NEURAL APPROACH IN MULTI-AGENT ROUTING FOR STATIC TELECOMMUNICATION NETWORKS

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Abstract: The problem of multi-agent routing in static telecommunication networks with fixed configuration is considered. The problem is formulated in two ways: for centralized routing schema with the coordinator-agent (global routing) and for distributed routing schema with independent agents (local routing). For both schemas appropriate Hopfield neural networks (HNN) are constructed.

Keywords: centralized and distributed multi-agent routing schemas, Hopfield neural network, data streams distribution

Introduction

It is usual to consider only one "source-destination" pair of nodes, while stating and solving the routing problem in the local and global telecommunications networks (TCN). In this case of searching for an optimal route, different sourse-nodes (information resourses) and destination-nodes (TCN-clients) parallel work capability is not taking for granted. Therefore, in case of collective (multy-agent) employment of TCN for searching information resourses it is possible that network conflicts and TCN congestions could take place, and casts the TCN efficiency to decrease until the operability is lost. This problem was described in works [Timofeev, 2002] and [*Newton,* 2002].

In this paper we consider a model of multi-agent allocation of information streams in TCN with all participant-agents, involved into the searching and necessary data transfer process, took into account. In statement of the problem there two optimized allocation of data streams criteria are extracted: global, in case of which common TCN capacity is optimized, and local, when data stream allocation is being optimized for each source-destination pair.

By way of efficient computing models for solving such kind of routing problems it is convenient to use Hopfield neural networks (HNN) as it was shown in works [Timofeev, Syrtsev, 2002] and [Timofeev, Syrtsev, 2002a].

For both of optimization criteria an employment capability has been investigated and HNN models were constructed in this work.

Firstly multi-agent data stream routing problems are being considered in two cases:

- for centralized scheme with coordinator ("global optimization"), when decision acceptance is effected by special driving coordinator agent.;
- for distributed scheme ("local optimization"), when every TCN agent is accepting decision independently;

In this paper the possibility of solving stated problems for TCN with limited traffic capacity of communication channels is being considered and proved. Suggested solutions of multi-agent data stream routing in centralized and distributed schemas problems are basing on using HNN adapted to the statement conditions.

In the conclusion main work results are resumed and expanded directions of following investigation are considered.

1. Problem Formulation

As mathematical model of static network, so-called TCN, that doesn't change with the course of time, we will consider a graph

$$G(V, E(V)) \tag{1}$$

where V – set of nodes, E – ordered set of directed arcs. For given graph, modeling TCN, let's consider a set of its' nodes pairs D_0.

$$D_0 = \{(s,d) \mid s,d \in V, s \neq d\} \tag{2}$$

Where the first element of paired nodes is necessary data source-node and second one is the destination-node of demanded data. Thus, every multi-agent data stream routing problem is capable to be described as eventual set of such TCN node pairs from D_0. Let's denote this set as D ($D \subset D_0$) and consider it to be fixed, i.e. independent from time.

Let us introduce the following notions:

Π_d – set of paths for pair $d \in D$,

Π – ordered set of all paths for all pairs from D,

Φ_d – rate of information stream between nodes of pair $d \in D$,

$\Phi = (\Phi_1, \Phi_2, \ldots)$ – information streams rate distribution vector,

$\Phi(D) = \sum_{d \in D} \phi_d$ (or Φ_D) – total information streams rate D,

x_p – total information stream rate at path $p \in \Pi$,

$x = (x_1, x_2, \ldots)$ – information streams distribution vector,

δ_{lp} – percentage rate of information stream rate x_p, which pass through arc *l*,

ρ_l – utilization of arc $l \in E(V)$, where $\rho_l = \sum_{p \in \Pi} \delta_{lp} x_p$,

$\rho = (\rho_l, \rho_2, \ldots)$ – utilizations distribution vector,

$T_l(p_l)$ – weighted cost of arc *l*,

T_p – average cost of path $p \in \Pi$,

$T = (T_1, T_2, \ldots)$ – paths costs distribution vector.

