CLASSIFICATION-BASED METHOD OF LINEAR MULTICRITERIA OPTIMIZATION

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Abstract: The paper describes a classification-based learning-oriented interactive method for solving linear multicriteria optimization problems. The method allows the decision makers describe their preferences with greater flexibility, accuracy and reliability. The method is realized in an experimental software system supporting the solution of multicriteria optimization problems.

Keywords: linear multicriteria optimization, interactive methods, decision support systems.

1. Introduction

The problems of multicriteria optimization are multicriteria decision making problems with infinite number of alternatives [Steuer, 1986]. The interactive methods are the most widely spread methods [Gardiner and Vanderpooten, 1997] for solving problems of multicriteria optimization. Every iteration in such method consists of two phases: a computation and a decision one. One or more non-dominated solutions are generated with the help of a scalarizing problem at the computation phase. At the decision phase these non-dominated solutions are presented for evaluation to the decision maker (DM). In case the DM does not approve any of these solutions as a final solution (most preferred solution), he/she supplies information concerning his/her local preferences with the purpose to improve these solutions. This information is used to formulate a new scalarizing problem, which is solved at the next iteration.

The efficiency of each interactive method depends to a great extent on the type of the information, which the DM sets in order to improve the local preferred non-dominated solution, on the time for scalarizing problem solution, on the possibilities to learn the DM with respect to the multicriteria problem being solved, on the type and number of the non-dominated solutions being compared with the local preferred solution.

When solving linear problems of multicriteria optimization, linear programming problems are used as scalarizing problems. They are easy solved problems. Hence, in the interactive methods solving multicriteria linear problems, the time for scalarizing problems solution does not play a significant role. In the development of these methods main attention is paid to the type of information, which the DM can put forward in the attempt to improve the local preferred non-dominated solution. In a large part of the interactive methods recently known, this information comprises basically the aspiration levels of the criteria [Wierzbicki, 1980], which the DM wishes to achieve. The aspiration levels define the so-called reference point in the criteria space. These interactive methods use scalarizing problems belonging to the group of scalarizing problems of the reference point. The classification-oriented scalarizing problems (with some exclusions, like STEP scalarizing problem [Miettinen, 1999]) have been more rarely used in the interactive methods for linear multicriteria problems solving up to nowadays. The possibilities to learn the DM during the time of linear multicriteria problem solution figure another significant feature of the interactive methods. In addition to DM's freedom to move in the non-dominated space, these capabilities are expressed in the determination of more than one non-dominated solution in the computing phase. These solutions are presented to the DM for evaluation [Korhonen and Laakso, 1986]. It should be noted that in modern interactive methods solving multicriteria linear problems, it is accepted by default, that the DM can evaluate more than two non-dominated solutions without problems. However, in the comparison and evaluation of more than two non-dominated solutions, especially when the criteria number is large and when the non-dominated solutions do not differ significantly, the DM can meet considerable difficulties in the selection of a local (global) preferred non-dominated solution [Jaszkiewicz and Slowinski, 1995].

On the basis of new classification-oriented scalarizing problems, an interactive method is proposed in the paper, which to a high degree combines the positive aspects of the interactive methods for solution of linear multicriteria optimization problems developed up to now. The basic characteritics of this interactive method are the following:

- possibility to enlarge the information, with the help of which the DM can put down his/her local preferences, setting desired or acceptable directions and intervals of change in the criteria values, in addition to the criteria desired or acceptable levels;

- possibility for comparatively quick learning of the DM concerning the specific multicriteria linear problems solved, which results from representing more non-dominated solutions for evaluation at each iteration, as well as from DM's free movement in the whole area of these solutions;
- comparatively easy evaluation by the DM of the solutions presented, on account of the fact that they are close one to another.

2. Problem formulation

The linear problem of multicriteria optimization (denoted by LMK), can be formulated as follows:

(1) $$\text{"max"}\{f_k(x)\}, \qquad k \in K$$

subject to:

(2) $$\sum_{j \in N} a_{ij} x_j \le b_i, \qquad i \in M$$

(3) $$0 \le x_j \le d_j, \qquad j \in N$$

where the symbol "max" means that all the objective functions should be simultaneously maximized.
$K = \{1,2,...,p\}$, $M = \{1,2,...,m\}$, and $N = \{1,2,...,n\}$ are the index sets of the linear criteria (objective functions), the linear constraints and the variables (the solutions) respectively;
$f_k(x),\ k \in K$ are the linear criteria (objective functions);
$f_k(x) = \sum_{j \in N} c_j^k x_j$;
$x = (x_1, x_2, ..., x_j, ..., x_n)^T$ is the vector of variables (solutions).
The constraints (2)-(3) define the acceptable set of the variables (solutions). This set will be denoted by X.
Several definitions will be introduced for greater precision.
Definition 1. The solution x will be called efficient solution of the problem LMK, if there does not exist another solution $\bar{x}$, such that the following inequalities be satisfied:
$f_k(\bar{x}) \ge f_k(x),$ for each $k \in K$ and
$f_k(\bar{x}) > f_k(x),$ for at least one index.
Definition 2. The vector $f(x) = (f_1(x),..., f_p(x))^T$ is called a non-dominated solution of the problem in the criteria space, if x is an efficient solution of the corresponding problem in the variables space.
Definition 3. Desired or acceptable directions of change in the values of some of the criteria are the directions, in which the DM wishes to be improved or agrees to be deteriorated the values of these criteria in the last non-dominated solution obtained, so that this solution is improved according to his/her local preferences.
Definition 4. Desired or acceptable intervals of change in the values of some of the criteria are the intervals, in which the DM wishes to find the improved or deteriorated values of these criteria with respect to their corresponding values in the last non-dominated solution obtained.
The problems of multicriteria optimization do not possess an optimal solution. Hence, it is necessary to select such a solution among the non-dominated solutions, which suits best the DM's global preferences. This choice is personal and it depends entirely on the DM.

