AN INTERACTIVE METHOD OF LINEAR MIXED INTEGER MULTICRITERIA OPTIMIZATION

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Abstract: The paper describes a learning-oriented interactive method for solving linear mixed integer problems of multicriteria optimization. The method increases the possibilities of the decision maker (DM) to describe his/her local preferences and at the same time it overcomes some computational difficulties, especially in problems of large dimension. The method is realized in an experimental decision support system for finding the solution of linear mixed integer multicriteria optimization problems.

Keywords: linear mixed integer multicriteria optimization, interactive methods, decision support systems.

1. Introduction

The reference point interactive methods and the classification-based interactive methods are the most widely spread methods [Gardineer and Vanderpooten, 1997], [Miettinen, 1999], [Vassilev et al., 2003] for solving linear problems of multicriteria optimization. This kind of interactive methods is also used [Climaco et al., 1997], [Vassileva, 2001] for solving linear mixed integer problems of multicriteria optimization.

Single-criterion problems of linear programming are used in the interactive methods solving linear problems of multicriteria optimization. They belong to the class of P-problems [Garey and Johnson, 1979] and therefore the time for solving a single-criterion problem does not play a significant role in the interactive methods of linear multicriteria optimization. In the development of the interactive methods main attention is paid to the possibilities offered to the DM to describe his/her preferences. Single-criterion problems of mixed integer programming are applied in the interactive methods solving linear mixed integer problems of multicriteria optimization. They are NP-hard problems and hence the time for single-criterion mixed integer problems solution has to be obligatory taken into account when designing interactive methods.

A learning-oriented interactive method is proposed in the present paper on the basis of new linear mixed integer classification-based scalarizing problems, intended to solve linear mixed integer problems of multicriteria optimization. The method increases DM's possibilities to describe his/her local preferences and also overcomes some computational difficulties, connected with single-criterion mixed integer problems solving.

2. Problem Formulation

The linear mixed integer problem of multicriteria optimization (denoted by I), can be formulated as follows:

(1) $$\text{"max"}\{f_k(x)\}, \qquad k \in K$$

subject to:

(2) $$\sum_{j\in N} a_{ij}x_j \le b_i, \qquad i \in M$$

(3) $$0 \le x_j \le d_j, \qquad j \in N$$

(4) $$x_j - \text{integers}, \; j \in N'; \; N' \subseteq N,$$

where $f_k(x), k \in K$ are linear criteria (objective functions), $f_k(x) = \sum_{j=N} c_j^k x_j$, "max" denotes that all the objective functions should be simultaneously maximized; $x = (x_1, x_2 ..., x_j, ...x_n)^T$ is the variables vector; $K = \{1,2,..., p\}$, $M = \{1,2,..., m\}$, $N = \{1,2,..., n\}$ and $N' \subseteq N$ are sets of the indices of the linear criteria (objective functions) of the linear constraints, of the variables and of the integer variables respectively. Constraints (2)-(4) define the feasible region X_1 for the variables of the mixed integer problem.

Several definitions will be introduced for greater precision.

Definition 1: The solution x is called an efficient solution of problem (I), if there does not exist any other solution $\bar{x}$, so that the following inequalities are satisfied:

$f_k(\bar{x}) \geq f_k(x),$ for every $k \in K$ and

$f_k(\bar{x}) > f_k(x),$ for at least one index $k \in K$.

Definition 2: The solution x is called a weak efficient solution of problem (I), if there does not exist another solution $\bar{x}$ such that the following inequalities hold:

$f_k(\bar{x}) > f_k(x),$ for every $k \in K$.

Definition 3: The solution x is called a (weak) efficient solution, if x is either an efficient or a weak efficient solution.

Definition 4: The vector $f(x) = (f_1(x),..., f_p(x))^T$ is called a (weak) non-dominated solution in the criteria space, if x is a (weak) efficient solution in the variables space.

Definition 5: A near (weak) non-dominated solution is a feasible solution in the criteria space located comparatively close to the (weak) non-dominated solutions.

Definition 6: A current preferred solution is a (weak) non-dominated solution or near (weak) non-dominated solution, if selected by the DM at the current iteration.

Definition 7: The most preferred solution is the current preferred solution, which satisfies the DM to the highest extent.

