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MATRIXES OF RELATIONS BETWEEN PAIRS OBJECTS AND TRANSFORMATIONS BETWEEN VARIOUS KINDS

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Abstract: Ways of representation of relations between pair's objects are described at a complete choice. Methods of revealing and kinds of relations between objects are considered. The table of conformity between various forms of representation of relations is resulted.

Keywords: pair comparisons, the ranging, identical transformations, mark estimation, the expert.

Introduction

In practice frequently there are problems of decision-making in which some properties of objects are more convenient for expressing not in terms of parameters and their values, and in terms of relations between objects on some property [Миркин, 1980]. Therefore the widespread problem at processing the expert information is the problem of definition of relations on the set of objects. Thus there are various ways of representation of the specified relations and calculation of conformity between them also has essential value at the decision of problems of decision-making.

Numerous results of regular researches of a problem of comparison of two objects and allocation of "best" of them are known. These results testify that such operation is complex for the expert if the object is characterized by a plenty of parameters. Already at presence of three characteristics of objects experts use simplifying a problem {task} of heuristics which can result in contradictions. These restrictions are peculiar to the person by virtue of specific characteristics of his operative memory [Ларичев, 1980]. Thus, it agrees [Larichev, 1980], in problems of a complete choice of an opportunity of the expert are very great, as he uses a Gestalt (a complete image) object as one structural unit of the information. A Gestalt, as a rule, the richman of a corresponding set of parameters. In this connection the decisions accepted on the basis of complete representation, frequently do not coincide with decisions of the same problems formal methods.

Problem Definition

Let the set n of objects is considered

$$a_i \in A, \;\; i \in I = \{1,...,n\}, \tag{1}$$

which parameters are not allocated. In view of the nature of a practical problem the complete choice is carried out in connection with that it is impossible to measure parameters of objects, they are unknown for some reasons or are insignificant for decision-making. The expert is offered to determine relations (preferences, similarities - distinctions, affinity or others) between objects, making use of personal experience or some other indirect certificates.

One of the basic ways of representation of relations between objects of set (1) is matrixes of pair comparisons (MPC):

$$P = (p_{ij}), \;\; i, j \in I = \{1,...,n\}. \tag{2}$$

Elements $p_{ij}, i, j \in I,$ of matrixes of a kind (2) are the real numbers reflecting in some scale results of comparison by the expert of objects with indexes $i, i \in I,$ and $j, j \in I.$

Symmetric elements of matrixes p_{ij} also p_{ji} get out equal if objects corresponding to them are equivalent from the point of view of the expert. If the object with an index $i, i \in I,$ in opinion of the expert, is "best", than the object with an index $j, j \in I,$ that attitude{relation} between symmetric elements of a matrix is established $p_{ij} > p_{ji}, \;\; p_{ij}, p_{ji} \in P, \;\; i, j \in I.$ Except for these obvious conditions on elements of a matrix of a kind (2) the additional (calibrated) restrictions which unequivocally connect in pairs symmetric elements p_{ij} p_{ij} , as a rule, are imposed and p_{ji}.

Depending on conditions of a problem, value of elements $p_{ij}, i, j \in I,$ of matrixes of a kind (2) can have various senses. The matrix P can characterize relative "weight" of objects if the vector of preferences on set of objects (1) is determined, can specify relative importance of parameters of objects at decision-making or testify to relative competence of experts of pair $(i, j), \;\; i, j \in I.$

Method of steam rooms comparison and kinds of MPC

One of the most widespread and the most reliable ([Миркин, 1974]) methods of revealing of relations on the set set of objects (1) is the method of steam rooms (the term - paired [Паниотто, 1986], [Юшманов, 1990]) comparisons ([Дэвид, 1978], [Литвак, 1982]). At use of this method results of examination will be worn out in MPC of a kind (2) or are represented as focused the column of pair comparisons which tops are objects, and arches characterize relations between them. Relations on the set set of objects come to light also by use of methods of plural comparison ([Паниотто, 1982]), rangings ([Миркин, 1974]), attributing of points ([Кини, 1981]) and other methods. Thus MPC are most the common way of representation of relations on set of objects ([Миркин, 1980]).

