

MODEL OF ACTIVE STRUCTURAL MONITORING AND DECISION-MAKING FOR DYNAMIC IDENTIFICATION OF BUILDINGS, MONUMENTS AND ENGINEERING FACILITIES

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Abstract: Structural monitoring and dynamic identification of the manmade and natural hazard objects is under consideration. Math model of testing object by set of weak stationary dynamic actions is offered. The response of structures to the set of signals is under processing for getting important information about object condition in high frequency band. Making decision procedure into active monitoring system is discussed as well. As an example the monitoring outcome of pillar-type monument is given.

Keywords: math model of structural active monitoring, set of weak stationary dynamic actions.

Introduction

Structural monitoring and dynamic identification are extremely important topics in the maintenance of civil engineering structures in general, but especially in the case when the objects are dangerous from ecological angle of view or very valuable from historical point of view as example monuments. Taken into account that technical systems are damaged by overloading, fatigue, aging and environmental influences the method of dynamic identification provide the possibility to investigate the dynamic behavior of a given structure by means of non-sensitive for constructive tests, and consequently enable to assess the structure "health" and the possible need for structural maintenance. For such a goal there is proposed math model of structural active monitoring, which was realized into the automatic monitoring system. The power of a test signal should not be high. At least after testing the environment should not be essentially changed. The following experimentation has to correspond to these conditions: there is a set of weak signals, energy of each one is commensurable with a background noise level, they are dispatched and followed strictly periodically in time in encoded shape, and the response of an object is accrued in correspondent to to this periodicity. Here is used a black box model of the tasted system, when for analyses is used only the system response on outside influence. But the above mentioned experimentation has some imperfection. Instability of test signals source leads to the accentual accumulated signal disfigure.

The real signal sources ensure stability only in some domain of modification parameters of these signals. In this sense it is possible only to state, that the vector of parameters, which defined testing signal, is random with a priori known distribution. This circumstance can be considered as a possibility to research a priori statistic of parameter stability of a source, or as a possibility formally entered into the model of the researcher's heuristics. Further we shall show, that the drift in time leads to "fading" a high-frequency component in a response function of structure, while the fluctuations of testing signal energy are not so essential. In work [1] the model of a uniform distribution of the start moment is fragmentary reviewed. Under consideration is more global model and we shall construct algorithm for a signal restoring. A set of physically feasible signals with fluctuating amplitude and initiation delay can model the active monitoring. These models deserve attention, as the active monitoring is a regime testing of a structure by a testing signal, on response which one can make a conclusion about its state, and about possible modifications in its state. Term "regime" means, first of all, experimentation in time. As soon as absolutely precise temporal experimentation we can not provide then it is necessary for us to model stochastic character of this process. The model of a binomial flow that embodies determined component of testing process and stochastic character of parameter instability of this flow is further given. The paper introduces into this solution problem and a methodology for the identification of the main structural parameters (as a time history of the structure response function, in order to identify the main eigenfrequencies and associated modal shapes of the structure and so on) by means of an active monitoring. The response of structures to weak stationary dynamic actions is under consideration. There are proposed practical examples for illustration the main aspects of theory.

The mathematical model of active monitoring

In this formula the symbol * indicates the operator of convolution, t_d - transport delay in concrete experiment, i.e. time from start of a signal before its receipt on a receiver recorder T - the time between sending of signals and is simultaneous the value defining length of the signal carrier, i.e. time, when the signal is distinct from zero point. This value should be less than T . Explicitly this model is considered in treatise [1].

In this treatise the processing algorithm of observations permitting to augment a signal - noise ratio is offered and was shown, that at uniform distribution of the moments instability of sounding start the high-frequency components of a signal is "disappear". In this treatise we shall consider more general case of binomial flow with beta distributed partial density and we shall offer algorithm of a signal restoring

Let's consider that the partial density function of the moment of a k signal start $e_k(\tau)$ looks like:

$$e_k(\tau) = e_k(\tau + (k - 1) \cdot T) = e(\tau_k); \tau \in (0, \varepsilon); k = 1, 2, \dots, K. \tag{2}$$

It means, that the form of distribution is identical to all moments of the introduction, and for the k signal the shift on an interval is carried out $\tau + (k - 1) \cdot T$

With allowance for of transport delay t_d partial density of the moment of a k signal reception will be:

$$e_k(\tau) = e_k(\tau + (k - 1) \cdot T + t_d) = e(\tau_k); \tau \in (0, \varepsilon); k = 1, 2, \dots, K. \tag{3}$$

And hereinafter (3) under τ_k is realized $\tau_k = \tau + (k - 1) \cdot T + t_d$

Let's consider a case of not intersected partial density, i.e. case when $T > \varepsilon$, and completely solved i.e. with not intersected carriers, signals.

