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GENERALIZED SCALARIZING PROBLEMS *GENS* AND *GENSLEX* OF MULTICRITERIA OPTIMIZATION[1]

Mariyana Vassileva

Abstract: Generalized scalarizing problems, called GENS and GENSLex, for obtaining Pareto optimal solutions of multicriteria optimization problems are presented in the paper. The basic properties of these scalarizing problems are described. The existence of single-criterion problems with differentiable objective functions and constraints, which are equivalent to GENS and GENSLex scalarizing problems, are pointed out.

Keywords: Multicriteria optimization, Interactive methods, Multicriteria decision support systems.

ACM Classification Keywords: H.4.2 Information Systems Applications: Types of Systems: Decision Support.

Introduction

Various real problems can be modelled as multicriteria optimization problems. In multicriteria optimization problems several criteria are simultaneously optimized in the feasible set of alternatives. In the general case, there does not exist one alternative, which optimizes all the criteria. There is a set of alternatives however, characterized by the following: each improvement in the value of one criterion leads to deterioration in the value of at least one other criterion. This set of alternatives is called a set of the Pareto optimal alternatives (solutions). Each alternative in this set could be a solution of the multicriteria optimization problem. In order to select one alternative, it is necessary to have additional information set by the so-called decision maker (DM).

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The information that the DM provides reflects his/her global preferences with respect to the quality of the most preferred alternative.

The general problem of multicriteria optimization (MO) can be represented in the following way:

$$\max \{ f_k(x), k \in K \}$$

$$\text{subject to: } x \in X$$

where:

- $f_k(x)$, $k \in K=\{1,2,\ldots,p\}$ are different criteria (objective functions) of the type $f_k : R^n \to R$, which must be simultaneously maximized;
- $x = (x_1,\ldots,x_j,\ldots,x_n)$ is the vector of variables, belonging to the non-empty feasible set $X \subset R^n$;
- $Z=f(X) \subset R^p$ is the feasible set of the criteria values.

The scalarizing approach is one of the main approaches in solving MO problems. The basic representatives of the scalarizing approach ([Wierzbicki, 1980], [Sawaragi, Nakayama and Tanino, 1985], [Steuer, 1986], [Narula and Vassilev, 1994], [Buchanan, 1997], [Miettinen, 1999], [Vassileva, 2004], [Ehrgott and Wiecek, 2004]) are the interactive algorithms. The MO problem in these algorithms is treated as a decision-making problem and the emphasis is placed on the real participation of the DM in the process of its solution. Each interactive algorithm consists of two procedures in the general case – an optimization one and an evaluating one, which are cyclically repeated until the stopping conditions are satisfied. During the evaluating procedure the DM estimates the current Pareto optimal solution obtained, either approving it as the final (the most preferred) one, or setting his/her preferences in the search for a new solution. On the basis of these preferences a scalarizing problem is formed and solved in the optimization procedure and a new Pareto optimal solution is obtained with its help, which is presented to the DM for evaluation and choice. The main feature of each scalarizing problem is that every optimal solution is a Pareto optimal solution of the corresponding MO problem. The scalarizing problem is a single-criterion optimization problem, which allows the application of the theory and methods of single-criterion optimization. A number of scalarizing problems and a set of interactive algorithms developed on their basis have been proposed up to now. The different algorithms offer different possibilities to the DM in the control or in stopping the process of the final solution finding. On its hand, this searching process can be divided into two phases. In the first phase (the learning phase), the DM usually defines the region, in which he expects to find the most preferred solution, whereas in the second phase (the concluding phase) he is looking for this solution namely in this region.

The present paper describes generalized scalarizing problems, called *GENS* and *GENSLex*. They are extensions of the generalized scalarizing problem GENWS [Vassilev, 2004] and enables the obtaining of Pareto optimal solutions. Almost all scalarizing problems known up to now can be obtained from *GENS* and *GENSLex* problems, as well as new scalarizing problems with different properties can be generated from these problems.

Generalized Scalarizing Problems *GENS* and *GENSLex*

For easier description of the topic further on, the following definitions will be introduced:

Definition 1: The solution $x \in X$ is called a Pareto optimal solution of the multicriteria optimization problem, if there does not exist another solution $\bar{x} \in X$, satisfying the following conditions:

$$f_k(\bar{x}) \geq f_k(x),\ k \in K \text{ and } f_k(\bar{x}) > f_k(x) \text{ for at least one index } k \in K.$$

Definition 2: The vector $z = f(x) = (f_1(x),...,f_p(x))^T \in Z$ is called a Pareto optimal solution in the criterion space, if $x \in X$ is a Pareto optimal solution in the decision variable space.

