FUZZY MEMBERSHIP FUNCTIONS IN A FUZZY DECISION MAKING PROBLEM

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Abstract: *Authors analyses questions of the subjective uncertainty and inexactness situations in the moment of using expert information and another questions which are connected with expert information uncertainty by fuzzy sets with rough membership functions in this article. You can find information about integral problems of individual expert marks and about connection among total marks "degree of inexactness" with sensibility of measurement scale. A lot of different situation which are connected with distribution of the function accessory significance and orientation of the concrete take to task decision making are analyses here.*
Keywords: *expert, fuzzy sets.*

Introduction

Practically all real values measured by different procedures (in particular expert methods) are approximated, with rare exception. Except an inaccuracy of measurement there are some characteristics of measured values such as indetermination, multiplicity and incorrectness [Nariniani]. All mentioned above characteristics we shall combine by the term "fuzziness" Fuzzy can be: values of parameters, value of functions, ratio between objects etc.

Different kinds of uncertainty are to some extent characteristics of practically any situation of making decision in which the expert information is used. The nature of uncertainty is essentially different. Firstly, it's necessary to point out the objective uncertainty which is peculiar to all real constants and is connected with our world's "organization" itself [Nariniani, 1994]. Secondly, the subjective uncertainty is peculiar to human's nature on the whole and to his abilities to evaluate the information in particular. The reasons of the origin of the second type of uncertainty are: expert's lack of knowledge about the characteristics of the objects; his unsatisfied confidence degree in correctness of his marks; knowledge contradiction; in distinction of the information presentation and the semantic uncertainty corrected with different are indirect meanings of the natural language, indefiniteness of the keywords and definitions [Nariniani, 1994; Orlovski, 1981]. Beside, indefiniteness appears in the process of marks integration coming from different experts.

The result of the research [Larichev, 2002; Borisov, 1989] show that the main difficulty is caused by the necessity to appraise numerical values of the objects (variants, criteria) or to give numerical evaluation on the ratio scale between them. But the person fully much more confident [Orlovski, 1981; Larichev, 2002] if the has an opportunity to give unclear marks in the form of the intervals or with the help of rough sets pointing out the degree of belonging of the objects to these sets. In particular, more often in a form of package the additive criterion is used and its objectivation is connected with defining of weight coefficients of the objects [Voloshyn, 2003]. This approach seemed to be even more justified because in prevailing number of the cases it is enough to have the approximated characteristic of the data set and the expert info does not demand high accuracy.

So, the application of fuzzy sets and the function of belonging allow formalizing factors of uncertainty and unclearness with might happen in expert evaluation situation by some means. The function of belonging was interpreted in various works differently: as "subjective probability" [Orlovski, 1981], expert's confidence degree in object's belonging to the concept described by fuzzy set [Borisov, 1989], the opportunity of its interpretation by this concept [Melechov, 1990] and so on. For all this was traditionally considered reflection. But "unnatural" distinctiveness, and simple concept that according to its destination, is called to reflect distinction and inexactness of subjective marks. According to the author's opinion it is incorrect to demand precise meanings of the function of belonging from the expert. It is more logical to demand rough marks of the function of belonging because the situation of subjective evaluation is connected with the principle inexactness ("objective", "situational", "semantic" and so on [Nariniani, 1994]) in the definition of the value of function of belonging.

In connection with this author analyses the problems of setting and processing of expert information with the help of rough functions of belonging.

The forms of presentation of rough function of belonging

Let's X as the universal set of n-measured alternatives that describes totality of all possible variants of expert's choice. Let's describe rough concept by fuzzy set $A = \{(x,\ \mu_A(x))\}$, $x \in X$, $\mu_A(x)$, where $\mu_A : x \to [0,1]$, - function of belonging.

The expert is suggested to set the information about the value of function of belonging:

a) in the form of the interval $\mu_A(x) \in [\mu^H{}_A(x), \mu^B{}_A(x)]$;

b) pointing the absolute inexactness $\mu_A(x) = \mu^0{}_A(x) \pm \Delta\mu_A(x)$, where $\mu^0{}_A(x)$ - "exactness of measureness";

c) pointing relative inexactness $\mu_A(x) = \mu^0{}_A(x) \pm \varepsilon\mu^0{}_A(x)$, $\varepsilon \in (0,1]$.

We accept as well to get the expert's information about inexact function of belonging is a form of

d) triad $\{(x,\ \mu_A(x),\ \gamma(\mu_A(x)))\}$, where γ, $\gamma\in[0, 1]$, - expert's confidence degree is his mark.

Rough function of belonging is interpreted as sphere of expert's insensibility (inexactness, uncertainty) while defining function of objects belonging x, x∈X, to set A. And the sweep of the interval (expert's confidence degree in his mark) characterizes quantitative measure of this inexactness. Formally this uncertainty might be defined with the help of so called "granularity" [Nariniani, 1994] which reflects inexactness mark of the concrete parameter according to the granular size which is indivisible (and, of course, inexact) total mark of this parameter. In our case this parameter is the function of belonging that is defined on the interval [0,1], the smallest union of which defines the maximum granularity of the scale. The size of the granular is defined according to consideration of "limit of distinction"of granular for the expert.

