

A MATHEMATICAL APPARATUS FOR DOMAIN ONTOLOGY SIMULATION. LOGICAL RELATIONSHIP SYSTEMS¹

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Abstract: A mathematical apparatus for domain ontology simulation will be described in the series of the articles. The goal of this article is to define unenriched and enriched logical relationship systems that can be considered as mathematical models for domain ontologies. The extendable language of applied logic described in the previous articles of the series is used as the language of representation of these systems.

Keywords: Extendable language of applied logic, ontology language specification, kernel of extendable language of applied logic, unenriched logical relationship systems, enriched logical relationship systems, enrichment of logical relationship system.

ACM Classification Keywords: I.2.4 Knowledge Representation Formalisms and Methods, F4.1. Mathematical Logic

Introduction

In this article a class of mathematical models called logical relationship systems is defined. For representing these models the extendable language of applied logic described in [Kleshchev et al, 2005 a, 2005b] is used. Unenriched logical relationship systems simulate domain ontologies, their enrichments simulate domain knowledge, and enriched logical relationship systems simulate domains themselves.

1. An Unenriched Logical Relationship System without Parameters

A pair $O = \langle \Phi, \emptyset \rangle$, where Φ is a semantically correct applied logical theory, having at least one ambiguously interpreted name, will be called an unenriched logical relationship system O without parameters. The set of propositions for the reduction of Φ [Kleshchev et al, 2005 a] will be called the set of logical relationships. All the ambiguously interpreted names of the theory Φ will be called unknowns of the system O . The set of unknowns of the system O will be designated as X .

Example 1. An unenriched logical relationship system $O_1 = T_1(\text{ST}, \text{Intervals})$ without parameters representing a simplified model of an ontology for medical diagnostics of acute abdomen diseases.

The logical theory $T_1(\text{ST}, \text{Intervals}) = \langle \{\text{Definition of partitions}\}, \text{SS}_1 \rangle$, where SS_1 is the following set of propositions.

The sort descriptions for names.

(1.2.1) sort diagnosis: {healthy, pancreatitis}

A diagnosis means the diagnosis of the patient; the diagnosis can be either healthy or pancreatitis.

(1.2.2) sort partition for a sign: {blood pressure, daily diuresis, strain of abdomen muscles} \rightarrow partitions

A partition for a sign is a function that takes blood pressure, daily diuresis or strain of abdomen muscles and returns a partition of the patient's time axes.

(1.2.3) sort moments of examination: {blood pressure, daily diuresis, strain of abdomen muscles} \rightarrow ($\{ \} \{ [0, \infty) \}$)

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Moments of examination means a function that takes blood pressure, daily diuresis or strain of abdomen muscles and returns moments of examining these signs in the patient; time is measured by integer number of hours from the beginning of the patient's examination.

(1.2.4) sort blood pressure: moments of examination(blood pressure) \rightarrow {normal, high, low}

A blood pressure is a function (process) that takes a moment of the patient's blood pressure examination and returns the value of blood pressure at the moment; the value can be normal, high or low.

(1.2.5) sort strain of abdomen muscles: moments of examination(strain of abdomen muscles) \rightarrow {absence, presence}

A strain of abdomen muscles is a function (process) that takes a moment of the patient's strain of abdomen muscles examination and returns the value of strain of abdomen muscles at the moment; the value can be absence or presence.

(1.2.6) sort daily diuresis: moments of examination(daily diuresis) \rightarrow {normal, high, low}

A daily diuresis is a function (process) that takes a moment of the patient's daily diuresis examination and returns the value of daily diuresis at the moment; the value can be normal, high or low.

The unknowns of the system are diagnosis, partition for a sign, moments of examination, blood pressure, strain of abdomen muscles and daily diuresis.

2. Enriched Logical Relationship Systems Without Parameters

If $O = \langle \Phi_1, \emptyset \rangle$ is an unenriched logical relationship system without parameters and Φ_2 is such a set of restrictions on the interpretation of names that the logical theory $\Phi_1 \cup \Phi_2$ is semantically correct [Kleshchev et al, 2005a] then $S = \langle O, \Phi_2 \rangle$ will be called the enriched logical relationship system without parameters formed from O by the enrichment Φ_2 . We will also designate S as $\langle \Phi_1, \Phi_2 \rangle$.

Example 2. A possible enrichment for the unenriched logical relationship system O_1 without parameters of example 1.