Definition 3: The current preferred solution $z = (f_1,...,f_k,...,f_p)^T \in Z$ is a Pareto optimal solution in the criterion space, selected by the DM at the current iteration.

Definition 4: The most preferred solution is the current preferred solution, which satisfies the DM to the highest extent.

Definition 5: The criteria classification is called the implicit division of the criteria into classes, depending on the alterations in the criteria values at the current solution, which the DM wishes to obtain.

In order to obtain Pareto optimal solutions starting from the current preferred solution, *GENS* scalarizing problem is proposed. It has the following type:

Minimize

(1) $$T(x) = \max\Big(\max_{k\in K^{\geq}}\big(F_k^1 - f_k(x)\big)G_k^1 \; R_1 \max_{k\in K^{\leq}}\big(F_k^2 - f_k(x)\big)G_k^2 \; R_2 \max_{k\in K^{<}}\big(F_k^3 - f_k(x)\big)G_k^3$$
$$R_3 \max_{k\in K^{>}}\big(F_k^4 - f_k(x)\big)G_k^4\Big) + \sum_{k\in K^0}\big(F_k^5 - f_k(x)\big)G_k^5 +$$
$$+\rho\Big(\sum_{k\in K^{\geq}}\big(F_k^1 - f_k(x)\big)G_k^1 + \sum_{k\in K^{\leq}}\big(F_k^2 - f_k(x)\big)G_k^2 + \sum_{k\in K^{<}}\big(F_k^3 - f_k(x)\big)G_k^3 +$$
$$+\sum_{k\in K^{>}}\big(F_k^4 - f_k(x)\big)G_k^4 - \sum_{k\in K^{=}\cup K^{><}} f_k(x)G_k^6\Big),$$

subject to:

(2) $f_k(x) \geq f_k, k \in K^{>} \cup K^{=}$

(3) $f_k(x) \geq f_k - D_k, k \in K^{\leq}$

(4) $f_k(x) \geq f_k - t_k^-, k \in K^{><}$

(5) $f_k(x) \leq f_k + t_k^+, k \in K^{><}$

(6) $x \in X$

where:

- K is the set of all the criteria;
- $G_k^1, G_k^2, G_k^3, G_k^4, G_k^5$ are scaling, normalizing or weighting positive coefficients, $k \in K$;
- $F_k^1, F_k^2, F_k^3, F_k^4, F_k^5$ are parameters, connected with aspiration, current or other levels of the criteria values, $k \in K$;
- R_1, R_2, R_3 are equal to the arithmetic “+” or to a separator “ , ”;
- D_κ is the value, by which the DM agrees the criterion with an index $k \in K^{\leq}$ to be deteriorated (D_κ > 0);

- t_k^- and t_k^+ are the lower and upper bound of the feasible for the DM interval of alteration of the criterion with an index $k \in K^{><}$ (t_k^- > 0; t_k^+ > 0);
- f_k is the value of the criterion with an index $k \in K$ in the current solution obtained;
- $K^{\geq}$ is the set of criteria, the current values of which the DM wishes to be improved up to desired by him/her levels F_k^1;
- $K^{>}$ is the set of the criteria, the current values of which the DM wishes to be improved;
- $K^{\leq}$ is the set of the criteria, for which the DM agrees their current values to be deteriorated up to set by him/her feasible levels F_k^2, but not more than certain values D_κ (D_κ >0);
- $K^{<}$ is the set of criteria, for which the DM agrees their current values to be deteriorated;
- $K^{=}$ is the set of criteria, for which the DM agrees their current values not to be deteriorated;
- $K^{><}$ is the set of the criteria, for which the DM agrees their values to alter in defined intervals;
- K^0 is the set of criteria, for which the DM does not set explicit preferences concerning the change of their values;
- *ρ* is a small positive number.

The constraints (2) - (6) define a subset of *X*, containing Pareto optimal solutions.

__Theorem 1:__ The optimal solution of GENS scalarizing problem is a Pareto optimal solution of the multicriteria optimization problem.