In possible data transfer routers Π_d let us specify an incidence matrix

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots \\ \gamma_{21} & \gamma_{22} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \text{-}, \quad \text{где } \gamma_{pd} = \begin{cases} 1, \mathrm{p} \in \Pi_d \\ 0, \mathrm{p} \notin \Pi_d \end{cases} \tag{3}$$

To formalize and solve the problem of optimal routing let's introduce new conditions:

I. Average value of route is determined as sum of its' arcs loading values $\mathrm{T_p} = \sum_{\mathrm{l} \in \mathrm{E(V)}} \delta_{lp} T_l(p_l)$;

II. For each arc $l \in E(V)$ is fair that $T_l : [0, \infty) \to [0, \infty]$ and $T_l(0) < \infty$;

III. For each arc $l \in E(V)$, $T_l(p_l)$ is convex and either strongly monotonically increasing on interval, where $T_l(p_l) < \infty$, or $T_l(p_l) = \text{const}$;

IV. $T_l(p_l)$ function is continuous on all definitional domain, moreover on interval, where $T_l(p_l) < \infty$, it is continuously differentiable.

For centralized scheme with coordinator-agent, multi-agent routing problem decision is introducing itself as certain data streams allocation, by which common costs for them are minimal. Let F – common cost of data streams allocation. Then

$$F = \sum_{p \in \Pi} \frac{x_p}{\Phi(D)} T_p = \frac{1}{\Phi(D)} \sum_{l \in E(V)} \rho_l T_l(\rho_l). \tag{4}$$

For every route $p \in \Pi$ and every pair of nodes $d \in D$ following correlations a fair:

$$x_p \geq 0, \qquad \sum_{p \in \Pi_d} \gamma_{pd} x_p = \varphi_d. \tag{5}$$

Thus, the problem statement for centralized multi-agent routing will be represented as follows:

$$F \to \min \tag{6}$$

with the following constraints:

$$\Gamma^T x = \phi, \qquad x \geq 0. \tag{7}$$

In distributed scheme of multi-agent routing for each pair $d \in D$ problem is stated separately. In this case an optimal decision is the data stream allocation, at which its' cost for every pair by itself is minimal. Such decision is locally optimal.

Let us introduce concept of minimal stream between pair of nodes d as function: $A_d(x) = \min_{p \in \Pi d} T_p(x)$, $d \in D$. Then, as it was shown in work [*Altman, Kameda*], the decision of problem will be streams distribution vector x, suiting following correlations:

$$\left(T(x) - \Gamma \mathrm{A}(x)\right) \cdot x = 0, T(x) - \Gamma \mathrm{A}(x) \geq 0, \qquad \Gamma^T x - \varphi = 0, x \geq 0, \tag{8}$$

where A(x) = (A_1(x), A_2(x), …) – minimal streams vector.

3. Modification of capacity cost on communication channel for TCN with limited traffic capacity

For TCN, where channel traffic capacity is limited, function $T_l(p_l)$ will suit following constraints.

$$T_l(p_l) < \infty, 0 \leq p_l \leq p_{\max}, \quad T_l(p_l) = \infty, p_l > p_{\max}, \tag{9}$$

Where p_{max} – maximal traffic capacity of communication channel *l* .

In this case IV condition will be violated. It could be avoided in the event that auxiliary function $T_l^{(\varepsilon)}(p_l)$ is involved :

$$T_l^{(\varepsilon)}(p_l) = \begin{cases} T_l(p_l), p_l \notin [p_{\max} - \varepsilon, p_{\max}) \\ T_l(p_l)\left(1 - \frac{\pi}{2\varepsilon}(p_l - (p_{\max} - \varepsilon)) + tg\left(\frac{\pi}{2\varepsilon}(p_l - (p_{\max} - \varepsilon))\right)\right), p_l \in [p_{\max} - \varepsilon, p_{\max}) \end{cases} \tag{10}$$

Where amount ε>0 and arbitrarily small.

