EXPLORING LINEAR FUNCTIONS – REPRESENTATIONAL RELATIONSHIP

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Abstract: Experiences from a German-Ukrainian project for the joint development of dynamic learning environments are reported. General methodological and technological aspects are discussed as well as special items arising from the cross-cultural collaboration.

Keywords: dynamic mathematics software, dynamic learning environments

ACM Classification Keywords: K.3.1 Computer Uses in Education – Computer-assisted instruction (CAI)

Introduction

Dynamic learning environments are one way to foster an active learning process. Hence, in many countries research is done in order to develop dynamic learning environments. We report on our experiences from a German-Ukrainian cooperation on that topic.

Of course, the range of aspects in this context includes didactical and technological items. In our case, the underlying pedagogical models are a two-step problem-solving strategy and the concept I – You – We. Concerning the software, DG and GEONExT are under consideration.

When it comes to content, we have chosen the topic *linear functions*. It is a prominent topic involved in the curricula of both countries. Furthermore, it provides examples for a lot of interesting didactical issues.

Furthermore, however, the aim of cross-cultural deployment gives rise to special problems arising from different curricula and teaching traditions. In particular, the cross-cultural treatment sheds new light on the aspect of representational relationship.

ObDiMat

The considerations presented in this article arouse in the course of cooperation with Ukrainian and German partners.

In this joint project, the main objective is to develop dynamic learning environments that can be deployed both in Ukrainian as well as German schools. The larger framework was given by the network *ObDiMat* (formerly: *GEONExT Goes East*) that also includes further partners e.g. from Bulgaria and Czech Republic.

The work on this aim comprises both methodological research as well as the production of dynamic learning environment.

The experience in this joint work, in particular when heading for practical results, resulted in finding out four main strategies for the joint development of dynamic learning environments, which were described in detail in [Bauch et al., 2005]. Let us just give a brief survey here:

When a dynamic learning environment is given in one language a has to be transferred to another one, two alternatives to do so might be

1. Translate the original dynamic learning environment to the other language
2. Rewrite the given dynamic learning environment

The first alternative is the most straight-forward method. However, it may rather soon come to its limits due to reasons we will explain more detailed in a later section. If so, one cannot avoid the more costly way to rewrite the given dynamic learning environment. This is necessary for example when a change of the supporting software is desired or emphasis is shifted according to a different pedagogical model which is preferred.

When not an existing dynamic learning environment is considered but a new dynamic learning environment is to be developed, the two different methods that proved in our experience can briefly be described in keywords:

3. Set up an English master and then localize
4. Write the plot only and then realize in different versions

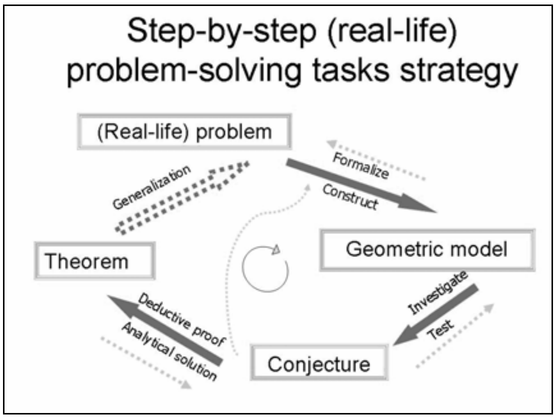
Again, these two alternatives differ in a similar way as above: Localizing an English version might bring along certain difficulties that require a more intensive rewriting. Such potential sticking points are tried to be foreseen and fixed in the proceeding number 4.

The discussion presented in the following refers to experiences coming along with joint work based on strategies 3 and 4.

Dynamic Learning Environments

To start with, let us introduce the pedagogical models and the technical basis of our project before we come to a discussion of the content.

The two pedagogical models, that are underlying our dynamic learning environments, are the ones preferred by the partners of this project. On the one hand, there is the concept of a *two-step problem solving strategy*. Its main features are evident from the following illustration:



The other pedagogical concept under consideration, called *I – you – we*, emphases the aspects of communication and collaboration. It features three main steps:

- *I* – individual work of the single student
- *You* – cooperation with a partner
- *We* – communication in the whole class.

Since these two models are concentrating on different dimensions of the learning process they are rather complimentary than conflicting. Thus, they can be combined with synergetic effects. This was outlined in [Bauch-Pikalova, 2003]. Similar advantage can be expected when other complimentary concepts are integrated.

DG and GEONExT

As it was with the preferred pedagogical models, also on the level of technology the two partners had developed dynamic mathematics software on their own: DG and GEONExT. Besides the basic features that are common for any dynamic mathematics software, however, their concepts differ in some essential aspects.