3. Scalarizing Problems

The linear mixed integer problems of multicriteria optimization do not possess a mathematically well-defined optimal solution. That is why it is necessary to choose one of the (weak) non-dominated solutions, which is most appropriate with reference to DM's global preferences. This choice is subjective and it depends entirely on the DM.

The interactive methods are the most widely used methods for solving linear mixed integer problems of multicriteria optimization [Climaco et al., 1997], [Vassileva, 2001]. Every iteration of such an interactive method consists of two phases: a computation and a decision one. One or more non-dominated solutions are generated with the help of a scalarizing problem at the computation phase. At the decision phase these non-dominated solutions are presented for evaluation to the DM. In case the DM does not approve any of these solutions as a final solution (the most preferred solution), he/she supplies information concerning his/her local preferences with the purpose to improve these solutions. This information is used to formulate a new scalarizing problem, which is solved at the next iteration.

The type of the scalarizing problem used lies in the basis of each interactive method. The scalarizing problem is a problem of single-criterion optimization and its optimal solution is a (weak) non-dominated solution of the multicriteria optimization problem. The clasification-based scalarizing problems are particularly appropriate in solving linear mixed integer multicriteria optimization problems, because they enable the decrease of the computational difficulties connected with their solution as well as the increase of DM's possibilities in describing his/her local preferences and also lead to reduction of the requirements towards the DM in the comparison and evaluation of the new solutions obtained. In the scalarizing problems suggested in this chapter, the DM can present his/her local preferences in terms of desired or acceptable levels, directions and intervals of alteration in the values of separate criteria. Depending on these preferences, the criteria set can be divided into seven or less criteria classes. The criterion $f_k(x), \quad k \in K$ may belong to one of these classes as follows:

$k \in K^{>}$, if the DM wishes the criterion $f_k(x)$ to be improved;

$k \in K^{\geq}$, if the DM wishes the criterion $f_k(x)$ to be improved by any desired (aspiration) value Δ_k;

$k \in K^{<}$, if the DM agrees the criterion $f_k(x)$ to be worsened;

$k \in K^{\leq}$, in case the DM agrees the value of the criterion $f_k(x)$ to be deteriorated by no more than δ_k;

$k \in K^{><}$, if the DM wishes the value of the criterion $f_k(x)$ to be within definite limits with respect to the current value f_k, $(f_k - t_k^- < f_k(x) \leq f_k + t_k^+)$;

$k \in K^{=}$, if the current value of the criterion $f_k(x)$ is acceptable for the DM;

$k \in K^{0}$, if at the moment the DM is not interested in the alteration of the criterion $f_k(x)$ and this criterion can be freely altered.

In order to obtain a solution, better than the current (weak) non-dominated solution of the linear mixed integer problem of multicriteria optimization, the following Chebyshev's scalarizing problems can be applied on the basis of the implicit criteria classification, done by the DM. The first mixed integer scalarizing problem [Vassileva, 2000] called *DAL* (desired and acceptable level) has the following type:

To minimize:

(5) $$S(x) = \max\left[\max_{k \in K^{\geq}}\left(\bar{f}_k - f_k(x)\right)/\left|f_k^{'}\right|, \max_{k \in K^{\leq}}\left(f_k - f_k(x)\right)/\left|f_k^{'}\right|\right]$$

under the constraints:

(6) $$f_k(x) \geq f_{k,}\ k \in K^{=},$$

(7) $$f_k(x) \geq f_k - \delta_k,\ k \in K^{\leq},$$

(8) $$x \in X_1,$$

where:

f_k, $k \in K$ is the value of the criterion $f_k(x)$ in the current preferred solution;

$\bar{f}_k = f_k + \Delta_k$, $k \in K^{\geq}$ is the desired level of the criterion $f_k(x)$;

$f_K^{'}$, $k \in K$ is a scaling coefficient:

$$f_k^{'} = \begin{cases} \varepsilon, \text{ if } \left|f_k^{'}\right| \leq \varepsilon \\ f_{k,} \text{ if } \left|f_k^{'}\right| > \varepsilon \end{cases},$$

where ε is a small positive number.