Lemma: For $T_l^{(\varepsilon)}(p_l)$ of aspect (10) I-IV conditions are held.

Proof: It is evident, that I and II conditions will be held. Let us prove, that $T_l^{(\varepsilon)}(p_l)$ is continuously differentiable on [0, p_{max}).

Let us consider function $z(p_l) = \frac{\pi}{2\varepsilon}(p_l - (p_{\max} - \varepsilon))$. It is continuously differentiable on whole definitional domain and strongly monotonically increases, moreover

$$z(p_{\max} - \varepsilon) = 0, \qquad z'(p_{\max} - \varepsilon) = z'(p_l) = \frac{\pi}{2\varepsilon}. \tag{11}$$

Let us consider $T_l^{(\varepsilon)}(p_l)$ in point {p_{max}-ε}. Allowing (11) we derive:

$$\begin{aligned} T_l^{(\varepsilon)}(p_{\max} - \varepsilon) &= T_l(p_{\max} - \varepsilon)\left(1 - z(p_{\max} - \varepsilon) + tg(z(p_{\max} - \varepsilon))\right) = \\ &= T_l(p_{\max} - \varepsilon)\left(1 - 0 + tg(0)\right) = T_l(p_{\max} - \varepsilon) \end{aligned} \tag{12}$$

From (10) follows, that in point {p_{max}} function $T_l^{(\varepsilon)}(p_l)$ is tending to ∞, i.e. $T_l^{(\varepsilon)}(p_l)$ is continuous on whole definitional domain. Subject to (10) and (11) we derive following phrase for auxiliary function derivate.

$$\left(T_l^{(\varepsilon)}(p_l)\right)' = \begin{cases} T'_l(p_l), p_l \in (0, p_{\max} - \varepsilon x) \\ T'_l(p_l)\left(1 - z(p_l) + tg(z(p_l))\right) + T_l(p_l) z'(p_l)\left(\frac{1}{\cos^2 z(p_l)} - 1\right), p_l \in (p_{\max} - \varepsilon, p_{\max}) \end{cases} \tag{13}$$

This implies, that function $T_l^{(\varepsilon)}(p_l)$ is continuously differentiable on (0, p_{max}))\{p_{max} -ε}. Let us consider limits of its' derivate from the right and left of the point {p_{max}-ε}.

$$\lim_{p_l \to p_{\max} - \varepsilon - 0} \left(T_l^{(\varepsilon)}(p_l)\right)' = (T_l(p_{\max} - \varepsilon))'$$

$$\lim_{p_l \to p_{\max} - \varepsilon + 0} \left(T_l^{(\varepsilon)}(p_l)\right)' = \lim_{p_l \to p_{\max} - \varepsilon + 0} \left(T_l(p_l)'\left(1 - z(p_l) + tg(z(p_l))\right) + T_l(p_l) z'(p_l) \left(\frac{1}{\cos^2 z(p_l)} - 1\right) \right) =$$

$$= (T_l(p_{\max} - \varepsilon))'(1 - 0 + tg(0)) + \frac{\pi}{2\varepsilon} T_l(p_{\max} - \varepsilon) tg^2(0) = (T_l(p_{\max} - \varepsilon))'.$$

As from this follows, that $\lim_{p_l \to p_{\max} - \varepsilon - 0} \left(T_l^{(\varepsilon)}(p_l)\right)' = \lim_{p_l \to p_{\max} - \varepsilon + 0} \left(T_l^{(\varepsilon)}(p_l)\right)'$, then function $T_l^{(\varepsilon)}(p_l)$ is continuously differentiable on $(0, p_{max})$. Hence the condition IV applies for auxiliary function (10).

