SIGNAL PROCESSING UNDER ACTIVE MONITORING

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Abstract: This paper describes a method of signal preprocessing under active monitoring. Suppose we want to solve the inverse problem of getting the response of a medium to one powerful signal, which is equivalent to obtaining the transmission function of the medium, but do not have an opportunity to conduct such an experiment (it might be too expensive or harmful for the environment). Practically the problem can be reduced to obtaining the transmission function of the medium. In this case we can conduct a series of experiments of relatively low power and superpose the response signals. However, this method is conjugated with considerable loss of information (especially in the high frequency domain) due to fluctuations of the phase, the frequency and the starting time of each individual experiment. The preprocessing technique presented in this paper allows us to substantially restore the response of the medium and consequently to find a better estimate for the transmission function. This technique is based on expanding the initial signal into the system of orthogonal functions.

Keywords: mathematical modelling, active monitoring, frequency and phase fluctuation.

ACM Classification Keywords: I.6.1 Simulation Theory.

Introduction

In June 2001 in Kiev, Ukraine, by Kiev Institute of Geophysics there was conducted an experiment to determine the properties of the monument “Ukraine” such as eigenfrequencies and damping decrement. In particular the responses of the monument to a series of low power ambient perturbation were measured [1-4]. This paper describes a method of preprocessing of the observed signal that allows to substantially restore the response of the medium and to find the properties of the monument more precisely.

The Mathematical Model of Active Monitoring with Frequency and Phase Fluctuations of Sound Signal

For the i-th experiment in a series of M experiments the model of active monitoring can be represented as follows:

$$y_i(t) = S\left(t, \tau_i, \vec{h}_i\right) * H(t) + n_i(t), \quad t \in \left(\tau_i, \tau_i + T\right) \tag{1}$$

where $y_i(t)$ is the response of the medium to the exploring signal $S\left(t, \tau_i, \vec{h}_i\right)$, which depends on the vector of parameters $\vec{h}_i$ in the i-th experiment. This signal is convoluted with the response of the medium $H(t)$ to the delta-function signal $\delta(t)$. Also, $n_i(t)$ is the additive background noise, which accompanies the experiment, T is the duration of one experiment, $\left(\tau_i, \tau_i + T\right)$ is the time interval of the i-th experiment realization, and $*$ is the symbol of the convolution operator. The experiment is created in such a manner that the energy $\mathrm{E}\left[S\left(t, \tau_i, \vec{h}_i\right) * H(t)\right]$ of the registered by sensors signal and the energy $\mathrm{E}\left[n_i(t)\right]$ of the natural background are commensurable in the selected metrics, i.e. the influence of the experiment on the environment is negligible. The model takes into account that nonlinear effects in the experiment might be neglected. Therefore the linear procedure for the interaction of the medium and the exploring signal in the form of convolution is selected.

The next model defines the full experiment:

$$y(t)=\sum_{i=1}^{M} y_i(t) \quad (2)$$

The Probing Signal Model

Consider a signal $S\left(t, \tau_i, \vec{h}_i\right)$ that depends on the vector of parameters $\vec{h}_i=\left\{h_{i1}, \ldots, h_{iN+2}\right\}$, first N components of which $h_{i1}, \ldots, h_{iN}$ are included into the model linearly. These parameters determine the form of the exploring signal in the i-th experiment, τ_i is the starting time of the i-th experiment, h_{iN+1} is the fluctuating frequency ω_{0i}, and h_{iN+2} is the fluctuating phase ψ_i.

One considers the signal to be a stationary, physically realizable wave, i.e. it satisfies the following conditions:

$$\int_0^\infty S\left(t, \tau_i, \psi_i, \vec{h}_i\right) dt = 0 \text{, for } \forall\, h_i;$$
$$S\left(t, \tau_i, \psi_i, \vec{h}_i\right)=\begin{cases} S\left(t-\tau_i-\psi_i, h_{1i}, \ldots, h_{iN+2}\right), & \text{for } t \geq \tau_i, \\ 0, & \text{for } t<\tau_i. \end{cases};$$
$$\int_0^\infty \left(S\left(t, \tau_i, \psi_i, \vec{h}_i\right)\right)^2 dt < \infty. \quad (3)$$
$$\int_0^\infty S\left(t, \vec{h}_i\right) dt = 0 \text{, for } \forall\, h_i.$$

The last equality in (3) means that the signal is a wave, i.e. the signal does not leave after-effects and consequently it does not change the constant constituent of the medium.