3. Classification-oriented scalarizing problems of desired or acceptable levels, directions or intervals

The classification-oriented scalarizing problems decrease the requirements towards the DM when comparing and evaluating the new solutions obtained. Relating to the information, required from the DM in the search of new solutions, these scalarzing problems are relatively near to the scalarizing problems of the reference point [Wierzbicki, 1980], but unlike them, here the DM is not obliged to determine the desired or acceptable levels for all the criteria. In the scalarizing problems, suggested in this chapter, the DM can represent his/her local preferences not only by desired or acceptable levels, but also by desired or acceptable directions and intervals of change in the values of separate criteria. In this way he/she can describe his/her local preferences with greater flexibility, accuracy and reliability. Depending on these preferences, the set of the criteria at each iteration can be indirectly divided into seven or less than seven classes, denoted as follows: $K^{>}$, $K^{\geq}$, $K^{=}$, $K^{<}$, $K^{\leq}$, $K^{><}$ and K^{0}. Each criterion $f_k(x)$, $k \in K$ may belong to one of these classes, as given below:

$k \in K^{>}$, in case the DM wishes the criterion $f_k(x)$ to be improved;

$k \in K^{\geq}$, if the DM wants the criterion $f_k(x)$ to be improved by a desired value $\Delta_k > 0$;

$k \in K^{=}$, in case the DM wishes that the current value of the criterion $f_k(x)$ not to be deteriorated;

$k \in K^{<}$, in case the DM agrees the criterion $f_k(x)$ to be deteriorated;

$k \in K^{\leq}$, if the DM wishes the criterion $f_k(x)$ to be deteriorated by an acceptable value $\delta_k > 0$;

$k \in K^{><}$, if the DM wishes the criterion $f_k(x)$ not to be altered beyond the limits of a given interval, determined as:

$f_k - t_k^- \leq f_k(x) \leq f_k + t_k^+$;

$k \in K^{0}$, in case the DM is not interested how the criterion $f_k(x)$ will be altered at this iteration.

In order to obtain a solution, which is better than the current non-dominated solution of the linear problem of multicriteria optimization, the following Tchebycheff type scalarizing problem L1 can be used [Vassileva et al., 2001] on the basis of the implicit criteria classification done by the DM:

Minimize:

(4)
$$S(x) = \max\left[\max_{k\in K^{\geq}}(\bar{f}_k - f_k(x))/|f_k^{'}|, \max_{k\in K^{<}\cup K^{\leq}}(f_k - f_k(x))/|f_k^{'}|\right] + \max_{k\in K^{>}}(f_k - f_k(x))/|f_k^{'}| +$$
$$+\rho\left[\sum_{k\in K^{\geq}}(\bar{f}_k - f_k(x)) + \sum_{k\in K^{<}\cup K^{\leq}}(f_k - f_k(x)) + \sum_{k\in K^{>}}(f_k(x) - f_k)\right]$$

under constraints:

(5) $f_k(x) \geq f_{k,} \quad k \in K^{>} \cup K^{=}$,

(6) $f_k(x) \geq f_k - \delta_k, \quad k \in K^{\leq}$,

(7) $f_k(x) \geq f_k - t_k^-, \quad k \in K^{><}$,

(8) $f_k(x) \leq f_k + t_k^+, \quad k \in K^{><}$,

(9) $x \in X$,

where f_k is the value of the criterion $f_k(x)$ in the current preferred solution;

$\bar{f}_k = f_k + \Delta_k$ is the desired level of the criterion $f_k(x)$;

$f_k^{'}$ is a scaling coefficient, defined as follows:

(10)
$$f_k^{'} = \begin{cases} \varepsilon, \ ако |f_k^{'}| \leq \varepsilon \\ f_{k,} \ ако |f_k^{'}| > \varepsilon \end{cases},$$

where ε is a small positive number.