(2.1) (moment: moments of examination(strain of abdomen muscles)) diagnosis = healthy \Rightarrow strain of abdomen muscles(moment) \in {absence}

If the patient is healthy, then at any moment of the patient's strain of abdomen muscles examination its value can be only absence.

(2.2) (moment: moments of examination(blood pressure)) diagnosis = healthy \Rightarrow blood pressure(moment) \in {normal}

If the patient is healthy, then at any moment of the patient's blood pressure examination its value can be only normal.

(2.3) (moment: moments of examination(daily diuresis)) diagnosis= healthy \Rightarrow daily diuresis(moment) \in {normal}

If the patient is healthy then at any moment of the patient's daily diuresis examination its value can be only normal.

(2.4) diagnosis = pancreatitis \Rightarrow length(partition for a sign(strain of abdomen muscles)) = 2

If the patient is ill with pancreatitis, then the number of dynamics periods for strain of abdomen muscles is equal to 2.

(2.5) diagnosis = pancreatitis \Rightarrow length(partition for a sign(blood pressure)) = 2

If the patient is ill with pancreatitis, then the number of dynamics periods for blood pressure is equal to 2.

(2.6) diagnosis = pancreatitis \Rightarrow length(partition for a sign(daily diuresis)) = 2

If the patient is ill with pancreatitis, then the number of dynamics periods for daily diuresis is equal to 2.

(2.7) (moment: moments of examination(strain of abdomen muscles) \cap interval(partition for a sign(strain of abdomen muscles), 1)) diagnosis = pancreatitis \Rightarrow strain of abdomen muscles(moment) \in {absence}

If the patient is ill with pancreatitis and a moment of examining strain of abdomen muscles belongs to the first dynamics period of the sign, then only value absence can be got.

(2.8) (moment: moments of examination(strain of abdomen muscles) \cap interval(partition for a sign(strain of abdomen muscles), 2)) diagnosis = pancreatitis \Rightarrow strain of abdomen muscles(moment) \in {presence}

If the patient is ill with pancreatitis and a moment of examining strain of abdomen muscles belongs to the second dynamics period of the sign, then only value presence can be got.

(2.9) (moment: moments of examination(blood pressure) \cap interval(partition for a sign(blood pressure), 1)) diagnosis = pancreatitis \Rightarrow blood pressure(moment) \in {normal}

If the patient is ill with pancreatitis and a moment of examining blood pressure belongs to the first dynamics period of the sign, then only value normal can be got.

(2.10) (moment: moments of examination(blood pressure) \cap interval(partition for a sign(blood pressure), 2)) diagnosis = pancreatitis \Rightarrow blood pressure(moment) \in {high}

If the patient is ill with pancreatitis and a moment of examining blood pressure belongs to the second dynamics period of the sign, then only value high can be got.

(2.11) (moment: moments of examination(daily diuresis) \cap interval(partition for a sign(daily diuresis), 1)) diagnosis = pancreatitis \Rightarrow daily diuresis(moment) \in {low}

If the patient is ill with pancreatitis and a moment of examining daily diuresis belongs to the first dynamics period of the sign, then only value low can be got.

(2.12) (moment: moments of examination(daily diuresis) \cap interval(partition for a sign(daily diuresis), 2)) diagnosis = pancreatitis \Rightarrow daily diuresis(moment) \in {normal}

If the patient is ill with pancreatitis and a moment of examining daily diuresis belongs to the second dynamics period of the sign, then only value normal can be got.

(2.13) diagnosis = pancreatitis \Rightarrow sup(interval(partition for a sign(strain of abdomen muscles), 1)) – inf(interval(partition for a sign(strain of abdomen muscles), 1)) \in R[24, 48]

If the patient is ill with pancreatitis, then the duration of the first dynamics period for strain of abdomen muscles is from 24 to 48 hours.

(2.14) diagnosis = pancreatitis \Rightarrow sup(interval(partition for a sign(strain of abdomen muscles), 2)) – inf(interval(partition for a sign(strain of abdomen muscles), 2)) \in R[1, 144]

If the patient is ill with pancreatitis, then the duration of the second dynamics period for strain of abdomen muscles is from 1 to 144 hours.

