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Author's Information

Vitaliy Snytyuk – Taras Shevchenko national university of Kyiv, postdoctorate researcher of cybernetics faculty; Glushkov's av. 2, building 6, Kiev, Ukraine; e-mail: svit@majar.com

ALGORITHM OF CONSECUTIVE DEFINITION OF RANKING OF THE OBJECTS NEAREST TO THE SET CYCLIC RELATION BETWEEN OBJECTS

Grigoriy Gnatienko

Abstract: The problem of a finding of ranging of the objects nearest to the cyclic relation set by the expert between objects is considered. Formalization of the problem arising at it is resulted. The algorithm based on a method of the consecutive analysis of variants and the analysis of conditions of acyclicity is offered.

Keywords: ranking, the binary relation, acyclicity, basic variant, consecutive analysis of variants

ACM Classification Keywords: K.3.2 Computer and Information Science Education.

Introduction

The problem of ordering of set of objects in degrees of display of some properties is one of the primary goals of expert reception of estimations [1]. The essence of a problem will consist in definition of the full order on set of compared objects under the set partial order.

Among problems of decision-making, the problem of linear ordering of objects is allocated with a plenty of concrete applications and a unconditional urgency of a theme. This problem traditionally is in the center of attention of researchers and the quantity of the works devoted to questions of construction optimum in this or that sense of linear orders on set of compared objects is very great [2].

Practical application of problems of ranging is very various [3]. Such problems arise, for example, at the decision of

- a problem of definition of sequence of loading and unloading of a transport spacecraft;
- a finding of sequence of elimination of malfunctions of various systems;
- the complex analysis of quality of production;
- the analysis of characteristics of production and allocation of the main parameters of quality;
- a finding of bottlenecks in some complex systems possessing such properties as stability, controllability, self-organizing;
- designing of liaison channels between units in information networks;
- expert reception of estimations of projects of development of branches of a national economy or scientific researches;
- planning of building of residential areas, etc.

Problems of qualitative and quantitative ranging are considered. At the decision of such problems wide application was received with a method of pair comparisons. The set of works [4] is devoted to the analysis of the specified problems.

The problem of definition of ranging of objects is the widespread problem of the theory of decision-making. This problem is solved various methods. At application of expert estimations for ranging objects the information both from one expert, and from expert group can be used.

As the person frequently supposes infringement of a condition of transitivity of relations at estimations of objects even at the decision of a problem of ranging one expert in relations between objects can appear cycles. In such cases there is a problem of definition of the ranking nearest to the relation set by the expert.

Problem Definition

Let it is necessary to find ranking (the linear order) n objects of set A, the nearest to the cyclic relation set by a matrix of pair comparisons

$$P = \left(p_{ij}\right),\ \ i, j \in I = \{1,...,n\}. \tag{1}$$

Elements of a matrix P express result of comparison of objects $a_i \in A,\ \ i \in I,$ with indexes $i, j \in I,$ and are defined thus:

$$p_{ij} = \begin{cases} 1,\ \ if\ \ a_i \succ a_j, \\ 0,\ \ if\ \ i = j, \\ -1,\ \ if\ \ a_i \prec a_j, \end{cases}$$

$$p_{ij} + p_{ji} = 0,\ \ \forall i, j \in I,\ \ i \neq j,$$

where "$\succ$" - a symbol of the relation of preference between objects.

Construction of the linear order generally demands entering enough the big changes in initial structure of preferences of a kind (1) on set of objects A. The problem of a finding of the order is a complex combinatory problem, NP - difficult in strong sense [2]. Therefore algorithms of local optimization, heuristic algorithms or the algorithms basing a method of branches and borders are applied to construction of ranking R*. Methods of the consecutive analysis of variants in the offered interpretation for this class of problems were not applied, though their use in this area of researches is perspective.

The linear order nearest to the set relation of a kind (1), we shall search as

$$R^* = Arg \min_{R \in \Re} d(P,R),$$

where $\Re$ - set of matrixes which correspond to linear orders n objects, d(R,P) – distance between ranking $R \in \Re$, which is under construction, and set cyclic relation P of a kind (1).

For measurement of distance between set relation P and ranking R, we shall use the most widespread in this class of problems metrics Hamming

$$d(P,R) = 0,5 \sum_{i \in I} \sum_{j \in I} \left| p_{ij} - r_{ij} \right|,$$

where p_{ij}, r_{ij} – accordingly elements of matrixes P and R.