These two assumptions mean, that the carrier of a signal is less than $T - \varepsilon$ and that the probability density $\pi(\tau_1, \dots, \tau_K, \Omega)$ of appearance in the observations domain $\Omega = K \cdot T$ precisely K signals with the moments τ_1, \dots, τ_K is introduce the multiplication of partial density and looks like:

$$\pi(\tau_1, \dots, \tau_K, \Omega) = \prod_{k=1}^K e_k(\tau_k) \tag{4}$$

Two of these assumptions completely correspond to conditions of experiment realization.

Fluctuations θ_k is accepted independent, with density functions, $p_k(\theta), \theta_k \in \Theta$.

The independence means, that the probability density of a vector parameters $\{\theta_1, \dots, \theta_K\}$ looks like:

$$p(\theta_1, \dots, \theta_K) = \prod_{k=1}^K p_k(\theta_k) \tag{5}$$

In our model is accepted, that the exploring signal $u(t)$ is physically feasible

After convolution with a transfer function of the environment $h(t)$ the signal $S(t)$ will be with the final carrier of length no more than $T - \varepsilon$, i.e. two conditions are executed: causalities (6) and stability (7)

$$S_k(t) = \begin{cases} \theta_k \cdot S(t - \tau_k), & t \in (\tau_k, \tau_k + \alpha T); \\ 0, & t \notin (\tau_k, \tau_k + \alpha T), \quad \alpha T < T - \varepsilon \text{ u } \alpha \in (0, 1) \end{cases} \tag{6}$$

$$\int_0^{\alpha T} S^2(t) dt < \infty \tag{7}$$

The last condition can be lead to following by normalization:

$$\int_0^{\alpha T} S^2(t) dt = 1 \tag{7a}$$

With normalization's allowance (7a) in distribution of fluctuations (5) for expectation $E\{\theta_k\}$ can be accepted, that

$$E\{\theta_k\} = 1. \tag{8}$$

The expectation of the environment response on flow of separately fluctuated signals with allowance for that $E\{n(t)\} = 0$ will look like

$$E\{y(t)\} = \int_{\Omega} \dots \int_{\Omega} \int_{\Theta} \dots \int_{\Theta} \sum_{k=0}^K \theta_k S(t - \tau_k) \cdot \prod_{k=1}^K e_k(\tau_k, \gamma, \eta, \varepsilon) \cdot p_k(\theta_k) \cdot d\tau_k \cdot d\theta_k + E\{n(t)\} =$$

$$= \sum_{k=0}^K \int_{\Theta} \theta_k \cdot p_k(\theta_k) \cdot d\theta_k \cdot \int_{\Omega} S(t - \tau_k) \cdot e_k(\tau_k, \gamma, \eta, \varepsilon) \cdot d\tau_k \tag{9}$$

Taking into account that the first integral, standing in the last expression in the sum sign, is expectation of fluctuation of each flow K signals and taking into account (8) last expressions becomes:

$$E\{y(t)\} = \sum_{k=0}^K \int_{\Omega} S(t - \tau_k) \cdot e_k(\tau_k, \gamma, \eta, \varepsilon) \cdot d\tau_k \tag{10}$$

In our case for quite satisfactory approximating for partial density of the moments of the signals introduction can be beta distributions with parameters $\gamma, \eta, \varepsilon$. Varying these parameters it is possible to receive approximating practically of any distribution on an interval of length [3].

$$e_k(\tau_k, \gamma, \eta, \varepsilon) = \begin{cases} \frac{1}{\varepsilon} \cdot \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot \left(\frac{\tau_k}{\varepsilon} - (k-1) \cdot T\right)^{\gamma-1} \cdot \left(1 - \frac{\tau_k}{\varepsilon} - (k-1) \cdot T\right)^{\eta-1}; & \tau_k \in \Delta_k; \\ 0, & \text{when } \tau_k \notin \Delta_k. \end{cases} \quad 0 < \gamma, 0 < \eta; \quad k = 1, 2, \dots, K. \quad \Delta_k = ((k-1) \cdot T, (k-1) \cdot T + \varepsilon) \tag{11}$$