__Proof :__

Let $K^{\geq} \neq \emptyset$ and/or $K^{>} \neq \emptyset$, or $K^0 = K$ and let $x^* \in X$ *be* an optimal solution of *GENS* scalarizing problem. Then the constraints (2) - (6) are satisfied for $x^* \in X$, together with the following condition:

(7) $T(x^*) \leq T(x)$, $x \in X$.

Let us assume that $x^* \in X$ is not a Pareto optimal solution of the multicriteria optimization problem. Then, another $x' \in X$ must exist, for which the constraints (2) – (6) are satisfied, as well as the conditions given below:

(8) $f_k(x') \geq f_k(x^*)$, $k \in K$ and $f_k(x') > f_k(x^*)$ for at least one index $k \in K$.

Inequality (8) follows from the definition of a Pareto optimal solution.

Using constraint (8) and the definitions of R_1, R_2, R_3, the objective function $T(x)$ of scalarizing problem *GENS* can be transformed, obtaining the following inequality:

(9)
$$T(x') = \max\Big(\max_{k\in K^{\geq}}\left(F_k^1 - f_k\left(x'\right)\right)G_k^1\ R_1 \max_{k\in K^{\leq}}\left(F_k^2 - f_k\left(x'\right)\right)G_k^2$$
$$R_2 \max_{k\in K^{<}}\left(F_k^3 - f_k\left(x'\right)\right)G_k^3\ R_3 \max_{k\in K^{>}}\left(F_k^3 - f_k\left(x'\right)\right)G_k^4\Big) +$$
$$+ \sum_{k\in K^0}\left(F_k^5 - f_k\left(x'\right)\right)G_k^5 +$$

$$
\begin{aligned}
&+\rho(\sum_{k\in K^{\geq}}\left(F_k^1-f_k\left(x^{'}\right)\right)G_k^1+\sum_{k\in K^{\leq}}\left(F_k^2-f_k\left(x^{'}\right)\right)G_k^2+\sum_{k\in K^{<}}\left(F_k^3-f_k\left(x^{'}\right)\right)G_k^3+\\
&\quad+\sum_{k\in K^{>}}\left(F_k^4-f_k\left(x^{'}\right)\right)G_k^4-\sum_{k\in K^{=}\cup K^{><}}f_k\left(x^{'}\right)G_k^6\Big)=\\
&=\max\Big(\max_{k\in K^{\geq}}\Big(\left(F_k^1-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^1\ R_1\\
&\quad\max_{k\in K^{\leq}}\Big(\left(F_k^2-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^2\ R_2\\
&\quad\max_{k\in K^{<}}\Big(\left(F_k^3-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^3\ R_3\\
&\quad\max_{k\in K^{>}}\Big(\left(F_k^4-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^4\ \Big)+\\
&+\sum_{k\in K^{0}}\Big(\left(F_k^5-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^5+\\
&+\rho(\sum_{k\in K^{\geq}}\Big(\left(F_k^1-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^1+\\
&\quad+\sum_{k\in K^{\leq}}\Big(\left(F_k^2-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^2+\\
&\quad+\sum_{k\in K^{<}}\Big(\left(F_k^3-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^3+\\
&\quad+\sum_{k\in K^{>}}\Big(\left(F_k^4-f_k\left(x^*\right)\right)+\left(f_k\left(x^*\right)-f_k\left(x^{'}\right)\right)\Big)G_k^4-\\
&\quad-\sum_{k\in K^{=}\cup K^{><}}(f_k\left(x^*\right)+\left(f_k\left(x^{'}\right)-f_k\left(x^*\right)\right)\Big)G_k^6)<\\
&<\max\Big(\max_{k\in K^{\geq}}\left(F_k^1-f_k\left(x^*\right)\right)G_k^1\ R_1\ \max_{k\in K^{\leq}}\left(F_k^2-f_k\left(x^*\right)\right)G_k^2\\
&\quad R_2\max_{k\in K^{<}}\left(F_k^3-f_k\left(x^*\right)\right)G_k^3\ R_3\max_{k\in K^{>}}\left(F_k^3-f_k\left(x^*\right)\right)G_k^4\Big)+\\
&+\sum_{k\in K^{0}}\left(F_k^5-f_k\left(x^*\right)\right)G_k^5+\\
&+\rho(\sum_{k\in K^{\geq}}\left(F_k^1-f_k\left(x^*\right)\right)G_k^1+\sum_{k\in K^{\leq}}\left(F_k^2-f_k\left(x^*\right)\right)G_k^2+\sum_{k\in K^{<}}\left(F_k^3-f_k\left(x^*\right)\right)G_k^3+\\
&\quad+\sum_{k\in K^{>}}\left(F_k^4-f_k\left(x^*\right)\right)G_k^4-\sum_{k\in K^{=}\cup K^{><}}f_k\left(x^*\right)G_k^6\Big)=\\
&=T\left(x^*\right).
\end{aligned}
$$