The same conditions hold for the response of the medium $H(t)$.

The latter circumstance allows us to determine the duration of one experiment T (taking into account statistical characteristics of the background noise). For this purpose the noise level ε is required to be much less or at least less than the natural background energy level. Here

$$\varepsilon=\sqrt{\int_T^\infty \left(S\left(t, \tau_i, \psi_i \vec{h}_i\right)\right)^2 dt}$$

Within some tolerance one can represent the signal as a linear combination of the truncated orthogonal basis functions (in some metric) on the interval of length T:

$$\varphi_k(t-\tau_i, k\omega_{0i})\chi\left(t, \tau_i, \tau_i+T\right); \;\; \omega_{0i}=\omega_0+\Delta\omega_i; \;\; T=\omega_0\pi.$$

$$S\left(t, \tau_i, \psi_i, \vec{h}_i\right)=\sum_{k=1}^{N} h_{ik}\varphi_k(t-\tau_i-\psi_i, k\omega_{0i})\chi\left(t, \tau_i, \tau_i+T\right) \quad (4)$$

Here ψ_i is the fluctuating phase with zero expectation; $\omega_{0i}=\omega_0+\Delta\omega_i$ is the random frequency with mean ω_0.

The set $\vec{\omega}=\left\{\omega_1, \omega_2, \ldots, \omega_M\right\}$ is considered to be the set of outcomes of the random variable $\omega_i=\omega_0+\Delta\omega_i$ fluctuating around ω_0, here fluctuations carry out on the closed interval $\left[\omega_1^*, \omega_2^*\right]$ and

$\left|\omega_1^* - \omega_2^*\right| < \omega_0$. In our case we can approximate the partial density of the parameter *h* distributed on the interval $\Delta = [h_1, h_1 + \varepsilon]$ of length ε with a beta distribution with parameters $\gamma, \eta, \varepsilon$. Varying these parameters one can obtain different forms of approximation of the density (5), which has the following form [5]:

$$e(h,\gamma,\eta,\varepsilon) = \begin{cases} \dfrac{1}{\varepsilon}\dfrac{\Gamma(\gamma+\eta)}{\Gamma(\gamma)\Gamma(\eta)}\left(\dfrac{h}{\varepsilon}-h_1\right)^{\gamma-1}\left(1-\dfrac{h}{\varepsilon}-h_1\right)^{\eta-1}; h \in \Delta; \\ 0, \text{when} h \notin \Delta; \quad 0<\gamma, 0<\eta; \quad \Delta = \left((k-1)T, (k-1)T+\varepsilon\right) \end{cases} \tag{5}$$

The particular case of the beta distribution with the parameters $\gamma = \eta = 1$ is the uniform density. Let us pay especial attention to the uniform density since it has the maximum entropy among all the distributions on the closed interval.

Here entropy $H(\alpha)$ is a measure of uncertainty of an experiment α, in the result of which the events $A_1, A_2, \ldots, A_n$ with corresponding probabilities $p_1, p_2, \ldots, p_3$ occur. The (Shannon) entropy $H(\alpha)$ is defined as $H(\alpha) = -\sum_{i=1}^{n} p_i \log_2(p_i)$ [6].

In our case as a result of the *i*-th experiment the parameter of the probing signal appears in the *i*-th sub-region of the set (divided with the points $x_1, x_2, \ldots, x_n$) of all possible values of this parameter. We consider the probability of this event to be defined as a beta distribution with the parameters η, γ:

$$p_i = \frac{\Gamma(\eta+\gamma)}{\Gamma(\eta)\Gamma(\gamma)}\int_{x_{i-1}}^{x_i} x^{\eta-1}(1-x)^{\gamma-1}\,dx, \quad \eta>0,\ \gamma>0. \tag{6}$$

Entropy becomes the function of the parameters η, γ and the number n of intervals of the partition. Entropy is represented on figure 1, with $0.1 \le \eta \le 10$, $0.1 \le \gamma \le 10$ and $n = 20$.