(2.15) diagnosis = pancreatitis \Rightarrow sup(interval(partition for a sign(blood pressure), 1)) – inf(interval(partition for a sign(blood pressure), 1)) \in R[1, 24]

If the patient is ill with pancreatitis, then the duration of the first dynamics period for blood pressure is from 1 to 24 hours.

(2.16) diagnosis = pancreatitis \Rightarrow sup(interval(partition for a sign(blood pressure), 2)) – inf(interval(partition for a sign(blood pressure), 2)) \in R[1, 144]

If the patient is ill with pancreatitis, then the duration of the second dynamics period for blood pressure is from 1 to 144 hours.

(2.17) diagnosis = pancreatitis \Rightarrow sup(interval(partition for a sign(daily diuresis), 1)) – inf(interval(partition for a sign(daily diuresis), 1)) \in R[48, 72]

If the patient is ill with pancreatitis, then the duration of the first dynamics period for daily diuresis is from 48 to 72 hours.

(2.18) diagnosis = pancreatitis \Rightarrow sup(interval(partition for a sign(daily diuresis), 2)) – inf(interval(partition for a sign(daily diuresis), 2)) \in R[1, 144]

If the patient is ill with pancreatitis, then the duration of the second dynamics period for daily diuresis is from 1 to 144 hours.

If $S = \langle \Phi_1, \Phi_2 \rangle$ is an enriched logical relationship system without parameters then any model of the applied logical theory [Kleshchev et al, 2005a] $\Phi_1 \cup \Phi_2$ will be called a solution of S .

Example 3. A solution of the enriched logical relationship system without parameters formed from the unenriched system O_1 of example 1 by the enrichment of example 2 is given by propositions 3.1.10 – 3.1.15 of example 3 of the article [Kleshchev, 2005 b].

3. An Unenriched Logical Relationship System with Parameters

A pair $O = \langle \Phi, P \rangle$, where Φ is a semantically correct applied logical theory and P is a nonempty proper subset of the set of ambiguously interpreted names of the theory Φ , will be called an unenriched logical relationship system with parameters. The set P can be given by several ways: by an explicit enumeration of its elements, by a description of a set of names possessing certain properties, by the union of parts of P given by several ways. The set of propositions for the reduction of the theory Φ will be called the set of logical relationships and P will be called the set of parameters. Ambiguously interpreted names of the theory Φ which do not belong to the set P , will be called unknowns of the system O . The set of unknowns of the O will be designated as X . As it follows from the definition of the set P , the set X is not empty.

Example 4. The system $O_2 = \langle T_1(\text{ST, Intervals, Mathematical quantors}), P_2 \rangle$ is an unenriched logical relationship system with parameters where $T_1(\text{ST, Intervals, Mathematical quantors})$ is the applied logical theory of example 2 [Kleshchev, 2005b] and the set of parameters P_2 consists of the following names: signs, diseases, possible values, normal values, clinical picture, number of dynamics periods, values for a dynamics period, upper bound, lower bound. The unknowns of the system are diagnosis, partition for a sign, moments of examination and also all the names that are elements of a set of names that is an interpretation of parameter signs.

4. Enriched Logical Relationship Systems with Parameters

We will consider two classes of unenriched systems with parameters: pure and mixed.

If $O = \langle \Phi, P \rangle$ is a pure unenriched logical relationship system with parameters and α_P is such an interpretation function of the parameters that can be extended to a model of the logical theory Φ , then $S = \langle \Phi, P, \alpha_P \rangle$ will be called an enriched logical relationship system with parameters formed from O by the enrichment α_P . The interpretation function α_P will be called the set of parameter values. An unenriched logical relationship system with parameters will be called pure if its enrichments only of the form α_P are considered.

Example 5. The unenriched system O_2 of example 4 belongs to the class of pure logical relationship systems. The set of parameter values given by propositions 3.1.1 – 3.1.9 of example 3 [Kleshchev, 2005b] is its possible enrichment.

If $O = \langle \Phi_1, P \rangle$ is a mixed unenriched logical relationship system with parameters and Φ_2 is such a set of restrictions on the interpretation of names containing no parameters that the logical theory $\Phi = \Phi_1 \cup \Phi_2$ is semantically correct, α_P is such an interpretation of the parameters that can be extended to a model of the logical theory Φ , then $S = \langle \Phi, P, \alpha_P \rangle$ will be called an enriched logical relationship system with parameters formed from O by the enrichment $\langle \Phi_2, \alpha_P \rangle$. An unenriched logical relationship system with parameters will be called mixed if its enrichments only of the form $\langle \Phi_2, \alpha_P \rangle$ are considered. It should be emphasized that the propositions of Φ_2 contain no parameters. But these propositions can contain unknowns that are constituents of parameter values. We will say that an unknown is a constituent of a parameter value if either the value of the parameter is the unknown or the value of the parameter is a set, tuple or other structure consisting of components and either at

least one of these components is the unknown or the unknown is a constituent of at least one of these components.