DAL scalarizing problem has three properties, which allow to a great extent the overcoming of the computational difficulties, connected with its solving as a problem of integer programming and also decrease DM's efforts in the comparison of new solutions. The first property is connected with this, that the current preferred integer solution of the multicriteria problem (found at the previous iteration), is a feasible integer solution of *DAL* problem. This facilitates the exact as well as the approximate algorithms for solving *DAL* problem, because they start with a known initial feasible integer solution. The second property is connected with the fact, that the feasible region of *DAL* problem is a part of the feasible region of the multicriteria problem (I). Depending on the values of the parameters $\Delta_k\ / k \in K^{\geq}$, $\delta_k\ / k \in K^{\leq}$ the feasible region of *DAL* problem can be comparatively narrow and the feasible solutions in the criteria space, found with the help of approximate integer programming algorithms, may be located very close to the non-dominated surface of the multicriteria problem (I). The third property comprises DM's possibility to realize searching strategy of "not big profits – small losses" type. This is due to the fact, that such optimal solution is searched for with the help of *DAL* problem, which minimizes Chebyshev's distance between the feasible criteria set and the current reference point, the components of which are equal to the wished by the DM values of the criteria being improved and to the current values of the criteria being deteriorated. The (weak) non-dominated solution obtained and the current solution are comparatively close and the DM can easily make his/her choice.

The classification-oriented scalarizing problems are appropriate in solving integer problems of multicriteria optimization, because the computational difficulties, connected with their solving are decreased with their help and the requirements towards the DM in the comparison and evaluation of the new solutions obtained, are diminished. From a viewpoint of the information, required by the DM in new solutions seeking in this scalarizing problem, the DM has to define the desired or acceptable levels for a part or for all the criteria. With the help of the scalarizing problem below described, called *DALDI* (desired or acceptable level, direction and interval), the DM is able to present his/her local preferences not only by desired and acceptable levels, but also by desired and acceptable directions and intervals of alteration in the values of separate criteria. The mixed integer scalarizing problem *DALDI* has the following type:

Minimize:

(9) $$S(x) = \max\left[\max_{k\in K^{\geq}}\left(\bar{f}_k - f_k(x)\right)/\left|f_k^{'}\right|, \max_{k\in K^{<}\cup K^{\leq}}\left(f_k - f_k(x)\right)/\left|f_k^{'}\right|\right] +$$

$$+\max_{k\in K^{>}}\left(f_k - f_k(x)\right)/\left|f_k^{'}\right|,$$

under the constraints:

(10) $$f_k(x) \geq f_{k,}\ k \in K^{>} \cup K^{=},$$

(11) $$f_k(x) \geq f_k - \delta_k,\ k \in K^{\leq},$$

(12) $$f_k(x) \geq f_k - t_k^{-},\ k \in K^{><},$$

(13) $$f_k(x) \leq f_k + t_k^{+},\ k \in K^{><},$$

(14) $$x \in X_1,$$

The scalarizing problem *DALDI* has characteristics similar to *DAL* scalarizing problem, but still there are two differences between them.

The first difference consists in this, that *DALDI* scalarizing problem gives greater freedom to the DM when expressing his/her local preferences in the search for a better (weak) non-dominated solution. Besides desired or acceptable values of a part or of all the criteria, the DM has the possibility to set also desired or acceptable directions and intervals of change in the values of some criteria The second difference between *DAL* and *DALDI* scalarizing problems concerns the possibility to alter their feasible sets (make them "narrower"), so that the feasible solutions are positioned close to the non-dominated (efficient) solutions of the multicriteria problem The more the criteria are, which the DM wishes to be freely improved or freely deteriorated, the smaller this possibility is. The narrow feasible regions of the scalarizing problems *DAL* and *DALDI* enable the successful application of approximate single-criterion algorithms, which is especially important when these problems are integer. It should be noted that scalarizing problem *DAL* is better than scalarizing problem *DALDI* in this aspect.