It follows from (9) that $T\left(x^{'}\right)<T\left(x^*\right)$, which contradicts to (7). Hence, $x^*\in X$ is a Pareto optimal solution of the multicriteria optimization problem.

The scalarizing problem *GENS* guarantees that Pareto optimal solutions are generated. The common drawback [Miettinen, 1999] is how to select the coefficient ρ. An alternative way is to use a lexicographic approach. The following *GENSLex* problem in two phases is a lexicographic variant of scalarizing problem *GENS*.

The first problem *GENSLex1* to be solved is the following:

Minimize

(10) $T_1(x)$ =max $\left(\max\limits_{k\in K^{\geq}}\left(F_k^1 - f_k(x)\right)G_k^1\ R_1 \max\limits_{k\in K^{\leq}}\left(F_k^2 - f_k(x)\right)G_k^2\ R_2 \max\limits_{k\in K^{<}}\left(F_k^3 - f_k(x)\right)G_k^3\right.$

$$\left. R_3 \max\limits_{k\in K^{>}}\left(F_k^4 - f_k(x)\right)G_k^4\right) + \sum_{k\in K^0}\left(F_k^5 - f_k(x)\right)G_k^5$$

subject to:

(11) $f_k(x) \geq f_k, k \in K^{>} \cup K^{=}$

(12) $f_k(x) \geq f_k - D_k, k \in K^{\leq}$

(13) $f_k(x) \geq f_k - t_k^-, k \in K^{><}$

(14) $f_k(x) \leq f_k + t_k^+, k \in K^{><}$

(15) $x \in X$

Let us denote the optimal objective function value of (10) by T_1^*. The final solution is obtained by solving the following problem *GENSLex2*:

Minimize

(16) $T_2(x) = \sum\limits_{k\in K^{\geq}}\left(F_k^1 - f_k(x)\right)G_k^1 + \sum\limits_{k\in K^{\leq}}\left(F_k^2 - f_k(x)\right)G_k^2 + \sum\limits_{k\in K^{<}}\left(F_k^3 - f_k(x)\right)G_k^3 +$

$$+ \sum_{k\in K^{>}}\left(F_k^4 - f_k(x)\right)G_k^4 - \sum_{k\in K^{=}\cup K^{><}} f_k(x)G_k^6)$$

subject to:

(17) $T_1(x)$ =max $\left(\max\limits_{k\in K^{\geq}}\left(F_k^1 - f_k(x)\right)G_k^1\ R_1 \max\limits_{k\in K^{\leq}}\left(F_k^2 - f_k(x)\right)G_k^2\ R_2 \max\limits_{k\in K^{<}}\left(F_k^3 - f_k(x)\right)G_k^3\right.$

$$\left. R_3 \max\limits_{k\in K^{>}}\left(F_k^4 - f_k(x)\right)G_k^4\right) + \sum_{k\in K^0}\left(F_k^5 - f_k(x)\right)G_k^5 \leq T_1^*$$

and constraints (11) - (15).

<u>*Theorem 2:*</u> *The optimal solution of GENSLex scalarizing problem is a Pareto optimal solution of the multicriteria optimization problem.*

<u>*Proof :*</u>

Let $K^{\geq} \neq Ø$ and/or $K^{>} \neq Ø$, or $K^0 = K$ and let $x^* \in X$ be an optimal solution of *GENLex* scalarizing problem. Then the constraints (11) - (15) are satisfied for $x^* \in X$, together with the following conditions:

$$T_1(x^*) \leq T_1(x) \text{ and } T_2(x^*) \leq T_2(x),\ x \in X.$$

Let us assume that $x^* \in X$ is not a Pareto optimal solution of the multicriteria optimization problem. Then there must exist another $x' \in X$, for which the constraints (11) – (15) are satisfied, as well as the condition given below:

(18) $f_k(x') \geq f_k(x^*)$, $k \in K$

and $f_k(x') > f_k(x^*)$ for at least one index $k \in K$.

