = -x(j^M)$.

Step 16. If there are the relations forming cycles, that is $\exists j : \delta_j^1 \neq 0$, $j = 1,..., N$, that transition to a step 3. Recurrence of points 3-15 of the resulted algorithm until in the decision $x(j)$, $j \in J$, of a task (3)-(5) there are cycles.

Step 17. Relations $x(j)$, $j = 1,..., N$, determined in such a manner that $\delta_j^1 = 0$, $\forall j, j = 1,..., N$, is acyclic and corresponds to ranking on set of objects with indexes i, $i = 1,.., n$.

Conclusion

The resulted algorithm allows finding consistently for final quantity of steps the ranking of the objects nearest to the cyclic relation set by the expert between objects.

Computing experiments have confirmed efficiency of the resulted algorithm. Received with the help of algorithm of the decision are one of the rankings, the nearest to the cyclic relation set by the expert on set of objects.

The basic laws determined during computing experiments on research of resulted algorithm are:

a) During work of algorithm of value of the sum $\Delta^1 = \sum_{j=1}^{N} \delta_j^1$, monotonously decrease, and the sum $\Delta^2 = \sum_{j=1}^{N} \delta_j^2$, thus monotonously grows.

b) Value of the sum Δ^1 is multiple to three on construction.

c) On each step of algorithm the sum Δ^1 decreases for number, multiple to three, and value of the sum Δ^2 decreases for size three times smaller.

d) Such relation $\Delta^1 / 3 + \Delta^2 = const = 3 \cdot \Delta^M$, where $\Delta^M -$ quantity of all three n of objects takes place, relations between which form cycles. That is, on each step of algorithm the sum $\Delta^1 / 3 + \Delta^2$ is a constant and is equaled to the trebled quantity of all possible three on set n of objects, relations between which can form cycles.

Bibliography

[Литвак, 1983] Литвак Б.Г. Меры близости и результирующие ранжирования // Кибернетика. 1983. №1. С.57-63.

[Миркин, 1976] Миркин Б.Г. Анализ качественных признаков: Математические модели и методы. М.: Статистика. 1976. 166 с.

[Левин, 1987] Левин М.Ш. Современные подходы к оценке эффективности плановых и проектных решений в машиностроении. Обзорная информация. Сер.С-9. Автоматизированные системы проектирования и управления. М.: ВНИИТЭМР. 1987. Вып.3.-56с.

[Гнатиенко, 1993] Гнатиенко Г.Н., Микулич А.Ю. Методы метризации качественных ранжировок объектов. Киев.ун-т.-Киев.1993. Библиогр.: 6 назв. Рус. Деп.в УкрНИИНТИ 10.03.93. №432-Ук93.-10с.

[Гнатиенко, 2005] Гнатієнко Г.М. Процедура послідовного аналізу та відсіювання варіантів з урахуванням ациклічності розв'язку//Наукові праці Кіровоградського державного університету. Технічні науки.-Вип.16.-Кіровоград. 2005.-С.294-299.

[Макаров, 1982] Макаров И.М., Виноградская Т.М., Рубчинский А.А. и др. Теория выбора и принятия решений: Учебное пособие. М.:Наука, 1982.-328с.

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MATRIXES OF RELATIONS BETWEEN PAIRS OBJECTS AND TRANSFORMATIONS BETWEEN VARIOUS KINDS

Grigory Gnatienko

Abstract: Ways of representation of relations between pair's objects are described at a complete choice. Methods of revealing and kinds of relations between objects are considered. The table of conformity between various forms of representation of relations is resulted.

Keywords: pair comparisons, the ranging, identical transformations, mark estimation, the expert.

Introduction

In practice frequently there are problems of decision-making in which some properties of objects are more convenient for expressing not in terms of parameters and their values, and in terms of relations between objects on some property [Миркин, 1980]. Therefore the widespread problem at processing the expert information is the problem of definition of relations on the set of objects. Thus there are various ways of representation of the specified relations and calculation of conformity between them also has essential value at the decision of problems of decision-making.

Numerous results of regular researches of a problem of comparison of two objects and allocation of "best" of them are known. These results testify that such operation is complex for the expert if the object is characterized by a plenty of parameters. Already at presence of three characteristics of objects experts use simplifying a problem {task} of heuristics which can result in contradictions. These restrictions are peculiar to the person by virtue of specific characteristics of his operative memory [Ларичев, 1980]. Thus, it agrees [Larichev, 1980], in