With it is the help of the concept of granularity its case to define the absolute mark of the "limit of distinction" of the expert mark for cases a)-d). Let's define this mark as ξ, and the granular size as h. Then constant ξ in cases a) - d) is defined correspondingly:

$$\xi = h/(\mu^B{}_A(x) - \mu^B{}_A(x)),$$

$$\xi = h/\Delta\mu_A(x),$$

$$\xi = h/\gamma(\mu_A(x)).$$

The term "exactness degree" of the expert mark is used further on while working out methods of operating with "unclear" function of belonging.

Special cases of fuzzy values of the membership function

It is possible to select some ways of the definition of the fuzzy information, depending on conditions of the problem, kind of value measurements and other aspects:

1. « The uniform distributed fuzziness», when the task conditions are as the following: all set of acceptable fuzzy values are equivalent. Or when only the interval of fuzzy values is known and there is no any possibility to obtain more detailed information about this interval.

2. «Point fuzziness », when the outcomes of separate measurements of the fuzzy value are obtained by the way of point estimations which are not conterminous among themselves.

3. «Three points fuzziness », when the fuzzy value boundaries and the its most probable or most advisable value are known. Such way of the definition of fuzzy value can be described by the triangular membership function.

4. «Nonuniform distributed fuzziness», when the fuzzy value of the membership function is described by some no triangular function.

Methods of defining unclear group mark

In the article [Voloshyn,, 2002] two ways of working out of mathematical apparatus for operating unclear function of belonging are pointed out.

The first way – preceding scalarization of rough function of belonging with consequent usage of standard apparatus of unclear analysis.

Because of the principle position of the subjective factor in drawing the function of belonging (as "value of function of belonging is subjective possibility"), it is necessary to take into consideration the expert's "psychological parameters" ("realism", "independence", "truthfulness", "inclination to risk" and so on [Voloshyn,, 2002]). For example, in case of one expert

$$\mu_A(x) = \alpha\mu^{\min}{}_A(x) + (1-\alpha)\mu^{\max}{}_A(x), \quad (1)$$

here α, $0 \le \alpha \le 1$, - the coefficient of expert's "risking".

In case of a group of expert two-phase procedure of scalarizations suggest (for each expert and for group). In general case the expert marks are correlated to some weight coefficients which reflect there expert confidence degree, i.e. $\lambda_i \in [0;1]$, $\sum_{i=1}^{m}\lambda_i = 1$, where m – expert quantity. Competence coefficients are calculated, for example, according to the method [Voloshyn, 1993].

Let's define a set of objects indexes as $I = \{1,..,n\}$, here n – is objects quantity in set X, and set of experts indexes - $J = \{1,..,m\}$. Let's define individual function of belonging, scalarized by the formula (1) as $\mu_A(x_i, p^j{}_i)$.

One of the most wide spread ways of the choice of the best object [Melechov, 1990] consist in the choice of the object which has the maximum degree of belonging to fuzzy set A (criterion of minimax), i.e.

$$\mu_A(x_i) = \max_{i\in I}\min_{j\in J}(\lambda_j\mu_A(x_i, p^j{}_i). \quad (2)$$

In [Orlovski, 1981] the effectiveness of linear package for the task of choice is pointed out:

$$\mu_A(x_i) = \max_{i\in I}\sum_{j\in J}(\lambda_j\mu_A(x_i, p^j{}_i)) \quad (3)$$

It coefficient of experts competence are considered as distribution of possibilities we can use the criteria of Bayes-Laplas or Yurnits [Mushik, 1990], so

$$\mu_A(x_i) = \max_{i\in I}\sum_{j\in J}\lambda_j\mu_A(x_i, p^j{}_i)/m, \quad (4)$$

$$\mu_A(x_i) = \max_{i\in I}(\alpha\min_{j\in J}\lambda_j\mu_A(x_i, p^j{}_i) + (1-\alpha)\max_{j\in J}\lambda_j\mu_A(x_i, p^j{}_i). \quad (5)$$

The methods of scalarization of rough function of belonging as for one expert and so as a group of expert have one drawback in common. In a result mark the level of individual measure of inexactness while defining function of belonging by the ways a)- c) is not taken into consideration, so as the degree of collective inexactness while integrating individual opines.

There are real situation in which it's more preferable to get rough functions of belonging which use the expert information more "carefully". It is connected first of all, with the principle inexactness of waited result when there is sense to consider all the range of possible values of resultant functions of belonging. The most typical example is medical diagnosis and prognosis.

In the cases we consider working out the operations for "rough" functions of belonging, presented in form a)- d) as more advisable which allow to preserve information about the degree of individual "uncertainly" of functions of belonging.

So for the cases a)- d) be the way of definite transformations come to each other, let's consider different ways of formalization of individual rough functions of belonging, defined by the expert is a form of intervals (case a)) is a collective form of belonging. The procedures are realized in two phases as wall: in the first phase the integration of individual interval marks into collective rough mark in a form of interval takes place; in the second phase the criteria of optimum decision taking are defined.