Example 6. The mixed unenriched logical relationship system $O_3 = \langle T_3(\text{ST}, \text{Intervals}), P_3 \rangle$ with parameters representing a model of a simplified ontology of the domain "Masses and volumes of bodies".

The logical theory $T_3(\text{ST}, \text{Intervals}) = \langle \emptyset, SS_3 \rangle$, where SS_3 is the following set of propositions.

Value descriptions for names

(6.1.1) $\text{bodies} \equiv \text{cubes} \cup \text{balls} \cup \text{rectangular parallelepipeds}$

Bodies mean a set of geometric bodies having the form of a cube, a ball or a rectangular parallelepiped.

(6.1.2) $\text{pi} \equiv 3.1415$

Pi is the well-known mathematical constant.

Sort descriptions for names

(6.2.1) sort cubes: $\{N\}$

Cubes mean a set of cubes.

(6.2.2) sort balls: $\{N\}$

Balls mean a set of balls.

(6.2.3) sort rectangular parallelepipeds: $\{N\}$

Rectangular parallelepipeds mean a set of rectangular parallelepipeds.

(6.2.4) sort radius: $\text{balls} \rightarrow R(0, \infty)$

A radius is a function that takes a ball and returns the length of its radius.

(6.2.5) sort length of an edge: $\text{cubes} \rightarrow R(0, \infty)$

A length of an edge is a function that takes a cube and returns the length of its edge.

(6.2.6) sort length: $\text{rectangular parallelepipeds} \rightarrow R(0, \infty)$

A length is a function that takes a rectangular parallelepiped and returns its length.

(6.2.7) sort width: $\text{rectangular parallelepipeds} \rightarrow R(0, \infty)$

A width is a function that takes a rectangular parallelepiped and returns its width.

(6.2.8) sort height: $\text{rectangular parallelepipeds} \rightarrow R(0, \infty)$

A height is a function that takes a rectangular parallelepiped and returns its height.

(6.2.9) sort volume: $\text{bodies} \rightarrow R(0, \infty)$

A volume is a function that takes a body and returns its volume.

(6.2.10) sort possible substances: $\{N\}$

Possible substances mean a set of chemical substances.

(6.2.11) sort substance: $\text{bodies} \rightarrow \text{possible substances}$

A substance is a function that takes a body and returns the chemical substance that the body is made from.

(6.2.12) sort mass: $\text{bodies} \rightarrow R(0, \infty)$

A mass is a function that takes a body and returns its mass.

(6.2.13) sort density: $\text{possible substances} \rightarrow R(0, \infty)$

A density is a function that takes a chemical substance and returns its density.

Restrictions on the interpretation of names

(6.3.1) $(\text{body: bodies}) \text{mass}(\text{body}) = \text{density}(\text{substance}(\text{body})) * \text{volume}(\text{body})$

The proposition represents the well-known relationship among the meanings of terms mass, substance, density, and volume.

The set of parameters P_3 consists of the names possible substances and density. The unknowns are cubes, balls, rectangular parallelepipeds, radius, length of an edge, length, width, height, volume, substance and mass.

A possible enrichment of the system is given by the following propositions

(6.4.1) possible substances $\equiv \{\text{copper, tin}\}$

Chemical substances copper and tin are only considered.

(6.4.2) density $\equiv (\lambda (\text{substance: } \{\text{copper, tin}\}) / (\text{substance} = \text{copper} \Rightarrow 8.96) (\text{substance} = \text{tin} \Rightarrow 7.29) /)$

The proposition defines the density of copper and tin.

(6.4.3) (ball: balls) volume (ball) $= (4 / 3) * (\text{radius}(\text{ball}) \uparrow 3) * \pi$

The proposition defines the well-known formula for calculation of the volume of a ball using its radius.