The software DG is a stand-alone program for Microsoft Windows. With it, dynamic constructions for a wide range of topics in (analytic) geometry, algebra and analysis can be generated.

In contrast, GEONExT is a Java based software and available for a large variety of operating systems. It can be used as a stand-alone construction tool or be integrated as a Java applet into HTML pages. With a computer algebra system integrated, it is possible to benefit from the use of GEONExT when dealing with topics both from geometry and analysis.

These differences have substantial implications on the design of a dynamic learning environment as it will become obvious in the next section.

Dynamic Learning Environments

Using GEONExT, it is rather easy – from a technological point of view – to provide material that enables the student to perform an active learning process.

The basis of such a dynamic learning environment is given by a sequence of HTML pages. Hence, we are free to integrate text, graphics and, in particular, dynamic constructions as a Java applet. In this way, the student's active learning by discovery is fostered.

Obviously, on the student's side the technical requirements are few (some browser with Java enabled), whereas the possibilities on the side of the developer build up a huge spectrum. Depending on the technology used, the simple presentation of a dynamic construction can be upgraded by mouse-over effects, pop-up windows to provide hints and solutions as well as interaction between text and applet.

All this potential, on the other hand, requires a careful analysis and selection in order to obtain a solution that effectively supports and realizes didactical objectives.

Though our further discussion will be based on the technical realisation of a learning environment as it was just described, let us have a look at alternatives. To be more precise: How do learning environments look like when they are realized with DG? Here, the main difference is that texts, even longer ones, can directly be written into the drawing area of the construction. Also buttons can be added into the drawing area in order to create a sequence of interactive drawings connected by hyperlinks. This obviously affects the design of a dynamic learning environment. For a discussion of such aspects we refer the reader to [Pikalova-Bauch, 2005].

Observations

We now come to some crucial observations that were made when preparing the basis for the joint development of dynamic learning environments. Two prominent aspects are the following.

The curricula in the two countries differ notably. As a striking example, let us mention the following aspect: Trigonometric functions like sine and cosine are introduced in Ukrainian schools at a much earlier time than in Germany. Hence, for example, the standard proofs for the *Theorem of Pythagoras* are quite different: In Ukraine, the use of cosine plays a substantial role, whereas in Germany methods from elementary geometry are used.

Secondly, we are facing different teaching traditions in the two countries. As an effect, speaking about a theorem given a certain name can mean to deal with completely different situations. As an example, consider the so called *Theorem of Thales*. In Germany, it says: A triangle that has the diameter of a circle as one side and the third point lying on that circle always is rectangular. Opposite to this, Ukrainian students will immediately think of projective geometry when hearing the name Thales.

Closely related to these two aspects, difficulties may arise when trying to translate the index of a school book, e.g. [Pogorelov, 2004], a list of keywords from lessons or a catalogue of basic knowledge. When doing so, we found several German words which seem not to have a common equivalent in Russian school mathematics. Examples that caused such problems in translation were e.g. *Symmetrie* (= symmetry) and *Drehung* (= rotation). They can be found in German textbooks at rather prominent places. When translating plain text, you may use explanations as a substitute. However, what to do when the concept connected to such a keyword is a substantial component of a dynamic learning environment?

Cross-Cultural Aspects of the Topic *Linear Functions*

Having all these prerequisites in mind, here is a list of some aspects from the topic *linear functions* that we have selected for a dynamic learning environment:

1. lines in the plane
2. description of a line; the notion of the slope
3. different types of equations
4. system of linear equations

In this list, we have deliberately chosen formulations that can be agreed by both German and Ukrainian teachers. When going through this list, again both will surely think about aspects that are worth a discussion, e.g. to what extent geometrical aspects of the topic should be treated explicitly? Since this is related to the aspect of representational relationship, we will discuss it in the next section.

But there are also differences that can be traced back to teaching traditions when it comes to notation. In German schools, it is common to denote the equation of a linear function by $y=mx+t$, whereas Ukrainian students are used to write $y=kx+b$ for the same.