The expectation for model (1) with allowance for (2) - (11) becomes:

$$E\{y(t)\} = \sum_{k=0}^K \frac{1}{\varepsilon} \cdot \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot \int_{(k-1) \cdot T}^{(k-1) \cdot T + \varepsilon} [S(t - \tau_k)] \cdot \left(\frac{\tau_k}{\varepsilon} - (k-1) \cdot T\right)^{\gamma-1} \cdot$$

$$\cdot \left(1 - \frac{\tau_k}{\varepsilon} - (k-1) \cdot T\right)^{\eta-1} d\tau_k. \tag{12}$$

Condition of physical feasibility (causality (6) and the stability (7)) allows to present a signal as following series:

$$S(t - \tau_k) = X(t, \tau_k, \alpha \cdot T) \cdot \sum_{i=1}^{\infty} s_i \cdot \varphi_i(t - \tau_k) \tag{13}$$

Characteristic interval function (14) and orthonormalized on $(0, \alpha \cdot T)$ basis $\varphi(t) = \{\varphi_i(t)\}$.

$$X(t, \tau_k, \alpha \cdot T) = \{1, \text{ npu } t \in (\tau_k, \tau_k + \alpha \cdot T); 0, \text{ npu } t \notin (\tau_k, \tau_k + \alpha \cdot T)\} \tag{14}$$

Then

$$E\{y(t)\} = \frac{1}{\varepsilon} \cdot \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot \sum_{k=0}^K \left[\sum_{i=1}^{\infty} s_i \cdot \int_{\Omega} X(t, \tau_k, \alpha \cdot T) \varphi_i(t - \tau_k) \cdot \left(\frac{\tau}{\varepsilon} - (k-1) \cdot T\right)^{\gamma-1} \cdot$$

$$\cdot \left(1 - \frac{\tau}{\varepsilon} - (k-1) \cdot T\right)^{\eta-1} d\tau_k =$$

$$= \frac{1}{\varepsilon} \cdot \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot \sum_{k=0}^K \left[\sum_{i=1}^{\infty} s_i \cdot \int_{(k-1) \cdot T}^{(k-1) \cdot T + \varepsilon} \varphi_i(t - \tau_k) \right] \cdot \left(\frac{\tau}{\varepsilon} - (k-1) \cdot T\right)^{\gamma-1} \cdot$$

$$\cdot \left(1 - \frac{\tau}{\varepsilon} - (k-1) \cdot T\right)^{\eta-1} d\tau_k. \tag{15}$$

If to realize shift of each signal on $(k-1) \cdot T$ and then to apply $L_K[y(t)]$ - operator of K aliquot shift and summation of results of shift with factor $1/K$, we shall receive following result.

$$L_K \{y(t)\} = \overline{E\{S(t)\}} = \frac{1}{\varepsilon} \cdot \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot \sum_{i=1}^{\infty} s_i \cdot \int_0^{\varepsilon} \varphi_i(t - \tau) \cdot \left(\frac{\tau}{\varepsilon}\right)^{\gamma-1} \cdot \left(1 - \frac{\tau}{\varepsilon}\right)^{\eta-1} d\tau \quad (16)$$

Here $\overline{E\{S(t)\}}$ is estimation of a signal expectation.

$$S(t) \cong s(t) = X(t, 0, \alpha \cdot T) \cdot \sum_{j=1}^Q s_j \varphi_{i_j}(t) = X(t, 0, \alpha \cdot T) \cdot (\mathbf{f}(t), \mathbf{s}) \quad (17)$$

From (16) follows that the signal is approximately recover by calculation of a vectors \mathbf{s} . We shall make dot product of a vector \mathbf{s} on the function $\mathbf{f}(t)$ we shall receive estimation (17).

Let:

$$\phi_j(t) = \frac{1}{\varepsilon} \cdot \frac{\Gamma(\gamma + \eta)}{\Gamma(\gamma)\Gamma(\eta)} \cdot \int_0^{\varepsilon} \varphi_{i_j}(t - \tau) \cdot \left(\frac{\tau}{\varepsilon}\right)^{\gamma-1} \cdot \left(1 - \frac{\tau}{\varepsilon}\right)^{\eta-1} d\tau \quad (18)$$

Than

$$L_K \{y(t)\} = X(t, 0, \alpha \cdot T + \varepsilon) \cdot \sum_{j=1}^Q s_j \cdot \phi_j(t) = X(t, 0, \alpha \cdot T + \varepsilon) \cdot \mathbf{s}^T \cdot \mathbf{F}(t), \quad (19)$$

$$\mathbf{F}(t) = \{\phi_j(t)\}; j = \overline{1, Q}$$

Vector of factors \mathbf{s} , we shall determine from the last expression as follows.