DAL and *DALDI* problems are nonlinear mixed integer programming problems [Wolsey, 1998]. Equivalent linear mixed integer programming problems can be constructed, [Vassileva, 2000], [Vassilev et al., 2003] with the help of additional variables and constraints

4. GAMMA-I1 Interactive Method

On the basis of scalarizing problems *DAL* and *DALDI*, a classification-oriented interactive method, called GAMMA-I1 is proposed for solving linear mixed integer programming problems of multicriteria optimization. The problems of mixed integer programming are NP-problems, i.e. the time for their exact solution is an exponential function of their dimension. That is why, in solving integer problems, particularly problems of larger dimension (above 100 variables and constraints), some approximate methods are used [Vassilev and Genova, 1991], [Pirlot, 1996]. Since finding a feasible solution is as difficult as finding an optimal solution, the approximate integer methods in the general case do not guarantee the finding of an optimal integer solution and of an initial feasible integer solution too. If the initial feasible integer solution is known and the feasible region is comparatively "narrow", then with the help of the approximate integer methods some satisfactory and in part of the cases optimal integer solutions could be found. The scalarizing problems *DAL* and *DALDI* have known feasible initial integer solutions.

DALDI scalarizing problem allows enlargement of the information with the help of which the DM can set his/her local preferences. This information expansion leads to the extension of the feasible set of criteria alteration in the criteria space and of the integer variables in the variables space. Hence, the approximate integer solutions of *DAL* problem (obtained with the help of an approximate integer method) are located closer to the non-dominated (efficient) set of the multicriteria problem, than the approximate solutions of *DALDI* problem. Therefore, if solving linear mixed integer problems of (*I*) type of large dimension, when the scalarizing problems have to be solved approximately in order to reduce the waiting time for new solutions evaluated by the DM, it is better to use *DAL* scalarizing problem than scalarizing problem *DALDI*.

Two different strategies are applied in the development of GAMMA-I1 interactive method in the process of searching for new solutions, that are evaluated. The first strategy, called integer strategy, consists in seeking a (weak) non-dominated integer solution at each iteration by exact solution of the corresponding linear mixed integer scalarizing problem. The second strategy, called approximate integer strategy, comprises searching for near (weak) non-dominated integer solutions at some iterations, approximately solving a respective linear mixed integer scalarizing problem. During the learning phase and in problems of large dimension up to the very end, only near (weak) non-dominated solutions can be looked for.

The interactive GAMMA-I1 method is designed to solve linear mixed integer problems of multicriteria optimization. The two strategies above described are realized in the method during the search for new solutions for evaluation in order to overcome the computational difficulties (particularly in solving problems of large dimension). The method is oriented towards learning and the DM has to determine when the most preferred solution is found. The algorithmic scheme of GAMMA-I1 interactive method includes the following basic steps:

Step 1. An initial near (weak) non-dominated solution is found, setting $f_k = 1$, $k \in K$ and $\bar{f}_k = 2$, $k \in K$ and solving *DAL* problem.

Step 2. The current (weak) non-dominated solution or near (weak) non-dominated solutions obtained are presented for evaluation to the DM. If the DM evaluates and chooses a solution that satisfies his/her global preferences, Step 6 is executed, otherwise – Step 3.

Step 3. A question is set to the DM what new integer solution he wishes to see – a (weak) non-dominated or a near (weak) non-dominated solution. Step 5 is executed in the first case, and Step 4 - in the second.

Step 4. The DM is asked to define the desired or feasible levels of the values of a part or of all the criteria. Scalarizing problem of *DAL* type is solved and then go to Step 2.

Step 5. The DM is requested to define the desired or feasible levels, directions and intervals of alteration in the values of a part or of all the criteria. Scalarizing problem of *DALDI* type is solved and then go to Step 2.

Step 6. Stop the process of the linear mixed integer multicriteria problem solving.

In GAMMA-I1 interactive method the DM controls the dialogue, the computing process and the conditions for canceling the process of linear mixed integer multicriteria problem solution..

5. Conclusion

The interactive GAMMA-I1 method is realized in the experimental software system MOLIP, developed at the Institute of Information Technologies of the Bulgarian Academy of Sciences. This system is designed for interactive solution of linear and linear mixed integer multicriteria optimization problems with different number and type of the criteria, with different number and type of the variables and constraints. MOLIP system functions in the environment of Windows 98 and higher versions and may serve for learning purposes, as well as for the solution of different applied problems. Our experimental results confirm that the computational effort and time are reduced considerably using heuristic integer algorithms in the learning phase and when solving large problems.

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