(6.4.4) (cube: cubes) volume(cube) $= \text{length of an edge}(\text{cube}) \uparrow 3$

The proposition defines the well-known formula for calculation of the volume of a cube using the length of its edge.

(6.4.5) (rectangular parallelepiped: rectangular parallelepipeds) volume(rectangular parallelepiped) $= \text{length}(\text{rectangular parallelepiped}) * \text{width}(\text{rectangular parallelepiped}) * \text{height}(\text{rectangular parallelepiped})$

The proposition defines the well-known formula for calculation of the volume of a rectangular parallelepiped using its length, width and height.

Here propositions 6.4.1, 6.4.2 represent the parameter values of the system and propositions 6.4.3 - 6.4.5 represent restrictions on the interpretation of names.

If $S = \langle \Phi, P, \alpha_P \rangle$ is an enriched logical relationship system, then an interpretation α_X of unknowns will be called a solution of S if there is such a model α of the theory Φ that narrowing α to P is the same as α_P , and narrowing α to X is the same as α_X .

Example 7. A possible solution of the enriched logical relationship system with parameters of example 5 is the set of unknown values given by propositions 3.1.10 – 3.1.15 of example 3 of article [Kleshchev,2005b].

A possible solution of the enriched logical relationship system with parameters of example 6 can be represented by the following set of value descriptions for names.

(7.1.1) cubes $\equiv \{\text{ABCDA1B1C1D1}\}$

The only cube is considered.

(7.1.2) rectangular parallelepipeds $\equiv \emptyset$

The set of rectangular parallelepipeds is empty.

(7.1.3) balls $\equiv \emptyset$

The set of balls is empty.

(7.1.4) length of an edge $\equiv (\lambda (\text{cube: } \{\text{ABCDA1B1C1D1}\}) / (\text{cube} = \text{ABCDA1B1C1D1} \Rightarrow 3) /)$

The length of the edge of the cube is equal to 3.

(7.1.5) volume $\equiv (\lambda (\text{cube: } \{\text{ABCDA1B1C1D1}\}) / (\text{cube} = \text{ABCDA1B1C1D1} \Rightarrow 27) /)$

The volume of the cube is equal to 27.

(7.1.6) substance $\equiv (\lambda (\text{cube: } \{\text{ABCDA1B1C1D1}\}) / (\text{cube} = \text{ABCDA1B1C1D1} \Rightarrow \text{copper}) /)$

The cube is made from copper.

(7.1.7) mass $\equiv (\lambda (\text{cube: } \{\text{ABCDA1B1C1D1}\}) / (\text{cube} = \text{ABCDA1B1C1D1} \Rightarrow 241.92) /)$

The mass of the cube is equal to 241.92.

By this means, every enriched logical relationship system determines the set of its solutions. The set of all the solutions for an enriched system S will be designated as $A(S)$. Two enriched logical relationship systems S_1 и S_2 will be called equivalent if $A(S_1) = A(S_2)$.

If ζ is an enrichment of unenriched system O then $S = \langle O, \zeta \rangle$ is O enriched by ζ . An equivalence relation on the set of all possible enrichments of unenriched system O will be defined by the following way: two enrichments ζ_1 and ζ_2 are equivalent if the enriched systems $\langle O, \zeta_1 \rangle$ and $\langle O, \zeta_2 \rangle$ are equivalent. The set of the equivalence classes of all the possible enrichments for an unenriched system O will be designated as $En(O)$. If $k \in En(O)$ is an equivalence class for the set of all possible enrichments of an unenriched system O , then let $\langle O, k \rangle = \langle O, \zeta \rangle$, where $\zeta \in k$ is an arbitrary representative of the equivalence class k . In such a manner, an unenriched system O determines the set of enriched logical relationship systems $\{\langle O, k \rangle \mid k \in En(O)\}$.

In what follows, we will consider only such logical relationship systems O that for all $k \in En(O)$ every enriched system $\langle O, k \rangle$ has the following property: for any solution $\alpha_{x \in A} \in A(\langle O, k \rangle)$ and for any unknown $x \in X$ the value $\alpha_x(x)$ contains no ambiguously interpreted names. We will say that a value $\alpha_x(x)$ contains no ambiguously interpreted names if either $\alpha_x(x)$ is not an ambiguously interpreted name or if $\alpha_x(x)$ is a set, a tuple or any other structure consisting of components and none of the components is an ambiguously interpreted name or contains such names. Notice that a parameter value can contain ambiguously interpreted names (both parameters and unknowns).