Formalization of a Problem

As matrixes of relation P and R are slanting symmetric they can be written down as vectors C and X with elements

$$\begin{aligned} &c_t = p_{ij}, \quad x_t = r_{ij}, \\ &t = (i-1)n + j - (i+1)i/2, \ 1 \le i < j \le n. \end{aligned} \tag{2}$$

Then the distance between relations P and R will be written down as

$$d(P,R) = \sum_{j \in J} \left| c_j - x_j \right|, \quad j = 1,..., n(n-1)/2 = N, \quad J = \{1,..., N\}.$$

The problem of a finding of the linear order nearest to set on set of objects to the relation (1), is formalized as

$$\sum_{j \in J} \left| c_{ij} - x_{ij} \right| \ \to \ \min, \tag{3}$$

$$x_j \in X_j^0 = \{-1,1\}, \quad j \in J, \tag{4}$$

$$x \in D^A \subset X^0, \quad X^0 = \prod_{j \in J} X_j^0, \tag{5}$$

where D^A – set of vectors of a kind (2) which correspond{meet} to acyclic relations between objects.

Specificity of a problem (3) - (5) will be, that its decision $x \in X$ should satisfy to a condition of acyclicity as the relation which is set by matrix R, should belong to a class of linear orders.

We shall consider a chain of objects a_{i1}, a_{i2}, a_{i3} with the set relations of preference which we shall designate symbols $a_{i1}\ \pi\ a_{i2}\ \pi\ a_{i3}$, $i_1, i_2, i_3 \in L$, $\pi \in \{\succ, \prec\}$.

Basic sub-variant b, which is generated by the three of objects (a_{i1}, a_{i2}, a_{i3}), we shall name elements of a vector of a kind (2) with components

$$b = (c_{j1}, c_{j2}, c_{j3}), \ 1 \le j_1 < j_2 < j_3 \le N, \ \ c_{j1}, c_{j2}, c_{j3} \in c, \ \ c_{j1}, c_{j2}, c_{j3} \in \{-1,1\}, \tag{6}$$

which values answer relations of a kind

$$\left(a_{i1}\ \pi\ a_{i2}, \ \ a_{i2}\ \pi\ a_{i3}, \ \ a_{i3}\ \pi\ a_{i1}\right), \ \ a_{i1}, a_{i2}, a_{i3} \in A, \ \ \pi \in \{\succ, \prec\}.$$

The basic sub-variant is a minimal subset of objects from set A, on which it is possible to reveal a cycle.

Allowable basic sub-variant we shall name a basic sub-variant which is generated by the three of objects, relations between which satisfy to a condition of acyclicity.

Full variant (variant of length N) we shall name a vector which answers the full binary relation on set of objects.

Allowable variant x^D we shall name a full variant which answers the acyclic relation on set of all n objects, that is $x^D \in D^A$.

At check of an admissibility of basic variants of a kind (6) which are formed by objects a_{i1}, a_{i2}, a_{i3}, $1 \le i_1 < i_2 < i_3 \le n$, it is necessary to consider relations as ($a_{i1} \pi\, a_{i2}$, $a_{i2} \pi\, a_{i3}$, $a_{i1} \underline{\pi}\, a_{i3}$), where $\underline{\pi}$ - inversion of the relation π: $a_{i1} \underline{\pi}\, a_{i3} \Leftrightarrow a_{i3} \pi\, a_{i1}$, $\pi = \{\succ, \prec\}$.

Set $X^s = \{\cup X^s_j, j=1,...,N\}$, $X^s \subseteq X^0$, s=1,2,..., we shall name reduced (concerning initial set X^0).

For the decision of a problem (3)-(5) procedure of reduction of set of allowable decisions Z on a condition $X^s = (X^s_1 \times X^s_2 \times \ldots \times X^s_N)$ of acyclicity of the relation which corresponds to the decision of a problem of a finding of strict resulting ranging of objects of set A is used. The analysis of variants in view of a condition of acyclicity of the decision is carried out with use of the procedure described in [5].