For condition III to be applied it is enough to prove its' applying on interval (p_{max}-ε, p_{max}). On this interval the function $T_l^{(\varepsilon)}(p_l)$ will be convex and strongly monotonically increasing as the composition of two convex and affirmative functions.

Thus lemma is proved.

This implies, that multi-agent routing optimization methods being considered are applicable towards TCN with communication channels limited traffic capacity. Here optimal routers are constructed with some fallibility.

4. Centralized Multi-Agent Routing Scheme

Let us consider optimization problem (6), (7). As it was shown in work [*Altman, Kameda*], under conditions I-IV this problem has at least one decision. Here for all of the decisions the valuation of traffic distribution vector ρ will be the same.

In work [*Cichocki, Bargiela*] the schema of solving similar systems using neural network was considered, and in work [Timofeev, Syrtsev, 2002] the consideration if realizing this scheme using HNN took place. A sufficient condition for solving the optimization problem (6), (7) with linear constraints using HNN, is strongly monotonically increasing of minimized function ([*Cichocki, Bargiela*]). Let us consider system (4). As for particular set $D \subset D_0$ the value of function Φ_D is positive constant which is possible to be moved to the left part while not changing tasks conditions, i.e.:

$$F\Phi_D = \sum_{l \in E(V)} \rho_l T_l(\rho_l)\,. \qquad (14)$$

Let us introduce matrix Δ:

$$\Delta = \begin{pmatrix} \delta_{11} & \delta_{12} & \cdots \\ \delta_{21} & \delta_{22} & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}, \qquad (15)$$

where δ_{ij} – is part of data stream x_j intensity, which falls on arc *i.*

As $\rho_l = \rho_l(x)$, then task (6)-(7) could be reformulated in the following way:

$$(F\Phi_D) \to \min \qquad (16)$$

at following constraints:

$$\Gamma^T x = \Phi, \qquad \Delta x = \rho, \qquad x \geq 0\,. \qquad (17)$$

As possible problem (16) , (17) decisions we will search (ρ, x) vectors. Without generalization loss it is possible to consider, that $0 \leq \rho \leq 1$ and $0 \leq x \leq 1$ (similar method of adduction problem to this view is described in [*Cichocki, Bargiela*]).

Let us construct energy function E = E(ρ, x) for HNN, that solves an optimization problem (16), (17). As well we demand function E to be a quadratic form of (ρ, x).

Firstly let us consider function E_0:

$$E_0 = \frac{\alpha_{11}}{2}(\sum_{l\in E(V)} \rho_l T_l(\rho_l))^2 + \sum_{d\in D}\frac{\alpha_{2d}}{2}(\sum_{p\in\Pi}\gamma_{pd}x_p - \varphi_d)^2 + \sum_{l\in E(V)}\frac{\alpha_{2l}}{2}(\sum_{p\in\Pi}\delta_{lp}x_p - \rho_l)^2, \qquad (18)$$

where α_{ij} - are certain positive constants, where α_{11} – is sufficiently small value ([Timofeev, Syrtsev, 2002]). However E_0 is not a quadratic form, as the first sum includes non-linear components $T_l(\rho_l)$. Let us replace first sum in (18) with squared linear combination ρ. Then we get following energy function for HNN:

$$E_0 = \frac{\alpha_{11}}{2}(\sum_{l\in E(V)} c_l \rho_l)^2 + \sum_{d\in D}\frac{\alpha_{2d}}{2}(\sum_{p\in\Pi}\gamma_{pd}x_p - \varphi_d)^2 + \sum_{l\in E(V)}\frac{\alpha_{2l}}{2}(\sum_{p\in\Pi}\delta_{lp}x_p - \rho_l)^2 \qquad (19)$$

where $c_l \geq 0$ and sufficiently small.

It is important to notice, that the main demand, while constructing energy function for solving similar systems with linear constraints, is sufficiently small value of summand appropriate to the function being minimized ([*Cichocki, Bargiela*]) , in order to approximate solution would not heavily differ from the exact one. Therefore coefficients of linear combination are to be taken sufficiently small. At the same time they shouldn't be assigned to small, as far as it will slow down convergence of searching decision process.