An unenriched (enriched) logical relationship system will be called predicative if none of its ambiguously interpreted name is a functional name. An unenriched (enriched) logical relationship system will be called functional if none of its ambiguously interpreted name is a predicative name. If unenriched (enriched) logical relationship system is both predicative and functional then it will be called objective. If unenriched (enriched) logical relationship system is neither predicative nor functional then it will be called a system of a general form. The unenriched system of example 7 is a functional logical relationship system.

Special cases of unenriched logical relationship systems have been considered in [Kleshchev et al, 1999], those of enriched logical relationship systems without parameters have been considered in [Artemjeva et al, 1996] and those of systems with parameters have been considered in [Artemjeva et al, 1997].

5. Relations among Logical Relationship Systems

To define a relation R among unenriched logical relationship systems O_1, O_2, \dots, O_m , an analogous relation R' among enriched logical relationship systems $\langle O_1, k_1 \rangle, \langle O_2, k_2 \rangle, \dots, \langle O_m, k_m \rangle$, where $k_1 \in En(O_1), k_2 \in En(O_2), \dots, k_m \in En(O_m)$, and also a relation R'' on the sets $En(O_1), En(O_2), \dots, En(O_m)$ are introduced. In doing so, O_1, O_2, \dots, O_m are in the relation R if and only if for any $k_1 \in En(O_1), k_2 \in En(O_2), \dots, k_m \in En(O_m)$ from the fact that k_1, k_2, \dots, k_m are in the relation R'' it follows that the enriched systems $\langle O_1, k_1 \rangle, \langle O_2, k_2 \rangle, \dots, \langle O_m, k_m \rangle$ are in the relation R' .

Following the scheme above an equivalence relation between unenriched logical relationship systems will be defined. An unenriched logical relationship system O_1 will be called equivalent to another unenriched logical relationship system O_2 if there is such a one-to-one map E from the set $En(O_1)$ onto $En(O_2)$ that for all $k \in En(O_1)$ the enriched systems $\langle O_1, k \rangle$ and $\langle O_2, E(k) \rangle$ are equivalent [Kleshchev, 2005].

The following statement takes place: for any unenriched logical relationship system with functional parameters there is an equivalent unenriched logical relationship system having no functional parameters. Also the following statement is true: for any unenriched logical relationship system with predicative parameters there is an equivalent unenriched logical relationship system having no predicative parameters.

The following theorem about eliminating parameters of enriched logical relationship systems takes place: for any enriched logical relationship system with parameters and for a given parameter there is an equivalent enriched logical relationship system not containing this given parameter. The proof of the theorem does not differ from the proof of the analogous theorem in [Artemjeva et al, 1997]. The following statement is a corollary of the theorem: for any enriched logical relationship system with parameters there is an equivalent enriched logical relationship system without parameters.

Also the following theorem about eliminating parameters of unenriched logical relationship systems takes place: if $O = \langle \Phi_1 \cup \Phi_2, P_1 \cup P_2 \rangle$ is an unenriched logical relationship system with parameters where Φ_1 is the set of the propositions containing no parameters from the set P_2 , then for the mixed unenriched logical relationship system with parameters $O_1 = \langle \Phi_1, P_1 \rangle$ there is such a completely defined one-valued map h from the set $En(O)$ to $En(O_1)$ that for all $k \in En(O)$ the enriched system $\langle O, k \rangle$ is equivalent to the enriched system $\langle O_1, h(k) \rangle$. The system O_1 will be called quasiequivalent to the system O . This theorem is a corollary of the theorem about eliminating parameters of enriched logical relationship systems. It follows from the theorem, in particular, that in general case O and O_1 are not equivalent because $\{h(k) | k \in En(O)\} \subset En(O_1)$. In addition, if the set of parameters P_1 is empty then the quasiequivalent system $O_1 = \Phi_1$ is an unenriched logical relationship system without parameters. The quasiequivalence relation is reflexive and transitive but antisymmetric.

Now we will define a notion of isomorphism between unenriched logical relationship systems. Two enriched logical relationship systems will be called isomorphic if there is a one-to-one correspondence between the sets of their solutions. An unenriched logical relationship system O_1 will be called isomorphic to an unenriched logical relationship system O_2 if there is such a one-to-one correspondence E between the sets $En(O_1)$ and $En(O_2)$ that for all $k \in En(O_1)$ the systems $\langle O_1, k \rangle$ and $\langle O_2, E(k) \rangle$ are isomorphic.