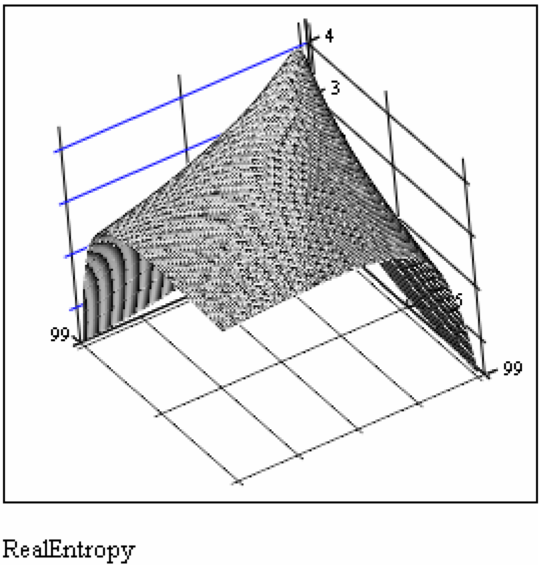

Figure 1. Entropy of the beta distribution as a function of parameters
$0.1 \le \eta \le 10$ *and* $0.1 \le \gamma \le 10$. Here $n = 20$.

We can see on figure 1 that the entropy is a uni-modal surface with the maximum at $\eta = \gamma = 1$, i.e. the maximum is reached when the distribution on the given interval is uniform.

As a result if we assume that the distribution of the parameters of the signal is uniform, the minimum information about the experiment is introduced and the estimate of the parameters will be the worst of all possible ones for such a length of the interval. This means that using any other a priory distribution we will only improve the estimate.

The same argument holds for the fluctuating phase $\vec{\psi} = \{\psi_i\};\ i = 1, \ldots, M$.

These parameters define the truncated system of basis functions:

$$\vec{\varphi}_i(t, \tau, \psi_i, \omega_{0i}) = \left\{\varphi_k(t - \tau - \psi_i, \omega_{0i}k)\chi\left(t, \tau_i, \tau_i + T\right)\right\},\ k = 1, \ldots, N \tag{5}$$

The parameters T , ω_{0i} , ψ_i are included into the model non-linearly.

Here $\chi\left(t, \tau_i, \tau_i + T\right) = \begin{cases} 1, & t \in \left(\tau_i, \tau_i + T\right), \\ 0, & t \notin \left(\tau_i, \tau_i + T\right); \end{cases}$ - is the characteristic function of the interval.

Let's consider the scenario when the values $\tau_i = \left(i - 1\right)T$ are deterministic.

We consider the case, when the set of vectors $\vec{h}_1, \ldots, \vec{h}_M$, and values ω_{0i}, ψ_i is sampled from the set of the possible values of the vector $\vec{h}, \omega_0, \psi$ with an a priori known distribution $P\left(\vec{h}, \omega_0, \psi\right)$ and the determined values $\tau_i = \left(i - 1\right)T$. I.e. the stochastic parameters $\vec{h}, \omega_0, \psi$ and the stochastic additive value $n\left(t\right)$ characterize the stochastic nature of the process $y\left(t\right) = \sum_{i=1}^{M} y_i\left(t\right)$. The information about the medium is included into the process via the deterministic function $H\left(t\right)$, the response of the medium to the delta function.

The response of the medium in the i -th experiment $y_i(t)$ is described by the following equation:

$$y_i\left(t\right) = \left(\sum_{k=1}^{N} h_{ik}\left(\int_{\tau_i}^{\tau_i + T} \varphi_k\left(t - \tau - \psi_i, \omega_{0i}k\right)H(\tau)d\tau\right)\right)\chi\left(t, \tau_i, \tau_i + T\right) + n_i\left(t\right). \tag{7}$$

Thus the result of the series of M experiments $y\left(t\right)$ is:

$$\begin{gathered} y\left(t\right) = \sum_{i=1}^{M} y_i\left(t\right) = \sum_{i=1}^{M}\left(\sum_{k=1}^{N} h_{ik}\left(\int_{\tau_i}^{\tau_i + T} \varphi_k\left(t - \tau - \psi_i, \omega_i k\right)H\left(\tau\right)d\tau\right)\right) \\ \times\chi\left(t, \tau_i, \tau_i + T\right) + n_i\left(t\right), \quad t \in \left(0, MT\right) \end{gathered} \tag{8}$$