$$\int_0^{\alpha \cdot T + \varepsilon} L_K \{y(t)\} \cdot \mathbf{g}^T(t) dt = \int_0^{\alpha \cdot T + \varepsilon} X(t, 0, T + \varepsilon) \cdot \mathbf{s}^T \cdot \mathbf{F}(t) \cdot \mathbf{g}^T(t) dt = \mathbf{s}^T \cdot \int_0^{\alpha \cdot T + \varepsilon} X(t, 0, T + \varepsilon) \cdot \mathbf{U}(t) dt. \quad (20)$$

$\mathbf{g}^T(t) = \{g_j(t)\}; j = \overline{1, Q}$ - Is the vector function composed from a subset Q of functions of orthonormalized on $(0, \alpha \cdot T + \varepsilon)$ basis.

$$\mathbf{U}(t) = \{u_{ij}(t)\}; i, j = \overline{1, Q}. u_{ij}(t) = (\phi_j(t), g_i(t)) \quad (21)$$

It is a matrix, elements by which one are the dot products of basic functions $\phi_j(t)$ and $g_i(t)$

Let's designate

$$\mathbf{I}^T = \int_0^{\alpha \cdot T + \varepsilon} L_K \{E\{y(t)\}\} \cdot \mathbf{g}^T(t) dt \quad (22),$$

We transpose the left and right parts of expression (20). We receive a set of equations concerning a vectors \mathbf{s} :

$$\mathbf{U}^T \cdot \mathbf{s} = \mathbf{I} \quad (23).$$

The algorithm of restoring of a signal in a series from K tests in binomial flow is obtained, using prior regards about distribution of the unstable moment of start of an exploring signal. The key moment here is the prior knowledge of the value ε , as it defines a choice of basis $\mathbf{g}(t)$.

Processing algorithm. The result of experiment $y(t)$ exposed to transformation $L_K \{y(t)\}$ Then the vector of scalar multiplication is created $(L_K \{y(t)\}, g_i(t))$ and matrix $u_{ij}(t) = (\phi_j(t), g_i(t))$. The vector \mathbf{s} is discovered from the equation (23). Substitute this vector in (17) and we receive a signal estimation.

Example. Such an approach for solution of the engineering problems in creation of high-rise extended pillar type monument in Kyiv was used [4].

With the purpose to get the monument spectral characteristics, logarithmic decrement of the oscillations of the object and to analyses of damping ability of the system, which was realized at the monument for oscillation reduction, the site tests were carried out. For registration of fluctuations three-directional geophone with gauges located on three mutually perpendicular axes was used. The special characteristics of gauges represent one-modal curve with the extreme point in $f=1$ Hz. Geophones were placed at a horizontal surface, on the level of 42 meters. They served as a part of interface of the monitoring registration and processing automated system. This system allows correcting the spectral characteristic up to uniform in the chosen range

of frequencies. The first part of experiment consisted in registration of monument reaction on a natural background as an input signal. This signal represents a superposition of the large number of the external factors from natural microseism noise and men made one up to signals from ground transport. The important moment is that the total spectrum of this signals is much wider then the response spectrum of the monument. For the monument it was obtained three modes on frequencies 0.48 Hz, 0.93 Hz and 1.47 Hz with corresponding amplitudes 1.0, 0.07 and 0.12. The frequency of 1.47 Hz with rather intensive amplitudes hypothetically is devoted to the mode of the top sculpture, the framework of which is less rigid then the framework of the self column. The second part of experiment was consisted in to get a logarithmic decrement of oscillation of the monument on the basic resonant frequency. For this purpose was used a damp of pendulum type. By compulsory swinging of this pendulum the monument was coupled in fluctuations and then the fluctuations faded by a natural way. The average value estimation of the logarithmic decrement of the oscillations was equaled 0.055. This figure shows that the metal column with granite shell has rather low capacity to dampen fluctuations. The damper, when it was put in operation during the tests, has increased the ratio of the logarithmic decrement of the oscillations up to the level was equaled 0.18-0.25. The damper construction gives the possibility to obtain greater ratio of logarithmic decrement of the oscillations via increasing of the friction coefficient the energy absorber. It's necessary to note that the spectrum of a structure is its steady characteristic. This function varies with change of mechanical parameters of a structure and can be used for detection of "age" changes of a structure while in exploitation. It's possible to consider that the fixed spectral monument characteristics further can be used as reference for detection of a beginning of the moment "age" changes during a structure-monitoring period.

Conclusion

Here is proposed and analyzed math model of an active monitoring system which is based on stochastic flow process of accruing data. For response signal correction is used premature probability of instability parameters of testing signals set generator. It is shown that the main source of instability testing signals is the time of signal departure and decision-making procedure is proposed.

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