It is clear that independently of defined values of R_1, R_2, R_3 and from (18) and (10 - 17) follows that:

$$T_1(x^{'}) \leq T_1(x^{*}) \text{ and } T_2(x^{'}) < T_2(x^{*})$$

or

$$T_1(x^{'}) < T_1(x^{*}) \text{ and } T_2(x^{'}) \leq T_2(x^{*}),$$

which contradicts with x^* being an optimal solution of *GENLex* scalarizing problem.

Scalarizing problem *GENS* is in the general case an optimization problem with a non-differentiable objective function. Every *GENS* scalarizing problem (defined values of R_1, R_2, R_3) can be reduced to an equivalent optimization problem with a differentiable objective function on the account of additional variables and constraints. The equivalency of each pair of optimization problems is in relation to the obtained values of the objective functions (criteria) and the main variables. Different types of equivalent problems are obtained at different values of R_1, R_2, R_3.

Every equivalent problem can be presented as follows:

$$\min\left(\mu + \sum_{k \in K^0} y_k + \rho \sum_{k \in K \setminus K^0} y_k\right)$$

and satisfies two groups of constraints.

The first group of constraints is equal for all types of equivalent problems and has the following form:

(19) $\alpha \geq \left(F_k^1 - f_k(x)\right)G_k^1, k \in K^{\geq}$

(20) $\beta \geq \left(F_k^2 - f_k(x)\right)G_k^2, k \in K^{\leq}$

(21) $\gamma \geq \left(F_k^3 - f_k(x)\right)G_k^3, k \in K^{<}$

(22) $\Omega \geq \left(F_k^4 - f_k(x)\right)G_k^4, k \in K^{>}$

(23) $\left(F_k^1 - f_k(x)\right)G_k^1 = y_k, k \in K^{\geq}$

(24) $\left(F_k^2 - f_k(x)\right)G_k^2 = y_k, k \in K^{\leq}$

(25) $\left(F_k^3 - f_k(x)\right)G_k^3 = y_k, k \in K^{<}$

(26) $\left(F_k^4 - f_k(x)\right)G_k^4 = y_k, k \in K^{>}$

(27) $\left(F_k^5 - f_k(x)\right)G_k^5 = y_k, k \in K^0$

(28) $-f_k(x)G_k^6 = y_k, k \in K^{=} \cup K^{><}$

(29) $f_k(x) \geq f_k, k \in K^{>} \cup K^{=}$

(30) $f_k(x) \geq f_k - D_k, k \in K^{\leq}$

(31) $f_k(x) \geq f_k - t_k^-, k \in K^{><}$

(32) $f_k(x) \leq f_k + t_k^+, k \in K^{><}$

(33) $x \in X$

α, β, γ, Ω, y_k / $k \in K$ - arbitrary.

The second group of constraints has different type and number of constraints depending on the values of R_1, R_2, R_3. The constraints from the second group for one equivalent problem of scalarizing problem *GENS*, which is obtained when R_1 is equal to the separator ",", R_2 and R_3 are equal to the arithmetic operation "+", have the following form:

(34) $\mu \geq \alpha$

(35) $\mu \geq \beta + \gamma + \Omega$

μ - arbitrary.

The constraints from the second group in the other equivalent problems can be stated in a similar way.

Scalarizing problems *GENSLex1* and *GENSLex2* are in the general case optimization problems with a non-differentiable objective functions and constraints. Every scalarizing problem of both types *GENSLex1* and *GENSLex2* (defined values of R_1, R_2, R_3) can be reduced to an equivalent optimization problems with a differentiable objective functions and constraints on the account of additional variables and constraints.