Substituting (18) by (19) is possible, as far as for two strongly increasing functions with same definitional domains (given by system (17)) extremums are reachable in the same points.

Thus HNN model consisting of |E|+|Π| neurons ([Timofeev, Syrtsev, 2002]) is synthesized and adapted to multi-agent centralized schema conditions.

5. Distributed Multi-Agent Routing Scheme

Let us consider a problem of local optimization (8). It was shown in work [*Altman, Kameda*], that if conditions I-IV are applied, the solution of this problem exists. Let us consider systems (8) first equation. By virtue of second and fourth inequations of the system, we get, that for any x following inequation is fair:

$$\left(T(x) - \Gamma A(x)\right)\cdot x \geq 0 \qquad (20)$$

This implies, that non-negative function (T(x)-Γy)x accepts zero values (subject to other constraints) in points of possible solutions of equations (20). It implies, that points of minimum for given function and solutions of optimization task (8) are coinciding. Subject to (20) and

$$T(x)x - \Gamma A(x)x = (F\Phi_D) - \Gamma A(x)x \qquad (21)$$

Let us reformulate task (8) in the following way:

$$\begin{gathered}(F\Phi_D) - \Gamma A(x)x \to \min, \\ T(x) - \Gamma A(x) \geq 0, \qquad \Gamma^T x - \varphi = 0, \qquad x \geq 0\end{gathered} \qquad (22)$$

according to works [*Cichocki, Bargiela*] and [Timofeev, Syrtsev, 2002] task (22) is reducable to the following task:

$$\begin{gathered}(F\Phi_D) - \Gamma A(x)x \to \min, \\ T(x) - \Gamma A(x) - z = 0, \qquad \Gamma^T x - \varphi = 0, \qquad x \geq 0, \qquad z \geq 0\end{gathered} \qquad (23)$$

We remark, that z_p=0, if $T_p(x) = (\Gamma A(x))_p$. Hence, subject to inequations $x_p \geq 0$ and $z_p > 0$ we get, that $T_p(x) > (\Gamma A(x))_p$ and x_p=0.

Let us construct HNN, that solves an optimizing problem (23). As possible solutions we will search vectors (x, z). The same way as in previous paragraph we will assume, that $0 \leq x \leq 1$ and $0 \leq z \leq 1$. From the first parity in system of constraints follows, that

$$\left(F\Phi_D\right) - \Gamma A(x)x = zx \qquad (24)$$

Thus, energy function E for HNN will have following presentaion:

$$E = \alpha_{11}(\sum_{p\in\Pi} z_p x_p) + \sum_{p\in\Pi} \frac{\alpha_{2p}}{2}\left(T_p(x) - \sum_{d\in D}\gamma_{pd}A_d - z_d\right)^2 + \sum_{d\in D}\frac{\alpha_{3d}}{2}\left(\sum_{p\in\Pi}\gamma_{pd}x_p - \varphi_d\right)^2 \quad (25)$$

Appropriate (25) model of HNN consists of $2|\Pi|$ neurons [Timofeev, Syrtsev, 2002] and adapted to conditions of the local optimization problem for multi-agent routing distributed schema.

Conclusion

Conducted investigations showed adaptability of NN for solving multi-agent data stream routing problem in static TCN, which configurations doesn't change in the course of time. Two models of HNN for global and local data stream distributing optimization were constructed. On demand of TCN agent-users. Besides this, capability of obtained NN solutions adapting to TCN with data stream limited traffic capacity has been investigated.

Common default of NN-routing for distributed and centralized schemas is large amount of beforehand calculations (filler of all possible routes, weights calculations, depending on large amount of parameters, etc.). Thus, it is being planned to avoid incipient hardships and lacks in following investigations.

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