The following statement takes place: for any unenriched logical relationship system with functional unknowns there is an isomorphic unenriched logical relationship system without any functional unknowns. Also the following statement is true: for any unenriched logical relationship system with predicative unknowns there is an isomorphic unenriched logical relationship system without any predicative unknowns. Moreover, for any functional (predicative) unenriched logical relationship system there is an isomorphic predicative (functional) unenriched logical relationship system.

Further, we will define a notion of homomorphism between unenriched logical relationship systems. An enriched logical relationship system S_2 will be called a homomorphic image of an enriched logical relationship system S_1 if there is a completely defined one-valued map h_1 from the set of solutions of the system S_1 to the set of solution of the system S_2 . In this case we will say that there is a homomorphism $h_1 : S_1 \rightarrow S_2$. An unenriched logical relationship system O_2 will be called a homomorphic image of an unenriched logical relationship system O_1 if there is such a completely defined one-valued map h from the set $En(O_1)$ to the set $En(O_2)$ that for all $k \in En(O_1)$ the system $\langle O_2, h(k) \rangle$ is a homomorphic image of the system $\langle O_1, k \rangle$. In this case we will say that there is a homomorphism $h : O_1 \rightarrow O_2$.

Finally, we will define a product of unenriched logical relationship systems. An enriched logical relationship system S will be called the product of enriched logical relationship systems S_1, S_2, \dots, S_m (of the product factors) if there are such homomorphisms $h_1: S \rightarrow S_1, h_2 : S \rightarrow S_2, \dots, h_m : S \rightarrow S_m$ that for any $\alpha'X, \alpha''X \in A(S)$ the statement $\alpha'X \neq \alpha''X \Rightarrow \langle h_1(\alpha'X), h_2(\alpha'X), \dots, h_m(\alpha'X) \rangle \neq \langle h_1(\alpha''X), h_2(\alpha''X), \dots, h_m(\alpha''X) \rangle$ is true. An unenriched logical relationship system O will be called the product of unenriched logical relationship systems O_1, O_2, \dots, O_m (of the product factors), $O = O_1 \otimes O_2 \otimes \dots \otimes O_m$, if there are such homomorphisms $h_1: O \rightarrow O_1, h_2: O \rightarrow O_2, \dots, h_m: O \rightarrow O_m$ that for any $k', k'' \in En(O)$ the statement $k' \neq k'' \Rightarrow \langle h_1(k'), h_2(k'), \dots, h_m(k') \rangle \neq \langle h_1(k''), h_2(k''), \dots, h_m(k'') \rangle$ is true and for all $k \in En(O)$ the system $\langle O, k \rangle$ is the product of the systems $\langle O_1, h_1(k) \rangle, \langle O_2, h_2(k) \rangle, \dots, \langle O_m, h_m(k) \rangle$.

Conclusions

In this article unenriched logical relationship systems are introduced on the basis of the applied logic languages. Every such a system represents a class of enriched logical relationship systems. Every enriched logical relationship system determines a set of its solution. In this article notions of equivalence, isomorphism, homomorphism, and product are introduced for unenriched logical relationship systems.

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GRID-ENABLING SATELLITE IMAGE ARCHIVE PROTOTYPE FOR UA SPACE GRID TESTBED¹

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Abstract: *The paper describes practical approach to implementation of satellite data archive using Globus Toolkit 4 components. The solutions consists in converting a hierarchy of remote data files available via FTP into Grid-enabled archive. All etries of such archive will be indexed using arbitrary but pre-defined XML schema. The information will be exposed via MDS4 Index service and the actual data will be exposed via GridFTP. The schema used in our solution is simple enough for understanding but in a real life applications we should use metadata standards such as ISO 19139 and ISO 19115 in particular. A working prototype of the archive described in this paper is deployed on the Grid testbed of Space Research Institute of National Academy of Science and National Space Agency of Ukraine (SRI-NASU-NSAU). The SRI-NASU-NSAU testbed is briefly described in this paper as well.*

Keywords: *Grid, distributed systems, parallel computing, satellite data, image processing, file archive, programming languages.*

ACM Classification Keywords: *C.2.4 Distributed Systems: Distributed applications*

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