Different types of equivalent problems of scalarizing problem *GENSLex1* are obtained at different values of R_1, R_2, R_3. Each equivalent problem can be presented as follows:

(36) $\min\left(\mu + \sum_{k \in K^0} y_k\right)$,

satisfying two groups of constraints. The first group of constraints is equal for all types of equivalent problems and has the following form:

(37) $\alpha \geq \left(F_k^1 - f_k(x)\right)G_k^1,\ k \in K^{\geq}$

(38) $\beta \geq \left(F_k^2 - f_k(x)\right)G_k^2,\ k \in K^{\leq}$

(39) $\gamma \geq \left(F_k^3 - f_k(x)\right)G_k^3,\ k \in K^{<}$

(40) $\Omega \geq \left(F_k^4 - f_k(x)\right)G_k^4,\ k \in K^{>}$

(41) $\left(F_k^5 - f_k(x)\right)G_k^5 = y_k,\ k \in K^0$

(42) $f_k(x) \geq f_k,\ k \in K^{>} \cup K^{=}$

(43) $f_k(x) \geq f_k - D_k,\ k \in K^{\leq}$

(44) $f_k(x) \geq f_k - t_k^-,\ k \in K^{><}$

(45) $f_k(x) \leq f_k + t_k^+,\ k \in K^{><}$

(46) $x \in X$

α, β, γ, Ω, $y_k / k \in K^0$ - arbitrary.

The second group of constraints has different type and number of constraints depending on the values of R_1, R_2, R_3. The constraints from the second group for one equivalent problem of scalarizing problem

GENSLex1, which is obtained when R_1 is equal to the separator ",", R_2 and R_3 are equal to the arithmetic operation "+", have the following form:

(47) $\mu \geq \alpha$

(48) $\mu \geq \beta + \gamma + \Omega$

μ - arbitrary.

Different types of equivalent problems of scalarizing problem *GENSLex2* are obtained at different values of R_1, R_2, R_3. Each equivalent problem can be presented as follows:

(49) $\min\left(\sum_{k \in K \setminus K^0} y_k\right)$

and satisfies two groups of constraints.

The first group of constraints is equal for all types of equivalent problems and has the following form:

(50) $\left(F_k^1 - f_k(x)\right)G_k^1 = y_k \, , k \in K^{\geq}$

(51) $\left(F_k^2 - f_k(x)\right)G_k^2 = y_k \, , k \in K^{\leq}$

(52) $\left(F_k^3 - f_k(x)\right)G_k^3 = y_k \, , k \in K^{<}$

(53) $\left(F_k^4 - f_k(x)\right)G_k^4 = y_k \, , k \in K^{>}$

(54) $-f_k(x)G_k^6 = y_k, k \in K^{=} \cup K^{><}$

(55) $f_k(x) \geq f_k, k \in K^{>} \cup K^{=}$

(56) $f_k(x) \geq f_k - D_k \, , k \in K^{\leq}$

(57) $f_k(x) \geq f_k - t_k^- \, , k \in K^{><}$

(58) $f_k(x) \leq f_k + t_k^+ \, , k \in K^{><}$

(59) $x \in X$

$y_k \,/\, k \in K \setminus K^o$ - arbitrary.

The second group of constraints has different type and number of constraints depending on the values of R_1, R_2, R_3. The constraints from the second group for one equivalent problem of scalarizing problem *GENSLex2*, which is obtained when R_1 is equal to the separator ",", R_2 and R_3 are equal to the arithmetic operation "+", have the following form:

(60) $\alpha \geq \left(F_k^1 - f_k(x)\right)G_k^1 \, , k \in K^{\geq}$

(61) $\beta \geq \left(F_k^2 - f_k(x)\right)G_k^2 \, , k \in K^{\leq}$

(62) $\gamma \geq \left(F_k^3 - f_k(x)\right)G_k^3 \, , k \in K^{<}$

(63) $\Omega \geq \left(F_k^4 - f_k(x)\right)G_k^4 \, , k \in K^{>}$

(64) $\left(F_k^5 - f_k(x)\right)G_k^5 = y_k, k \in K^0$

(65) $\mu \geq \alpha$

(66) $\mu \geq \beta + \gamma + \Omega$

(67) $\left(\mu + \sum_{k \in K^0} y_k\right) \leq T_1^*$

α, β, γ, Ω, μ, $y_k / k \in K^0$ - arbitrary.

Conclusion

The interactive algorithms solving different types of multicriteria optimization problems use different scalarizing problems. The features of each scalarizing problem are defined by the possibilities offered to the decision maker to set his/her preferences, as well as by the quality of the Pareto optimal solutions obtained. Altering the parameters of the generalized scalarizing problems *GENS* and *GENSLex*, a great part of the already known scalarizing problems can be obtained and also new scalarizing problems can be generated. In connection with this, generalized interactive algorithms with alterable scalarization and parameterization can be designed, which expand to a great extent the possibilities of the decision-maker in describing his/her preferences.

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