			Simple structure SS {0,1/2,1}	Simple structure SS {-1,0,1}	Simple structure SS {0,1,2}	Mark structure (M)	Sedate calibration (S)	Tournament calibration (T)	Probability's calibration (P)	Slanting symmetric calibration (C)
Qualitative relations	Simple structure (SS)	Simple structure SS {0,1/2,1}	—	$r_{ij}=2(p_{ij}-1/2)$	$r_{ij}=2p_{ij}$	Methods of metrization				
		Simple structure SS {-1,0,1}	$r_{ij}=(p_{ij}+1)/2$	—	$r_{ij}=2p_{ij}+1$					
		Simple structure SS {0,1,2}	$r_{ij}=p_{ij}/2$	$r_{ij}=p_{ij}-1$	—					
Quantitative relations	Mark estimation (The weighed structure) $p_{ij}\geq 0$, $\forall i,j\in I$, (M)		$r_{ij}=[sign(p_{ij}-p_{ji})+1]/2$	$r_{ij}=sign(p_{ij}-p_{ji})$	$r_{ij}=sign(p_{ij}-p_{ji})+1$	—	$r_{ij}=p_{ij}/p_{ji}$	$r_{ij}=p_{ij}+(max_{i,j}\ S_{ij}-S_{ij})/2$; $r_{ij}=p_{ij}+(p_{ij}*max_{i,j}\ S_{ij})/S_{ij}$; $S_{ij}=p_{ij}+p_{ji}$.	$r_{ij}=p_{ij}+(max_{i,j}\ S_{ij}-S_{ij})/2$; $r_{ij}=p_{ij}+(p_{ij}*max_{i,j}\ S_{ij})/S_{ij}$; $S_{ij}=p_{ij}+p_{ji}$; $r_{ij}=p_{ij}/(p_{ij}+p_{ji})$.	$r_{ij}=p_{ij}-S_{ij}$; $S_{ij}=p_{ij}+p_{ji}$; $t_{ij}=(p_{ij}-p_{ji})/2$.
	Multiplicate matrixes	Sedate calibration $p_{ij}*p_{ji}=1$, $p_{ij}>0$, $\forall i,j\in I$, (S)	$r_{ij}=[sign(p_{ij}-p_{ji})+1]/2$	$r_{ij}=sign(log_T p_{ij})$, $T>1$	$r_{ij}=sign(p_{ij}-p_{ji})+1$	$r_{ij}=T_1 p_{ij}/(T_2+p_{ij})$, $T_1>0$, $T_2\geq 1$	—	$r_{ij}=p_{ij}/(1+p_{ij})$; $r_{ij}=T\cdot p_{ij}/(1+p_{ij})$; $T>0$.	$r_{ij}=p_{ij}/(1+p_{ij})$.	$r_{ij}=log_T p_{ij}$; $T>1$.
	Additive matrixes	Tournament calibration $p_{ij}+p_{ji}=T$, $\forall i,j\in I$, (T)	$r_{ij}=[sign(p_{ij}-p_{ji})+1]/2$	$r_{ij}=sign(p_{ij}-p_{ji})$	$r_{ij}=sign(p_{ij}-p_{ji})+1$	$r_{ij}=p_{ij}$	$r_{ij}=p_{ij}/p_{ji}$	—	$r_{ij}=p_{ij}/(p_{ij}+p_{ji})$.	$r_{ij}=p_{ij}-p_{ji}$; $r_{ij}=(p_{ij}-p_{ji})/2$.
		Probability's calibration $p_{ij}+p_{ji}=1$, $0\leq p_{ij}\leq 1$, $\forall i,j\in I$, (P)	$r_{ij}=[sign(p_{ij}-p_{ji})+1]/2$	$r_{ij}=sign(p_{ij}-p_{ji})$	$r_{ij}=sign(p_{ij}-p_{ji})+1$	$r_{ij}=Tp_{ij}$, $T>0$	$r_{ij}=p_{ij}/p_{ji}$	$r_{ij}=Tp_{ij}$, $T>0$.	—	$r_{ij}=T(p_{ij}-p_{ji})$; $T>0$.
		Slanting symmetric calibration $p_{ij}+p_{ji}=0$, $\forall i,j\in I$, (C)	$r_{ij}=[sign(p_{ij}-p_{ji})+1]/2$	$r_{ij}=sign(p_{ij}-p_{ji})$	$r_{ij}=sign(p_{ij}-p_{ji})+1$	$r_{ij}=(p_{ij}+T_1)/T_2$, $T_1>max\ p_{ij}$, $T_2\neq 0$	$r_{ij}=T^{p_{ij}}$; $T>1$	$r_{ij}=p_{ij}(sign\ p_{ij}+1)/2+$ $+(max_{i,j}\ p_{ij}-\lvert p_{ij}\rvert)/2$; $r_{ij}=T(p_{ij}+max_{i,j}\ p_{ij})$; $T>0$.	$r_{ij}=(p_{ij}+max_{i,j}\ p_{ij})/$ $/(2\ max_{i,j}\ p_{ij})$	—

Table 1. Identical transformations between ways of representation of relations between objects.