Data-processing

Let's propose the following model for the processing procedure of the observed data:

$$\frac{1}{I}\sum_{i=1}^{M} y_i\left(t-(i-1)T,\vec{h}_i\right)+ \\ +\frac{1}{I}\sum_{i=1}^{M} n\left(t-(i-1)T\right)=\hat{E}\left[y(t)+n(t)\right]=\hat{E}\left[y(t)\right]+\hat{E}\left[n(t)\right];\ I=MT;\ t\in(0,T) \tag{9}$$

Here $\hat{E}\left[y(t)+n(t)\right]$ is the expectation estimator of the response of the medium and additive background noise, and $I=MT$ is the total time of monitoring.

Suppose $\hat{E}\left[n(t)\right]=0$, then the procedure of data processing reduces to the calculation of $\hat{E}\left[y(t)\right]$.

On one hand we have equation (9), on the other hand we can formally calculate the average of the superposition of the responses of the medium and additive background noise using the following formula (taking into account the fact that $\vec{h},\psi$ and $\Delta\omega$ are mutually independent):

$$E\left[y(t)\right]=\int_{R_{\vec{h}}}\int_{R_\psi}\int_{R_{\Delta\omega}}\left(\sum_{k=1}^{N} h_k\left(\int_0^T \varphi_k(t-\theta-\psi,(\omega_0+\Delta\omega)k)H(\theta)d\theta\right)\right)\chi\left(t,\tau,\tau+T\right) \\ \times dP(\vec{h})dP(\psi)dP(\Delta\omega)+E\left[n(t)\right] \\ =\sum_{k=1}^{N}\int_0^T H(\theta)\int_{R_{\vec{h}}} h_k\int_{R_\psi}\int_{R_{\Delta\omega}}\varphi_k(t-\theta-\psi,(\omega_0+\Delta\omega)k)\chi\left(t,\tau,\tau+T\right) \\ \times dP(\vec{h})dP(\psi)dP(\Delta\omega)d\theta \tag{10}$$

Let's denote:

$$\tilde{\varphi}_k(t-\theta)=\int_{R_h}\int_{R_\psi}\int_{R_{\Delta\omega}}\varphi_k(t-\theta-\psi,(\omega_0+\Delta\omega)k)\chi\left(t,\tau,\tau+T\right)dP(\vec{h})dP(\psi)dP(\Delta\omega) \tag{11}$$

and

$$\hat{h}_k=\int_{R_{\vec{h}}} h_k dP\left(\vec{h}\right). \tag{12}$$

Assuming independence of the fluctuating parameters, equation (9) takes the following form:

$$E\left[y(t)\right]=\sum_{k=1}^{N} h_k\int_0^T H(\theta)\tilde{\varphi}_k(t-\theta)d\theta \tag{13}$$

The ultimate purpose of the experiment was to evaluate the response of the medium to one powerful signal. This signal exceeds the noise level by many times. Depending on how much do we want the signal to exceed the noise, the number of the (individual) experiments M is chosen for accrual of the signal. However the procedure of accumulation leads to considerable distortions especially in the high-frequency domain. One can eliminate these distortions by solving equation (13) with respect to $H(t)$. The calculation of the vector of functions $\tilde{\varphi}(t)=\left\{\tilde{\varphi}_k(t)\right\}, k=\overline{1,N}$ is not difficult, since one can always receive a priori distributions of the parameters of the generated probing signals, and we can plug in the estimator $\hat{E}\left[y(t)\right]$ to the left hand side of equation (13).

Thus the problem is reduced to finding the solution of integral equation (13). One might look for the solution of the form:

$$H(t) = \sum_{q=1}^{Q} H_q \phi_q(t) \tag{14}$$

then

$$\hat{E}\left[y(t)\right] = \sum_{k=1}^{N} \hat{h}_k \sum_{q=1}^{Q} H_q \int_0^T \phi_q(\theta)\tilde{\varphi}_k(t-\theta)d\theta = \sum_{k=1}^{N} \hat{h}_k \sum_{q=1}^{Q} H_q \Psi_{k\,q}(t)\,. \tag{15}$$

Here

$$\Psi_{k\,q}(t) = \int_0^T \phi_q(\theta)\tilde{\varphi}_k(t-\theta)d\theta\,. \tag{16}$$

Solving the latter problem with respect to $\vec{H}$ we obtain the desirable result.