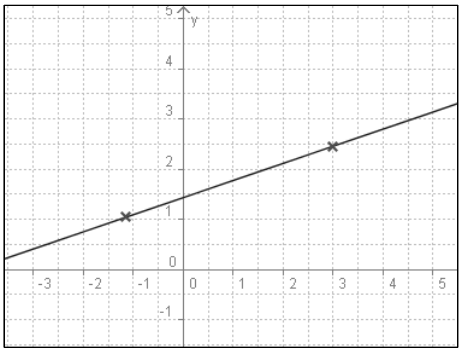
Representational relationship

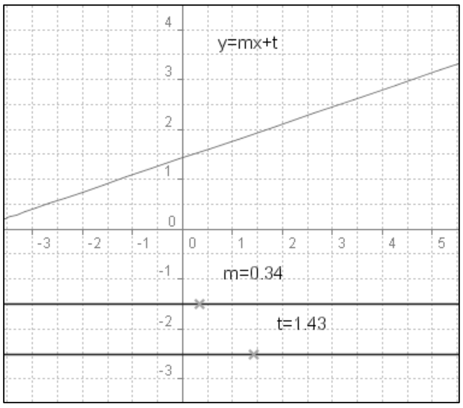
During our work on a dynamical learning environment for the topic linear functions, new impulses were given to considerations related to another important concept: representational relationship.

Spoken roughly, every description or depiction of a mathematical object or situation is a model of a concept lying behind.

Of course, this concept and its implications always play an important role in context with the design of a learning unit, no matter which medium it is meant for. However, the dynamic mathematics software delivers striking examples for the relevance of this concept:

Consider a dynamic construction that displays the plane together with a coordinate system and in it a freely movable line, given by two points (see illustration).





This line can be considered as a model for different concepts, e.g.

1. analysis: graph of a linear function
2. analytic geometry: line in a plane

Depending on the detailed design of the construction, the aspect emphasized or the possible interpretation may change. Let us look at some examples:

If it is possible to move the line in a position parallel to the y-axis, the interpretation as the graph of a linear function is excluded.

See the other way round: If the interpretation as the graph of a linear function is intended it might be useful to design the dynamic learning environment in a different way. One possibility might be to vary the parameters of the linear function (see illustration).

In our project to jointly develop a dynamic learning environment for the topic linear functions, we rather naturally came across aspects closely related to the concept of representational relationship.

As an example, consider the notion *slope of a line.* In this respect, German curricula stress the *slope triangle*, whereas in Ukrainian education the basic notion is the *slope angle*. Seemingly just a small difference, it has further implications on the conceptual level: In the line equation $y=ax$, the coefficient a is interpreted in one case as the ratio of two lengths (given by the legs of the slope triangle), in the other case it is the tangent of the slope angle. The reader can easily work out further implications, e.g. which further topics in mathematics education are connected to the one approach or the other.

Conclusion

In this article, we reported on a Ukrainian - German project dealing with the development of dynamic learning environments both on the level of methodological research as well as practical implementation.

Generally speaking, many aspects occurred as they do always in the context of dynamic learning environments. In particular, it proved that this field is a very sensitive one and offers both chances and risks. Chances e.g. in the sense that the integration of a new tool respectively a new medium offers the chance for new approaches in mathematics education and for the support of concepts like active learning and learning by discovery.

It is also accepted that the development and deployment of dynamic learning environments is not for free. In particular a short-term balance will show considerable initial outlays. However, and this has always to be taken into consideration, these expenditures will pay off in the long run. In particular, the long-term effects due to active learning and a deeper understanding supported by dynamic mathematics software recompenses the efforts.

This proves in our joint project. Of course, one hope of joint development is that it will pay off by the large number of potential users. More important, however, the cross-cultural cooperation rewards rather immediately with interesting new insights on several aspects and – on the long run – with the potential of a broader understanding of mathematical topics due to a larger spectrum of approaches that are included and presented by a qualified didactical treatment of the subject.

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METHODOLOGY FOR TRAINING TEACHERS IN BASIC AND SPECIFIC COMPUTER SKILLS

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Abstract: This article describes methodology for training teachers in Maths, Physics, Astronomy and Professional subjects in basic and specific computer skills.

Keywords: teachers, IT education for teachers, methodology for training, curriculums, education process, online tests

ACM Classification Keywords: K.3.0 Computers and Education - General

Introduction

The high school pupils have new requirements towards teaching style and methods. This fact requires that besides the teachers in Computer Science those teaching other subjects should be also IT competent and manage computer aided teaching. ITs are a modern tool for visualization of the taught material that makes the introduction of the curriculum much easier, more interesting, modern and comprehendible. On the other hand, ITs allow exploiting Bulgarian and world information sources as well as the contemporary communication devices in the teaching process.

The methodology offered has been developed by a team from Institute of Mathematics and Informatics under a contract with the Bulgarian Ministry of Education and Science.

Aim of the Project is to provide a methodology and training 250 instructors supposed to supervise training in basic and specific computer skills for

- teachers in Maths, Physics and Astronomy and
- teachers in professional subjects (mechanics, electronics, economics, engineering, etc.).