Given that the objective function (4) of scalarizing problem L1 is a non-differentiable function, an equivalent L2 linear problem could be solved instead of it (Murtaph (1981), [Padberg, 2000]:

(11)
$$\min\left(\alpha + \beta + \rho \sum_{k\in K^{\geq}\cup K^{>}\cup K^{<}\cup K^{\leq}} y_k\right)$$

subject to constraints:

(12) $\alpha \geq (\bar{f}_k - f_k(x))/|f_k^{'}|, \quad k \in K^{\geq}$,

(13) $\alpha \geq (f_k - f_k(x))/|f_k^{'}|, \quad k \in K^{<} \cup K^{\leq}$,

(14) $\beta \geq \left(f_k - f_k(x)\right)/\left|f_k^{'}\right|, \quad k \in K^{>},$

(15) $f_k(x) \geq f_k, \qquad k \in K^{>} \cup K^{=},$

(16) $f_k(x) \geq f_k - \delta_k, \qquad k \in K^{\leq},$

(17) $f_k(x) \geq f_k - t_k^{-}, \qquad k \in K^{><}$,

(18) $f_k(x) \leq f_k + t_k^{+}, \qquad k \in K^{><}$,

(19) $\bar{f}_k - f_k(x) = y_k, \qquad k \in K^{\geq}$,

(20) $f_k - f_k(x) = y_k, \qquad k \in K^{<} \cup K^{\leq}$,

(21) $f_k(x) - f_k = y_k, \qquad k \in K^{>}$,

(22) $x \in X$,

(23) α , β - arbitrary.

With the help of problem L2, a non-dominated solution of LMK linear multicriteria problem is obtained. In order to obtain more non-dominated solutions of LMK problem some problems could be used, that are parametric extensions of problem L2 (Murtaph (1981), [Padberg, 2000]. One parametric extension of problem L2, called L2P, may have the following form:

(24) $$\min\left(\alpha + \beta + \rho \sum_{k \in K^{\geq} \cup K^{>} \cup K^{<} \cup K^{\leq}} y_k\right)$$

under constraints:

(25) $f_k(x) + \left|f_k^{'}\right| \alpha \geq \bar{f}_k + \Delta f_k \, t, \qquad k \in K^{\geq}$,

(26) $f_k(x) + \left|f_k^{'}\right| \alpha \geq f_k - \Delta f_k \, t, \qquad k \in K^{<} \cup K^{\leq}$,

(27) $f_k(x) + \left|f_k^{'}\right| \beta \geq f_k + \Delta f_k \, t, \qquad k \in K^{>}$,

(28) $t \geq 0$

and constraints (15)-(23),

where Δf_k is a parameter.

4. GAMMA-L interactive method

GAMMA-L interactive method designed to solve linear problems of multicriteria optimization is developed on the basis of the scalarizing problems L2 and L2P. It is an interactive method oriented towards learning [Gardiner and Vanderpooten, 1997], which means that the existence of an implicit utility function of the DM is not presumed. The DM can seek freely non-dominated solutions in the set of the non-dominated solutions, evaluating on his own whether the current solution found is the most preferred or the final solution of the initial multicriteria problem.

The classification-oriented problems L2 and L2P enable to a different extent the expansion of DM's possibilities to describe his/her local preferences, connected to the improvement of the current non-dominated solution found. These scalarizing problems enable the DM set in addition to the desired or acceptable levels of the criteria also desired or acceptable directions and intervals of change in the criteria values.

The algorithmic scheme of GAMMA-L interactive method consists of the following main steps:

Step 1. Finding an initial non-dominated solution of the multicriteria problem by setting f_k =1, $k \in K$ and $\bar{f}_k$ =2, $k \in K$, and solving problem L2.

Step 2. Representing of the current non-dominated solution obtained to the DM for evaluation. If the DM considers, that this non-dominated solution satisfies his/her global preferences, Step 6 is executed, otherwise – Step 3.

Step 3. A request to the DM to determine his/her local preferences for improving the current non-dominated solution found by defining desired or acceptable levels, directions and intervals of change of a part or of all the criteria.

Step 4. A requirement towards the DM to estimate whether one or more new non-dominated solutions he/she wishes to consider in the evaluation. In the first case scalarizing problem L2 is solved and Step 2 is executed, and in the second case – Step 5 is accomplished.
Step 5. A question to the DM to determine the maximal number s of new non-dominated solutions that he/she wishes to obtain. Solving the scalarizing problem L2P and representing of less or equal to s new non-dominated solutions for evaluation and for choice of a current preferred solution. In case the DM decides that this non-dominated solution satisfies his/her global preferences, Step 6 is executed, otherwise – Step 3.
Step 6. Stop of the process of the linear multicriteria problem solving.
In GAMMA-L interactive method the DM controls the dialogue, the computing process and the conditions for canceling the process of linear multicriteria problem solving.

5. Conclusion

The interactive GAMMA-L method is included in the software insurance of the experimental system MOLIP developed at the Institute of Information Technologies of the Bulgarian Academy of Sciences. This system is designed for interactive solution of continuous and integer multicriteria optimization problems with different number and type of the criteria, with different number and type of the variables and constraints. The advantages of GAMMA-L method and of the interface modules of MOLIP system allow decision makers with different degree of qualification to describe comparatively easy his/her local preferences, to evaluate the new solutions obtained, to be trained in the specifics of the multicriteria problems solved and to find the most preferred solution of these problems with a large degree of reliability.

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