The Analysis of Illustrative Examples

Solving the fragment of the direct problem (11) for one harmonic allows us to investigate the evolution of the signal during the accumulation resulted from its frequency and phase fluctuations. For the harmonic with number *k* the analytical solution of the direct problem for the uniform distribution of frequency fluctuations (the case, which has the maximum entropy) is given by the following expression:

$$\varphi_k(t) = \frac{-2\sqrt{2T}\,\sin\left[\left(\omega_0 + \frac{\varepsilon_2+\varepsilon_1}{2}\right)k\,\frac{t}{T}\right]\sin\left[\frac{\varepsilon_2-\varepsilon_1}{2T}\,kt\right]}{k(\varepsilon_2-\varepsilon_1)t}\,\chi(t,0,T)\,. \tag{17}$$

From expression (17) we see that as the signal accumulates the harmonic is superimposed with the branch of the hyperbola whose coefficient equals to the number of the harmonic $\frac{1}{k(\varepsilon_2-\varepsilon_1)t}$, and the frequency of beating $\sin\left[\frac{\varepsilon_2-\varepsilon_1}{2T}kt\right]$ that is proportional to the number of the harmonic and the length of the time interval (on which the fluctuations of frequency are distributed). Figure 2 shows the corresponding harmonic number 31, which is undistorted (light line) and distorted (bold line) as the result of accumulation of the signal. Here $\omega_0 = \pi$ ε_1 =0, and ε_2 = 0.1.

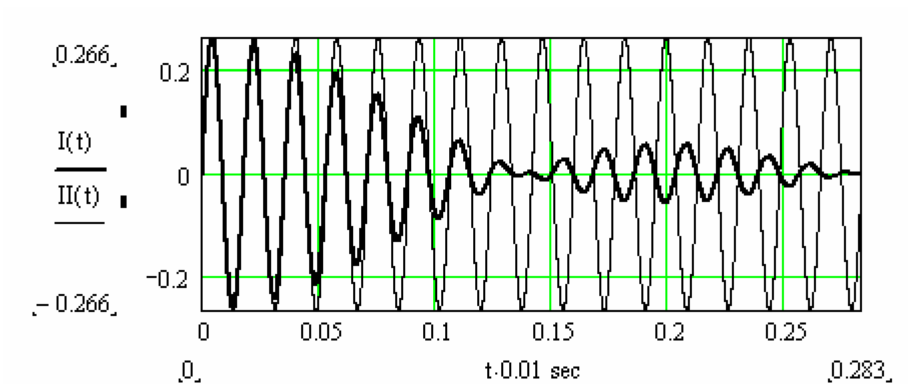

Figure 2. Undistorted harmonic (light line II(t)) and distorted harmonic (bold line I(t)) as the result of accumulation of the signal. Here $\omega_0 = \pi$ ε_1=0, and ε_2 = 0.1. The abscissa axis represents time in seconds; the ordinate axis corresponds to the amplitude of the signals in relative units.

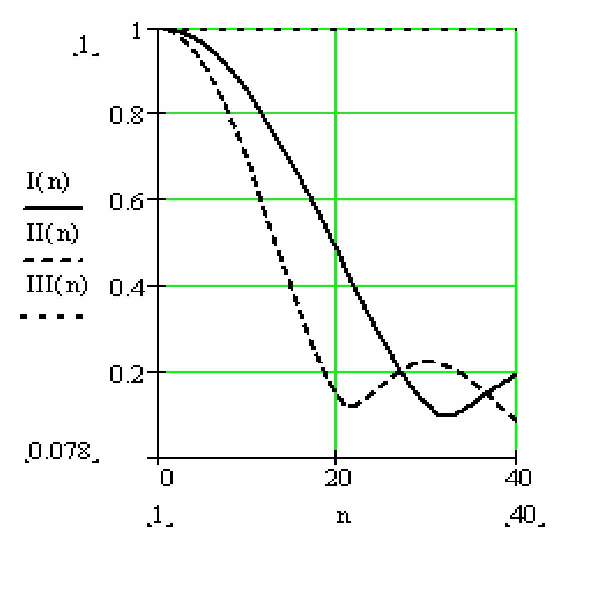

Figure 3. Change of the norm of the harmonic as a function of its number resulting from the accumulation of signals. I(n) and II(n) correspond to different values of the entropy H of the a priory distribution of the fluctuating frequency.