MPC of a kind (2) can be full (when all elements of a matrix P are completely determined) or incomplete, that is such in which not all elements $p_{ij} \in P,\ i, j \in I,$ are known. In the latter case there can be a problem of restoration of unknown elements of MPC ([Загоруйко, 1999]), and also definitions of "weight" of objects on incomplete MPC [Чеботарев, 1989].

In real examinations experts are not always consecutive in the preferences owing to complexity of a problem, uncertainty of relations between the objects, insufficient competence, personal bias and other. Natural consequence of subjectivity of experts is discrepancy, incompleteness and discrepancy of expert judgements. Therefore elements of a matrix of a kind (2) are sometimes represented in an interval kind or as functions of an accessory to indistinct set. However in this work we shall consider only dot values of elements of MPC.

Ways of representation of relations between objects

It agrees [Миркин, 1980], at comparison of objects of set (1) exists four basic ways of representation of results of such comparison as elements of MPC of a kind (2). The estimation the expert of the relation between objects can express:

#1) simply the fact of preference of the expert of one object to another or an equivalence between objects (simple structure) ([Кемени, 1972], [Кендэлл, 1975], [Литвак, 1982]);

#2) share of total intensity of preference of compared objects which falls at each of them ([Литвак, 1982]), so $p_{ij} + p_{ji} = T,\ i, j \in I,$ where $T \geq 0$ – some real number, identical to all $p_{ij} \in P,\ i, j \in I;$ more often $T = 1$ and then speak that the probability's calibration is applied; at $T = 0$ slanting symmetric calibration takes place, and at $T > 0$ – tournament calibration;

#3) mark estimation of the relation ([Кини, 1981]) $p_{ij} \in R,\ p_{ji} \in R,\ i, j \in I,$ where R – set of real numbers; unilateral or bilaterial borders of allowable attributing of points are sometimes established;

#4) in how many time one object surpasses another, that is $p_{ij} = 1/p_{ji},\ i, j \in I,$ – speak, that sedate calibration ([Миркин, 1980], [Белкин, 1990]) takes place.

The important characteristic of metrize's relations is their supertransitivity or a cardinal coordination in force of preference which will consist in performance of conditions: $p_{ij} > 0$ and $p_{ij} p_{jk} = p_{ik},\ i, j, k \in I.$

If relations between pairs objects are set in forms #1) or #2) the matrix of a kind (2) is slanting symmetric (anti-symmetric): $p_{ij} = -p_{ji},\ i, j \in I,$ or it is easily reduced to such. If relations are set in form #3) or #4) the matrix (2) is back symmetric: $p_{ij} = 1/p_{ji},\ i, j \in I,$ or it is reduced to it.

The formulas of transformation systematized and developed by the author between various ways of representation of paired relations between objects are resulted in table 1. In table 1 through $p_{ij}, i, j \in I,$ reference values of elements of a matrix of a kind (2) are designated. Through $r_{ij}, i, j \in I,$ - resulting values of these elements at the decision of a problem of their transformation into the required form of representation.

Conformity between forms of representation of relations between objects

The relations submitted in form #1), are set in qualitative (qualitmetrics) a scale. Last three forms of representation of relations between objects which express a quantitative measure of relations, name metrization and speak, that they display intensity of relations. Form #2) refers to still additive, and form #4) – as the multiplicate relation. Between forms #2), #3), #4) there is a biunique conformity ([Миркин, 1980], [Хованов, 1982]).

For processing results of measurement of quantitative sizes the device of mathematical statistics is used. For processing by statistical methods of results of measurements in qualitative scales, it is necessary the non-numerical information the metrize ([Литвак, 1982], [Бевз, 1989]) that is to ship in the system derivative of real numbers. The metrization (numbering ([Бевз, 1989], [Хованов, 1986]), arithmetization ([Хованов, 1982])) qualitmetrics scales refers to construction of conformity between form #1) and other forms. Not each relational system can be isomorphic metrization [Гильбурд, 1988]. On the other hand, the some people qualitmetrics scales can be метризованы various ways. Methods metrization of qualitmetrics relations are resulted, for example, in works [Бевз, 1989], [Гнатиенко, 1993].

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