Let's designate set of all possible values which can get elements of a basic variant, through B^0. Sets of a kind B^0 are formed of set X^s by association of three various columns of a matrix X^s: $X^0_{j1} \cup X^0_{j2} \cup X^0_{j3}$, $1 \le j_1 < j_2 < j_3 \le N$, $X^0_{j1}, X^0_{j2}, X^0_{j3} \in X^s$. Capacity of this set is equal $|B^0| = 6$.

Basic set $B^0 = B^0_1 \times B^0_2 \times B^0_3$, $B^0_i = (-1,1)^T$, i=1,…,3, we shall name set of elements of a matrix of which values of basic vectors get out.

Columns of basic set $B^0_i = (-1,1)^T$, i=1,2,3, we shall name subsets of basic set.

The reduced basic set B^s, $B^s \subset B^0$, s=1,2,..., we shall name a matrix which is formed of a matrix B^0 by removal from it separate elements.

It is known [6], that for matrixes of pair comparisons with elements of a kind (2) requirement of absence of cycles is equivalent to the requirement of absence of cycles of length three (T=3).

As the top triangular matrix of a matrix P^i, $i \in I$, contains the full information on all matrix indexes of objects need to be considered only on increase, that is $1 \le i_1 < i_2 < i_3 \le n.$ Indexes of elements of a vector of relations between objects also satisfy to conditions $1 \le j_1 < j_2 < j_3 \le N.$

Let's designate through ψ function of two arguments which values are calculated under the formula

$$j = \psi(i_1, i_2) = (i_1 - 1)n + i_2 - (i_1 + 1)i_1 / 2, \ 1 \le i_1 < i_2 < n.$$

Algorithm

In a problem of definition of the linear order nearest to the set cyclic relation, it is possible to present algorithm of the consecutive analysis and elimination of inadmissible elements in the following kind.

Step 1. Let's put initial values of the decision of a problem equal $x(j) = c(j), \quad j \in J.$

Step 2. The organization of three enclosed cycles: i:=1 до n-2; i_1:=i+1 до n-1; i_2:=i_1+1 до n. Variables of cycles i, i_1, i_2 are indexes of objects. In a body of these cycles the following steps are executed.

Step 3. Definition of indexes of elements *j, j₁, j₂* the current basic set B^0_j on indexes of objects *i*, i_1, i_2: $j=\psi(i,i_1)$, $j_1=\psi(i,i_2)$, $j_2=\psi(i_1,i_2)$.

Step 4. Definition of three of objects, relations between which form cycles. Quantity of all three n objects equally: : k3=n*(n-1)*(n-2)/6.

Quantity of cycles on set n objects it is equal $d=(n^3-4n)/24$ for even and $d=(n^3-n)/24$ for odd values n.

Step 5. Generation of a vector of indexes of participation of relations between objects in cycles:

$vc(j), \quad j \in J$. That is, value $vc(j), \quad j \in J$, is equal to quantity of ocurrences of the relation with an index $j, \quad j \in J$, in cyclic three.

Step 6. Definition of values of vectors of indexes $vic(j), \quad j \in J$, participations of the inverted relations $x(j) = c(j) - 2, \quad j \in J$.

The choice of an index of the relation between objects is carried out in view of three criteria:

$К_1$ – inversion of the relation does not generate new three;

$К_2$ – the total quantity of cycles for the inverted relation is minimal;

$К_3$ – the difference of quantity of cycles for the set relation and inverted is minimal.

Step 7. Definition of relations, which replacement on inverted, as much as possible reduces quantity of cycles.

Step 8. Choice of an index of the relation for decision-making on its final inverting.

Step 9. The termination of cycles on i, i_1, i_2.

Step 10. Recurrence of points 1-9 of the resulted algorithm until in the decision $x(j), \quad j \in J$, problems (3)-(5) exist cycles.

Conclusion

The resulted algorithm allows to find consistently for final quantity of steps the ranking of the objects nearest to the cyclic relation set by the expert between objects.

Computing experiments have confirmed efficiency of the resulted algorithm. Received with the help of algorithm of the decision are one of the rankings, the nearest to the cyclic relation set by the expert on set of objects.

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Author's Information

Grigoriy M. Gnatienko – Dr.Ph. Taras Shevchenko Kiev National University, Faculty of Cybernetics, Kiev, Ukraine. e-mail: G.Gnatienko@veres.com.ua