I(n) – n=200, $n = 200, \Delta = 1, H = 2.301$

II(n) - $n = 200, \Delta = 0.3, H = 1.778$

III(n) – norms of the undisturbed harmonics. Here Δ *is the interval of the fluctuating frequency.*

Example

Let us consider an example of the signal restoration procedure for a simple oscillator with a dumping term. Here the signal is a sinusoid of some frequency that is modulated with a decreasing exponent. The frequency of sinusoid is fluctuating.

Being simple enough the signal clearly reflects the essence of the processes. In a linear approximation one can describe most of the surrounding us objects with a system of the signals of this type. The Fourier's, Laplace's, Heaviside's images of the solution are well known, so the reader can imagine such a signal. In practice more complicated signals are used (the signal can be a function of time, frequency, phase or a function of all arguments simultaneously). The only difference of the considered example from solution (18) is that the signal has the finite length due to the presence of the characteristic function of the interval.

$$S(t, \vec{h}_i) = \theta_i \exp\{-\alpha_i t\} \sin\{\omega_i (t - \tau_i)\} \chi(t, \tau_i - \psi_i, \tau_i + T) \quad (18)$$

In (18) ψ_i is a phase shift in the i -th experiment.

In this case the vector of free parameters of the model, determining the signal, is $\vec{h}_i = \{h_{ik}\} = \{\tau_i, \theta_i, \alpha_i, \omega_i, \psi_i, T\}, \quad k = 0, \ldots, 5$. It has only six components, second of which h_{i1} comes into the model linearly.

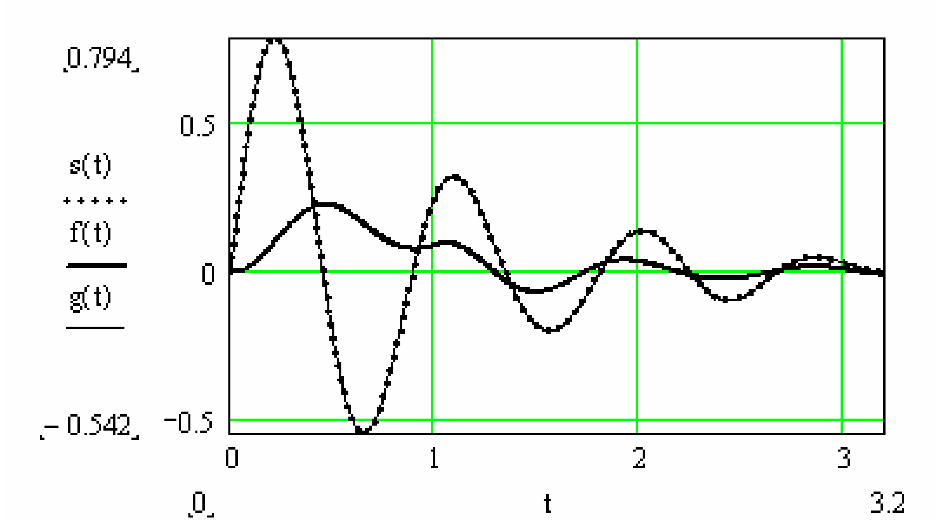

Figure 4. There are three curves on this picture: the first one, which is named $s(t)$, is the model of the sounding signal. The second one $f(t)$ is the signal misshaped by random frequency and phase fluctuations, the third one $g(t)$ is the reconstructed signal. The procedure of the signal reconstruction allows us to get the curve shape very close to the shape of the original one $s(t)$.

Conclusion

The results of the offered analysis of the procedure of active monitoring demonstrate considerable loss of information during accumulation of the medium response to the probing signal. It is important to emphasize, that the higher the signal frequency is, the greater the corresponding loss of information becomes. The preprocessing technique proposed in this paper allows us to conduct the correction of the experimental results and substantially restore the response of the medium.

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