The most is the integrative mark suck as:

$$\mu^{*(\min)}{}_A(x_i) = \min_{j\in J} \lambda_j \, \mu^{*(\min)}{}_A(x_i, p^j{}_i), \;\; \mu^{*(\max)}{}_A(x_i) = \min_{j\in J} \lambda_j \, \mu^{*(\max)}{}_A(x_i, p^j{}_i). \quad (6)$$

Defining the resultant interval in suck a way $[\mu^{*(\min)}{}_A(x_i), \mu^{*(\max)}{}_A(x_i)]$ there are no preferences inside the sphere, i.e. all the values are equally possible. Evidently, the more sphere sweep is according to (6), lest accurate the resultants function of belonging is defined.

The other approach to word defining if resultant interval on a set of individual intervals consists in defining its measure as centers of gravity of the sets accordingly $\mu^{(\min)}{}_A(x_i, p^j{}_i)$ и $\mu^{(\max)}{}_A(x_i, p^j{}_i)$, $i \in I$, $j \in J$, according to the formulas:

$$\mu^{*(\min)}{}_A(x_i) = \sum_{j=1}^{m} \lambda_j \mu^{(\min)}{}_A(x_i, p^j{}_i)/m, \;\; \mu^{*(\max)}{}_A(x_i) = \sum_{j=1}^{m} \lambda_j \mu^{(\max)}{}_A(x_i, p^j{}_i)/m. \quad (7)$$

It the distribution inside the individual intervals is not equal, for example, is defined by b), so while defining the resultant intervals it is also taken into consideration accordingly.

Finally the resultant intervals of functions of belonging are defined by usage of the definite criteria of “inexactness degree” of value which are based on total marks of the concrete interval with “granularity” of measurement scale. For example, the resultant membership function is defined by points or the help of the interval according to the principle:

$$\mu^{*}{}_A(x_i) = \begin{cases} 1/2(\mu^{*(\max)}{}_A(x_i) + \mu^{*(\min)}{}_A(x_i)), & \mu^{*(\max)}{}_A(x_i) - \mu^{*(\min)}{}_A(x_i) \sim h \\ [\mu^{*(\max)}{}_A(x_i), \;\; \mu^{*(\min)}{}_A(x_i)], & \mu^{*(\max)}{}_A(x_i) - \mu^{*(\min)}{}_A(x_i) >> h \end{cases},$$

$i \in I$.

For the tasks of medical and legal diagnosis “displacement” of values distribution of function of belonging toward, the left border occurs; and for technical and social diagnosis – toward the right. So we’ve got “the second derivative”of unclear function of belonging that is defined by the group of system experts according to the context of the problem [Larichev, 2002] .

Further on the criteria (2)-(5) are used with the resultant intervals of possible values of functions of belonging in each perimeter.

The procedures on the analogy are suggested for unclear binary relations with rough functions of belonging.

Constructing of the generalized fuzzy membership function of indistinct set on distinct sets of a linguistic variable

Let group of k experts creates the membership function by linguistic variable I = {1..., k} - the set of indexes of the experts and, accordingly, sets, on which one is created the function of an accessory.

Let group from k experts creates the membership function $\mu^i{}_A(x)$, $i\in I$, of x from some fuzzy set A on sets of T^i, $i\in I$, by linguistic variable; I = {1..., k} - the set of experts indexes and accordingly sets, which one creates the membership function. Let's consider, that $\exists i_1, i_2$: $i_1, i_2 \in I$, $T^{i1}=T^{i2}$, $i_1 \neq i_2$; $\exists i_1, i_2$: $i_1, i_2 \in I$, $T^{i1} \neq T^{i2}$, $i_1 \neq i_2$., among sets are used by the experts for constructing of individual membership functions, at any rate there are existed two of them with indexes $i_1, i_2 \in I$ not conterminous among themselves.

To determine the generalized (resulting, group) membership functions $\mu^*{}_A(x)$) at first it is necessary to construct generalized set T of this function, that is association of set units T^i, $i\in I$.

Definition 1. The scaling of set T is the changing of its structure by means of changing of its linguistic variables or swap about their following.

The constructing of generalized set T can be carried out by scaling of sets T^i, $i\in I$. At the second stage of generalized set T can be used the reduction to a base scale.

Definition 2. Let's call a base scale of generalized set T the association of individual sets

Then the individual membership functions $\mu^i_A(x)$, $i \in I$ appointed by experts are put in correspondence to a base scale.

Definition 3. Let's call the interpolation of the membership function values $\mu^i_A(x)$, $i \in I$ restoring these values on a base scale of set T. As the expert did not set it because of these values absence in its individual set

The interpolation of a unit x_j is carried out by the rule: $\mu(x_j)=(\mu(x_{j-1})+\mu(x_{j+1}))/$, where the values $\mu(x_{j-1})$ and $\mu(x_{j+1})$ are preset or already computed. For some linguistic variables the value $\mu(x_j)$ can be undefined.

Conclusion

The development of mathematical apparatus for operating with unclear functions of belonging is very actual because it would allow solving the tasks of decisions taking on the basis of expert information with the help of the methods more adequate to the nature of expert mark which is characterized by different types of inexactness and uncertainty.

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